Computational Biomechanics 2016

Lecture I:

Introduction, Basic Mechanics 1

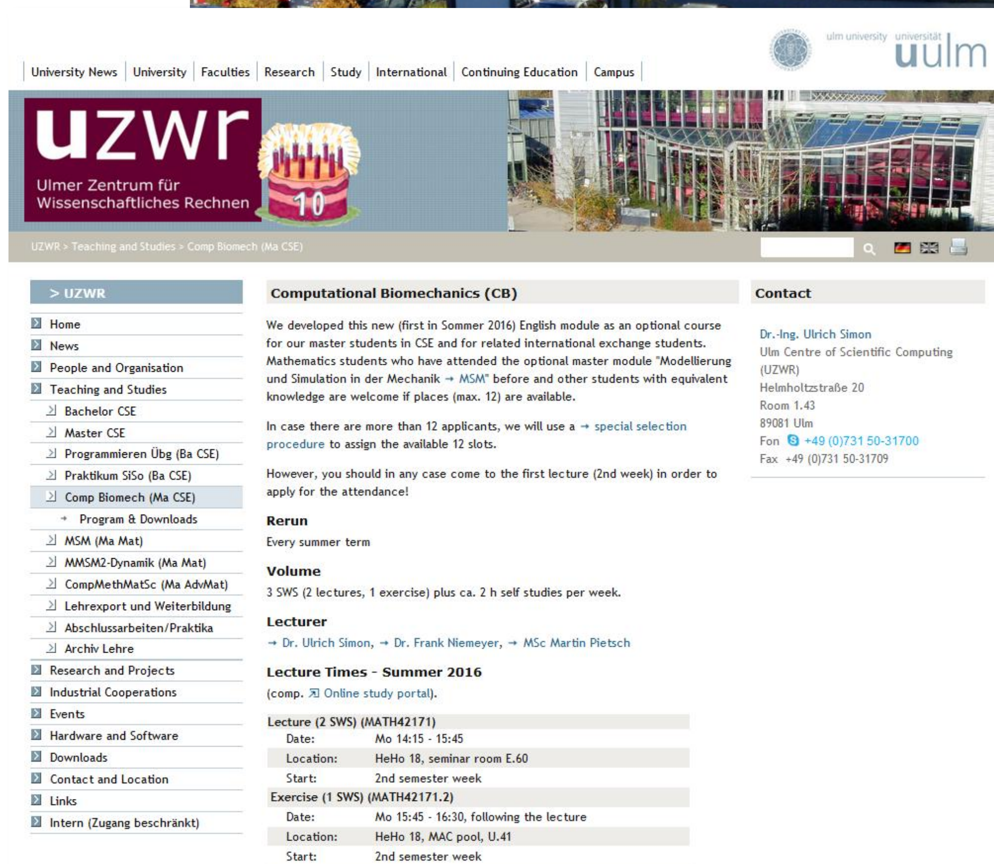
Ulli Simon, Frank Niemeyer, Martin Pietsch

Scientific Computing Centre Ulm, UZWR
Ulm University

0 Organisation

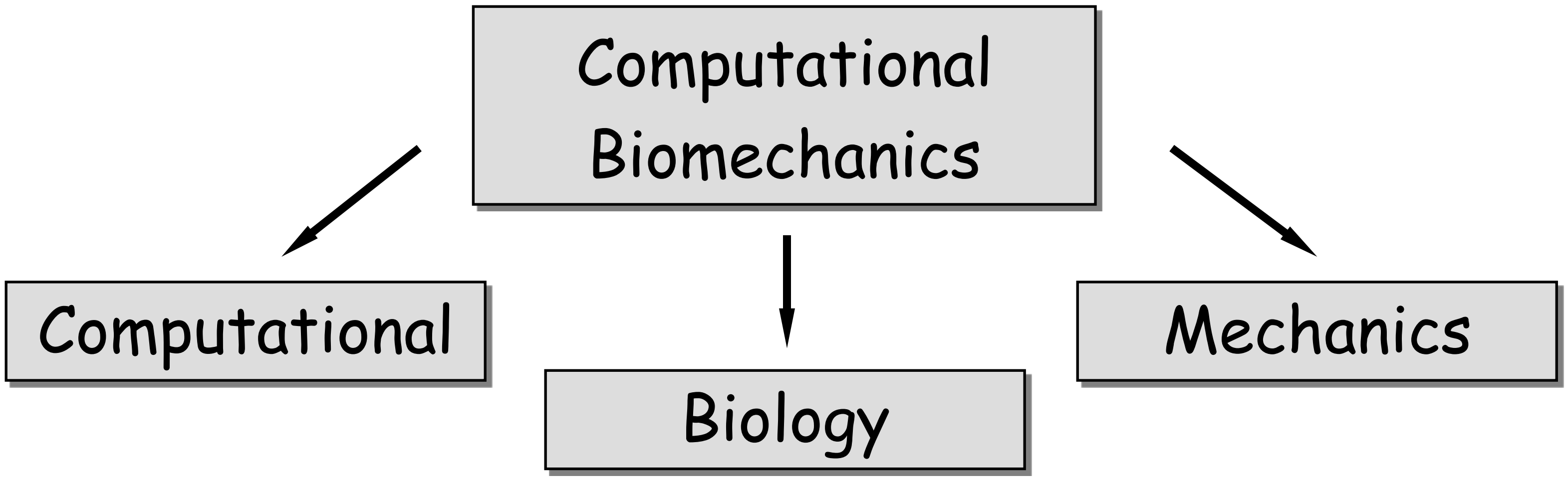
Scientific Computing Centre Ulm

→ www.uzwr.de



- Rerun
- Lecture times
- Exam
- Max. 12 students

1 General Information



Biomechanics: Solving biological questions using methods of mechanical engineering (Technische Mechanik), incl. experiments.

Mechanobiology: Reaction of biological structures on mechanical signals. Mechanotransduction: Melekular cell reaction.

Research Fields

Orthopadic Biomechanics: Bone-implant contact, fracture healing, (artificial) joints, musculoskeletal systems, …

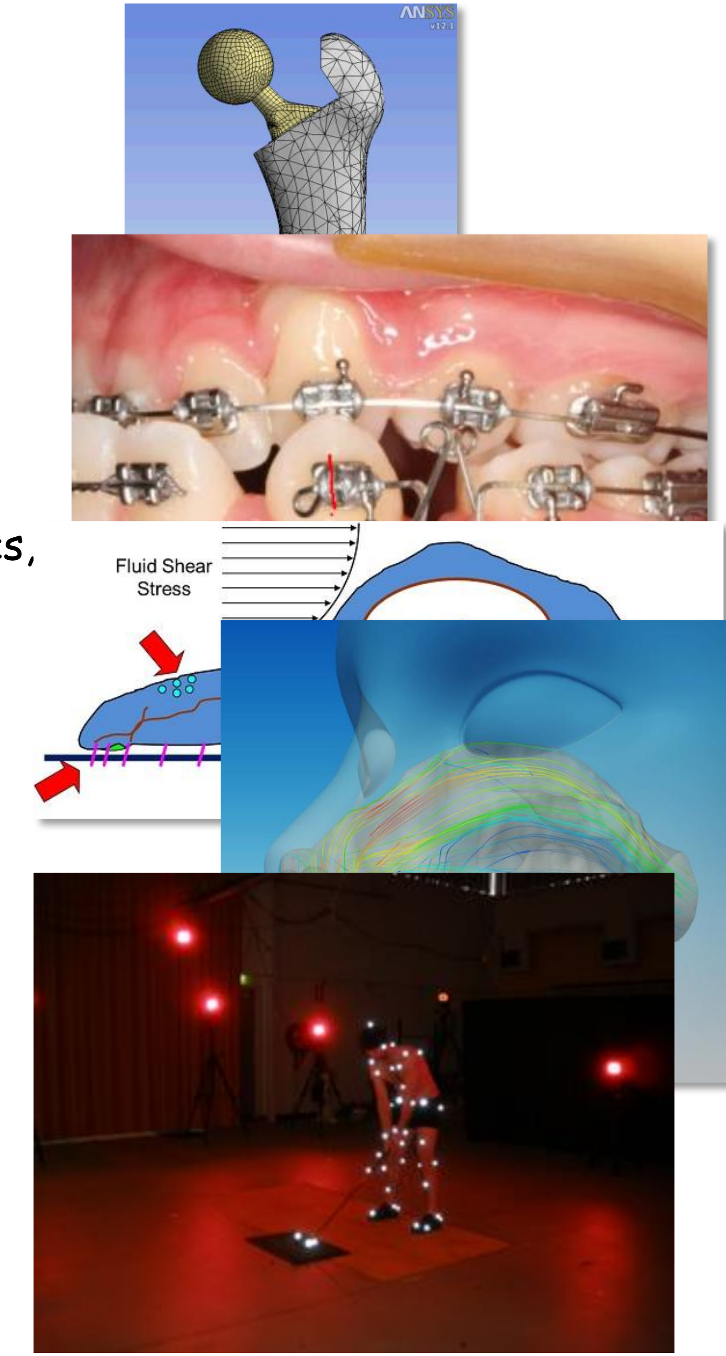
Dental Biomechanics: Dental implants, orthodontics, dental movements, braces, brackets, …

Cell Biomechanics: Cell experiments (cell gym) and simulations to study mechenotransduction

Fluid Biomechanics: Respiratory systems, blood flow, heart, …

Sport Biomechanics: Optimizing performance, techniques and equipment of competitive sports

Tree Biomechanics, Traffic Safety, Accident Research, …



Numerical Methods

Boundary Value Problems: Finite Elements, static structural analyses, displacements, stresses strains,

Initial Value Problems: Dynamics, musculoskeletal systems, movements, Inverse Problem: Calculating muscle forces from measured movements

Multiscale Modeling: To handle higly complex systems

Model Reduction: dito

Fuzzy Logic: Fracture healing in Ulm

Contents

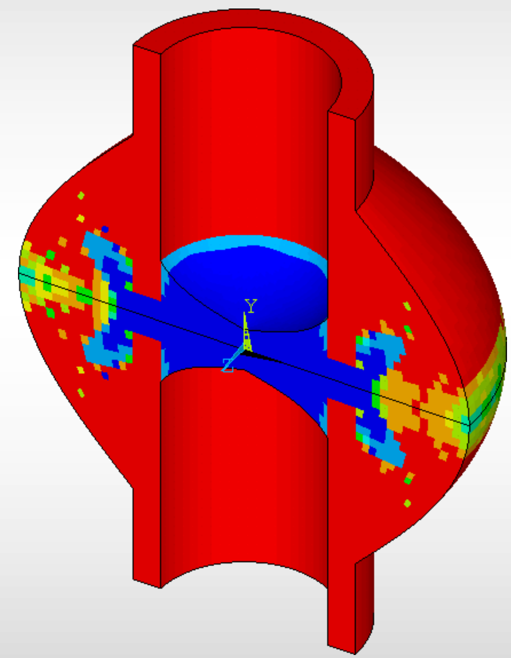
Detailed Schedule Summer 2016 (preliminary)

No	Day	Date	Topic	Lecturer
01	Mo	11 Apr	- canceled -	-
02	Mo	18 Apr	Intro to Biomech; Mech 1: Statics	Ulli
03	Mo	25 Apr	Mech 2: Elastostatics; Mat. Props. Biol. Tissues	Ulli
04	Mo	02 May	Intro FEA, Intro to Ansys WB, Exercise "L"	Ulli
05	Mo	09 May	From 3D Image to FE Mesh	Frank
06	Mo	16 May	- Holiday: Pentecost -	-
07	Mo	23 May	Mech 3: Kinematics & Dynamics; Exercise: Simulink	Ulli
08	Mo	30 May	Forward & Inverse Dynamics: ADAMS, Anybody	Ulli
09	Mo	06 Jun	Remodeling 1: Biology, APDL/Phyton Intro	Frank
10	Mo	13 Jun	Remodeling 2; Exercise "Trabecular Bone"	Frank
11	Mo	20 Jun	Frakture Healing 1: Biology, Algorithms	Frank
12	Mo	27 Jun	Fracture Healing 2: Levelset	Martin
13	Mo	04 Jul	CFD 1: Theory, Numerics, Modelling	Martin
14	Mo	18 Jul	CFD 2: Human Nose Air Flow	Martin
15	-	- tba -	Oral Examinations	All

Appetizer:

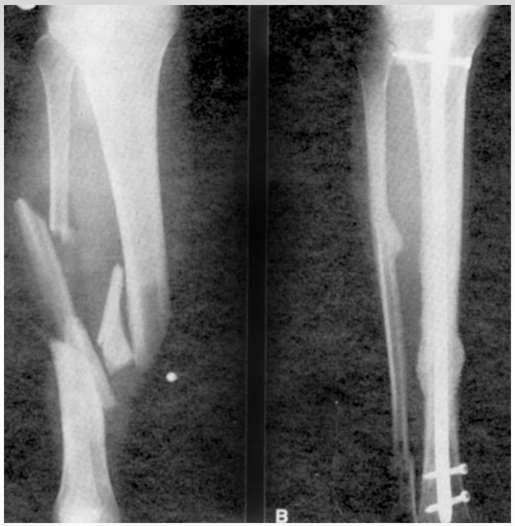
Simulation of Bone Healing

Computermodell für die Knochenheilung



Introduction: Bone Healing

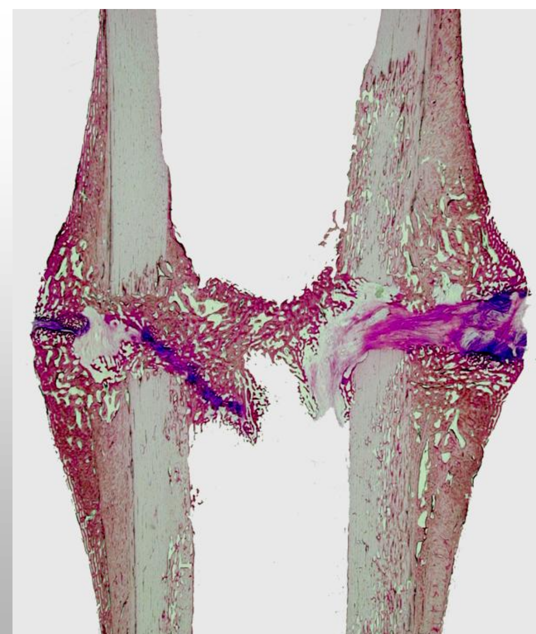
Mechanical situation



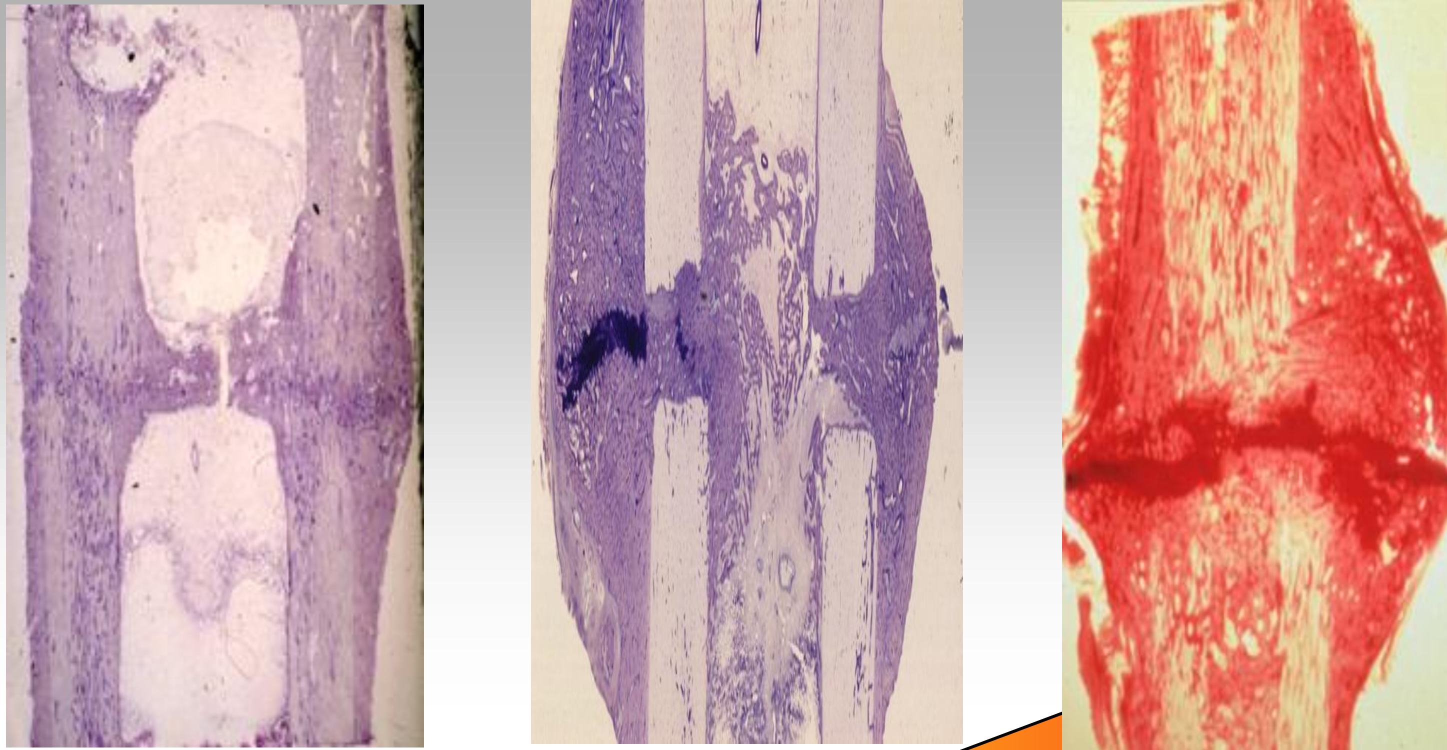
Local Blood supply

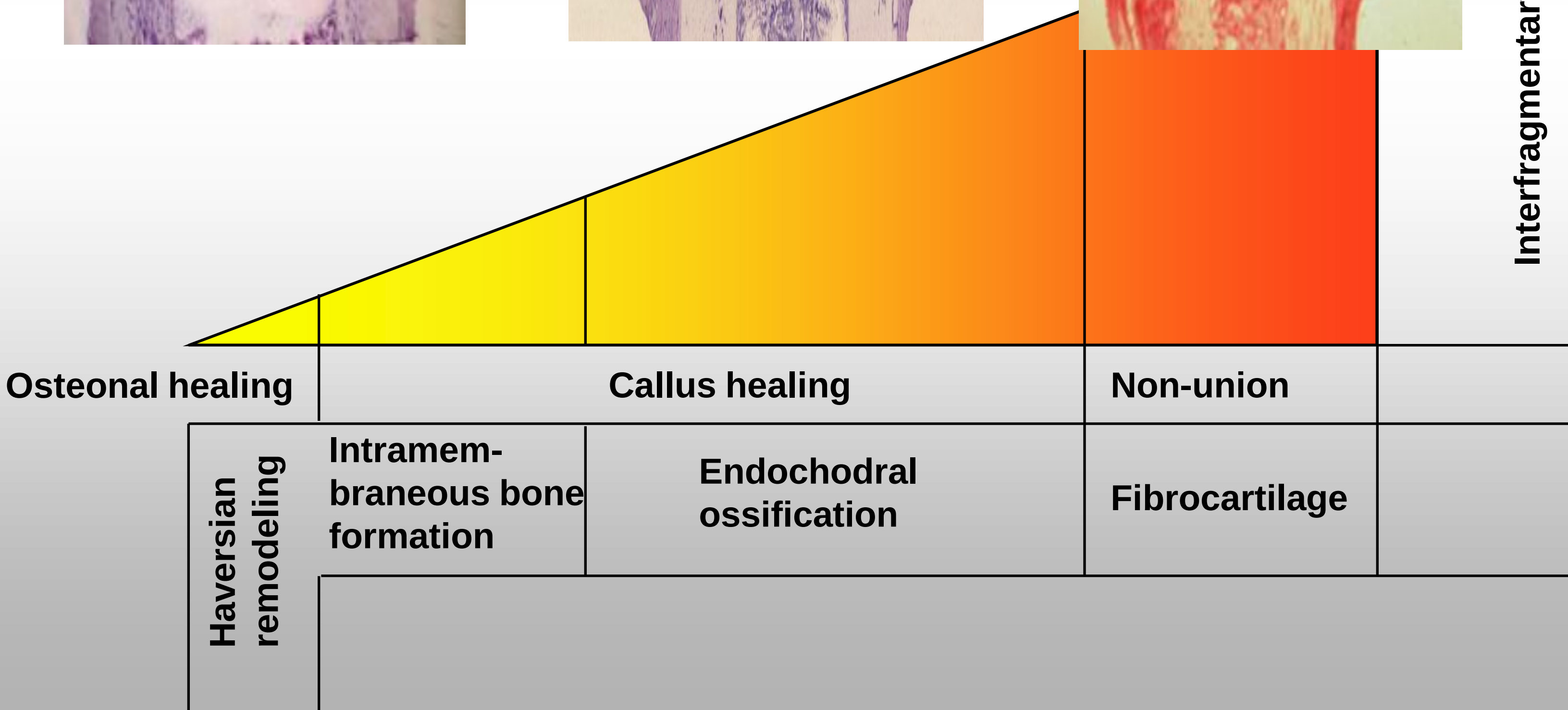
Rhinelander (1968)*

Healing process

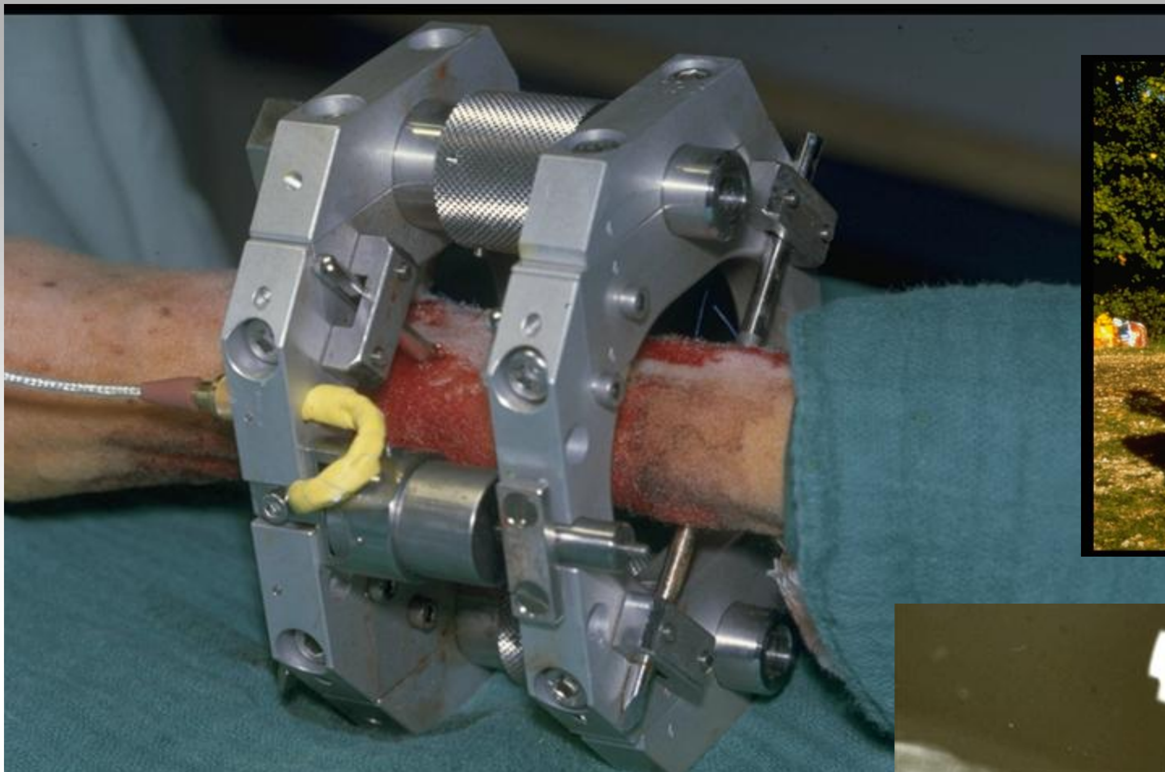


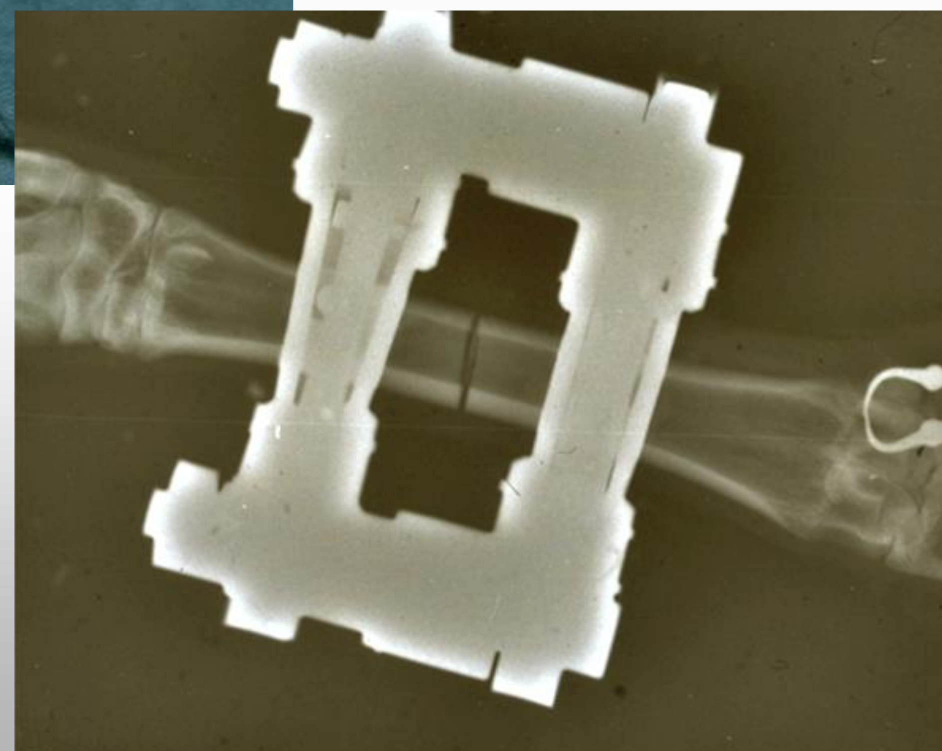
*) From Bone in Clinical Orthopedics by permission of AO Publishing.





Interfragmentary Movement
Osteonal healing
Callus healing
Non-union
Haversian remodeling
Intramem-braneous bone formation
Endochodral ossification
Fibrocartilage

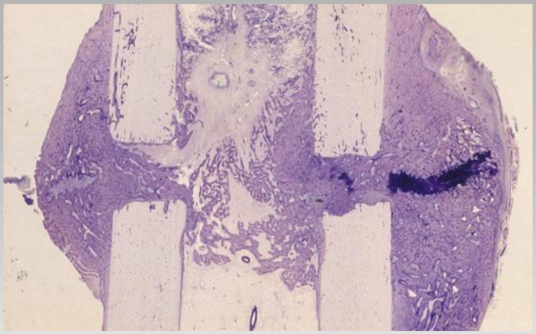


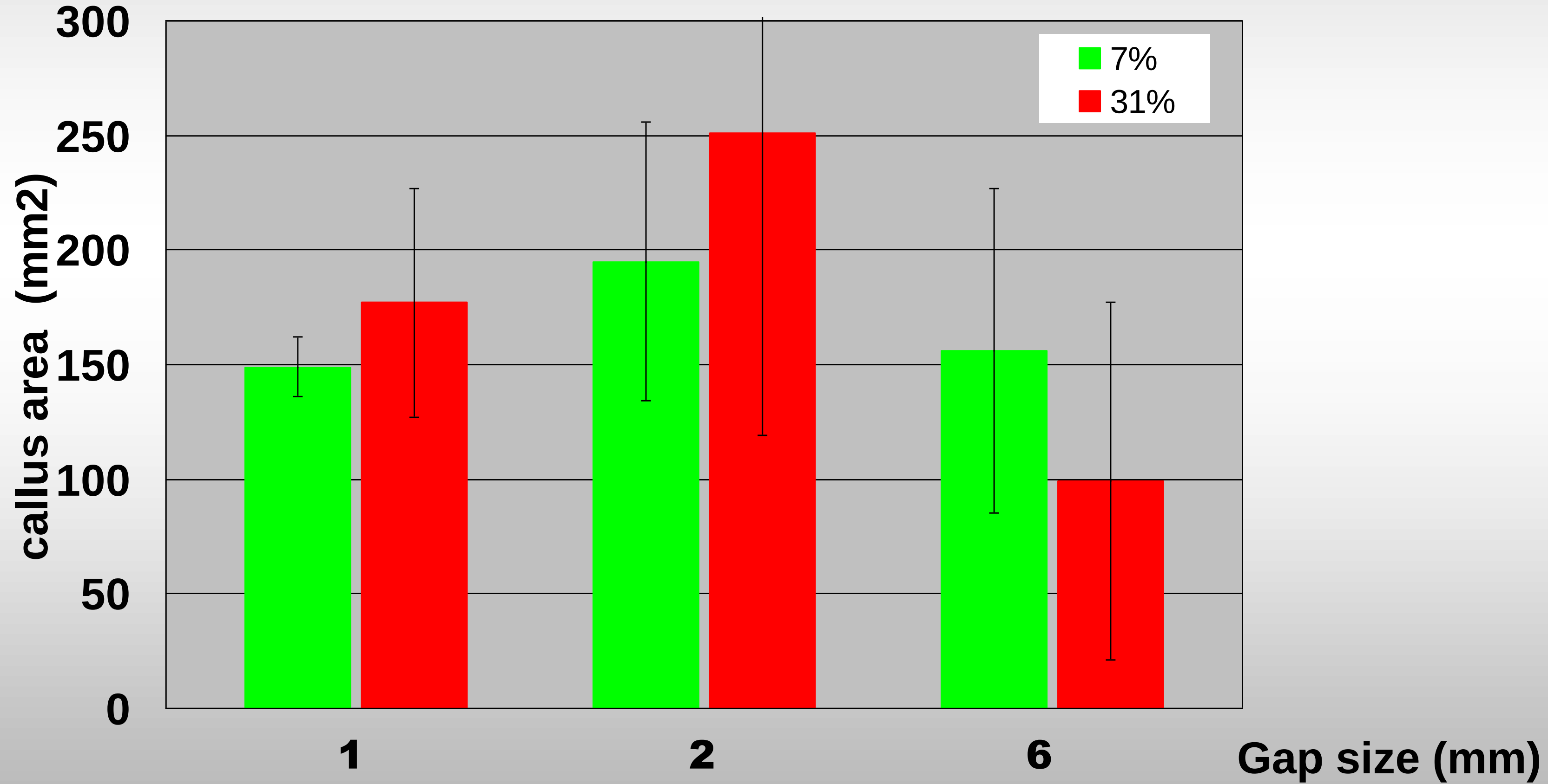


Animal Experiments

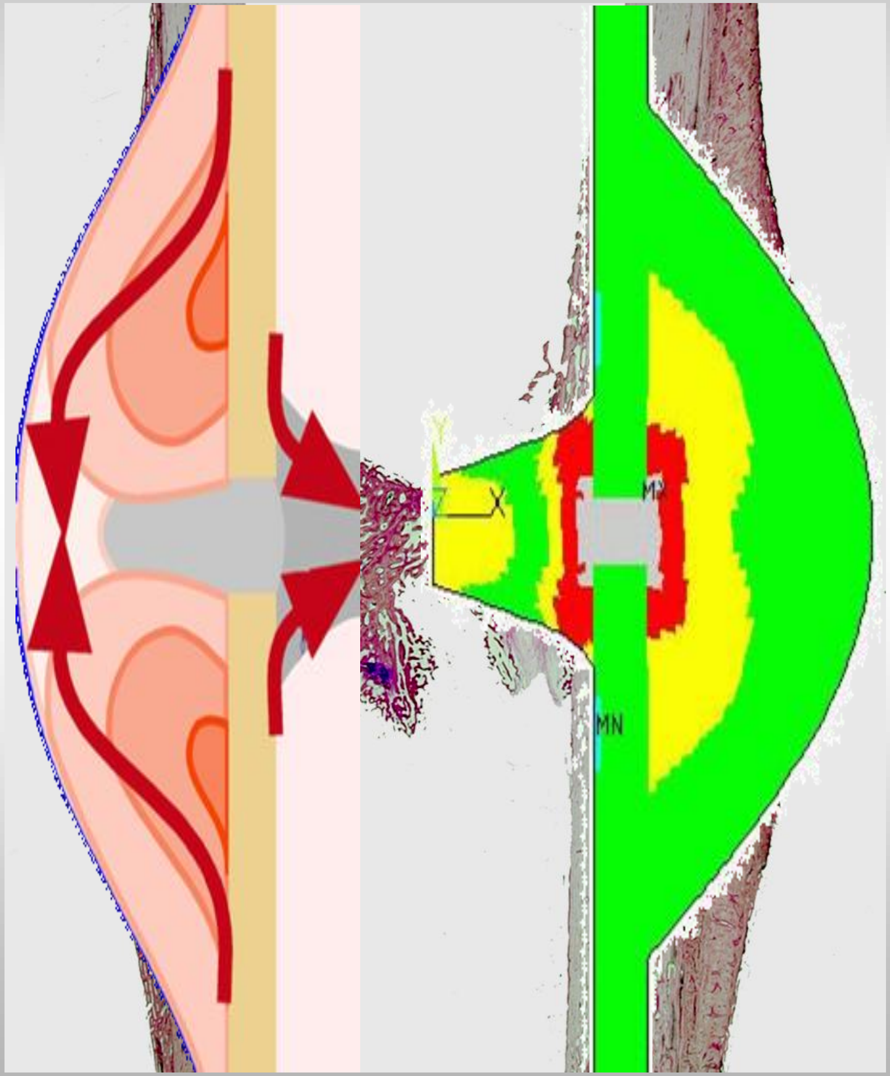
- Sheep methatarsus
- Fixateur exteren
- Gap sizes: 1, 2, 6 mm
- Interfrag. Strain: 7%, 31%

Exp. Results: Callus Area





Introduction: Callus Healing Process

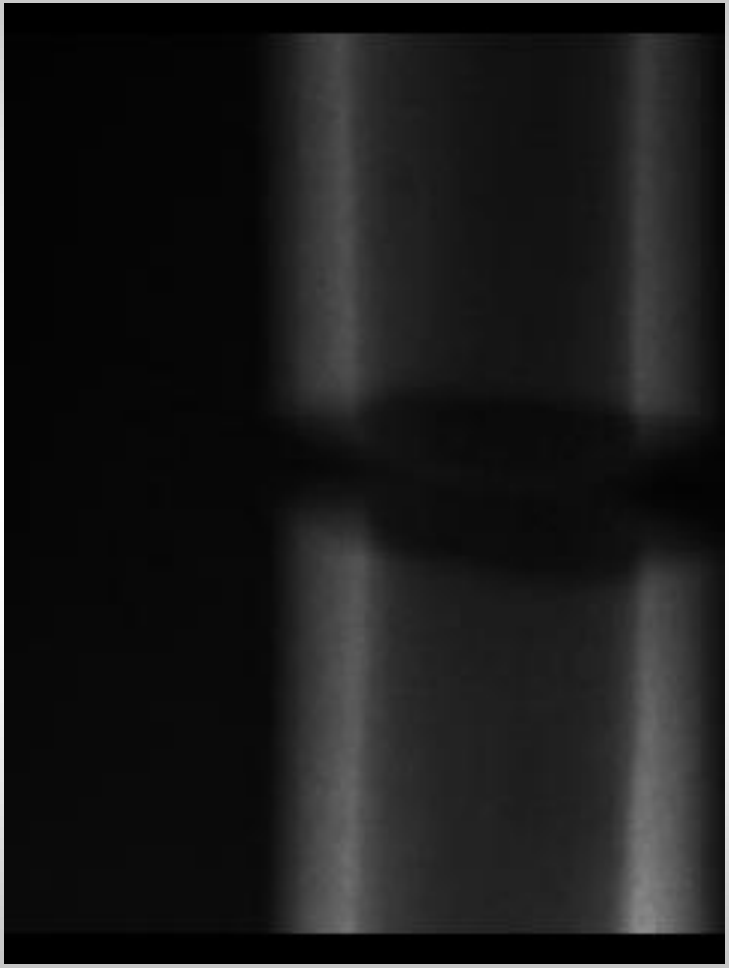


New bone formation only with low strain state !

Biological Solution:

1. Callus: larger diameter
2. Via cartilage to bone

Wirklichkeit

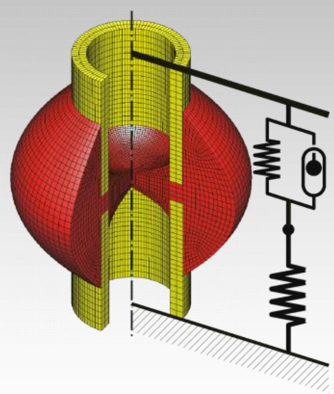


Modell

Methods: Finite Element Model

Geometry:

- Idealized osteotomy (sheep)
- 2D, axi-symmetric
- Predescribed, fixed healing area



Discretization:

- 3.000 el.

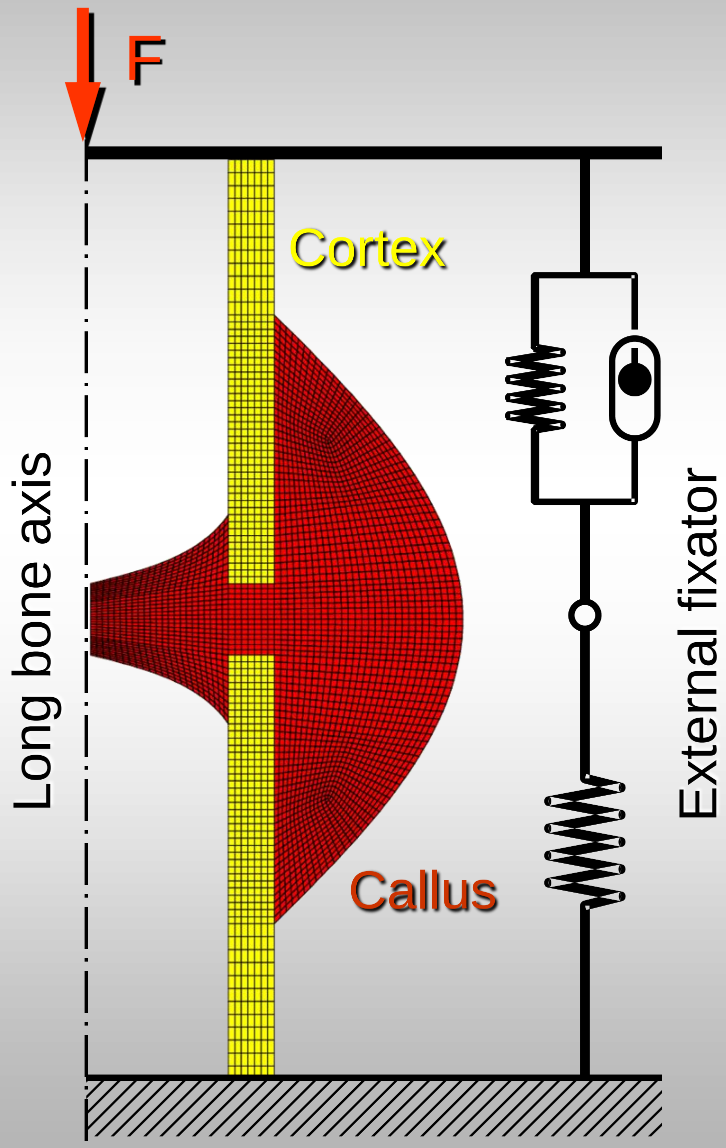
Material properties:

Tissue type	*Modul*	*Poisson ratio*
Cortical bone	10,000 MPa	0.36
Woven bone	4,000 MPa	0.36
Fibro cartilage	200 MPa	0.45
Connective tissue	3 MPa	0.30

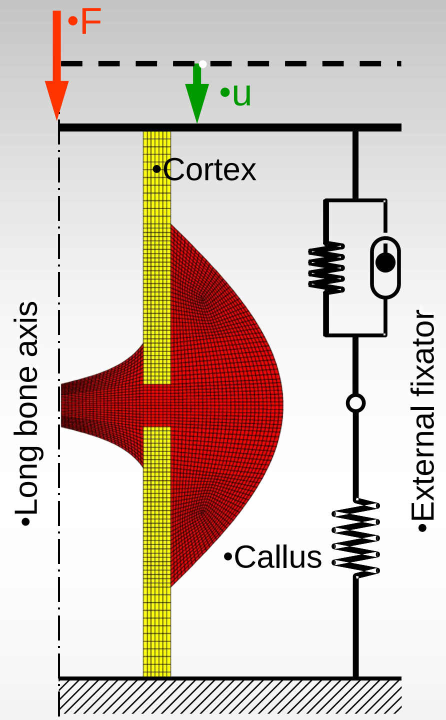
Load / Boundary conditions (Claes 97):

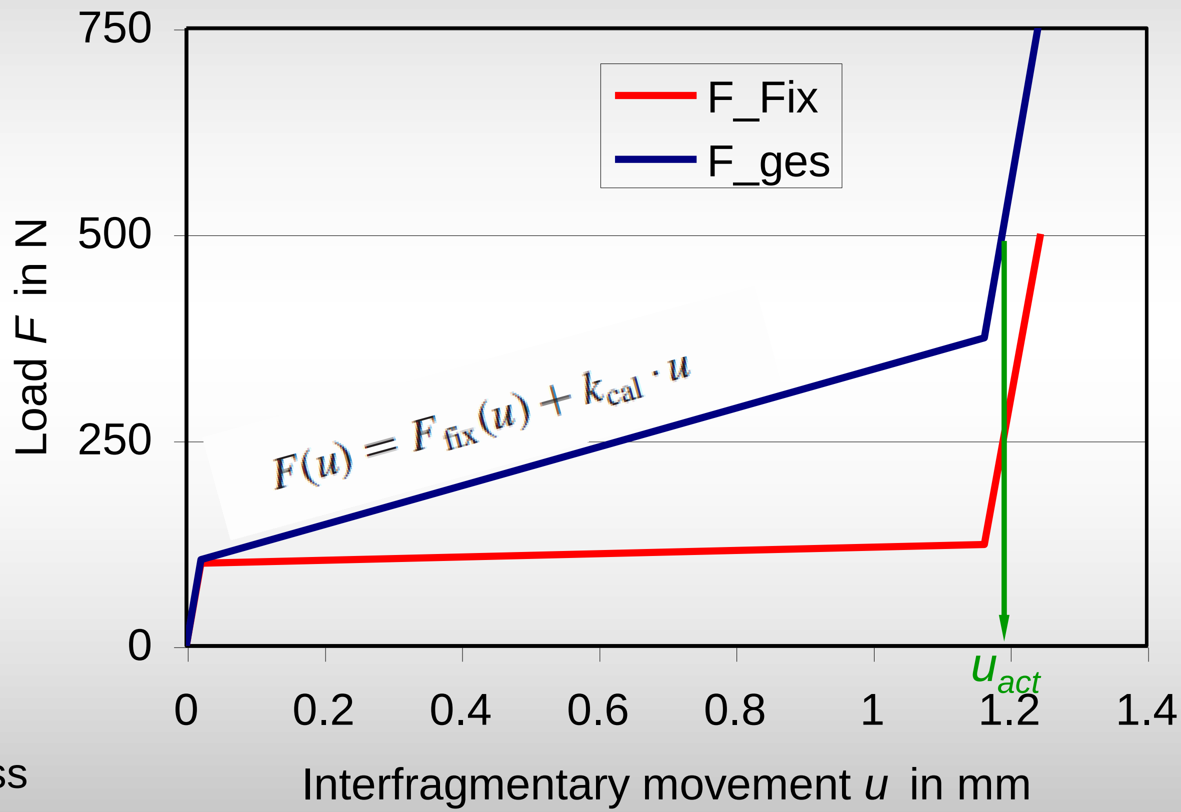
Case A: Initial movement = 0.25 mm

Case B: Initial movement = 1.30 mm



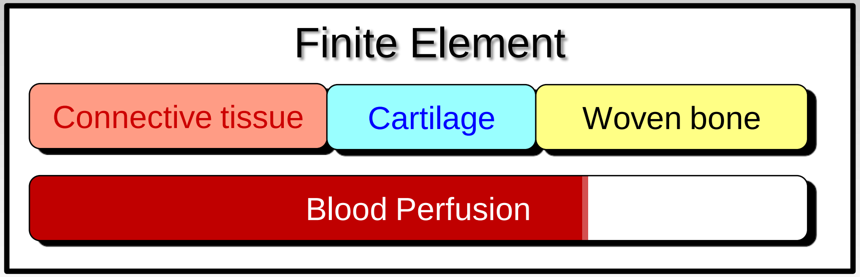
Seperation





- FEA of linera part: callus from unit displacement u_{unit}
- Calculation of callus stiffness
- Calculation of F(u)
- Using inverse u(F) to find u_{actual}
- Scale callus strains with u_{actual}/u_{unit}

Methods: Tissue Mixture



Volumetric Tissue Concentrations

Perfusion Index

Constraint

$$\sum_{\mathrm{tiss}} c_{\mathrm{tiss}} \overset{!}{=} 1, \quad \mathrm{tiss} = (\mathrm{soft}, \mathrm{cart}, \mathrm{bone}).$$

Resultant State Variables

$0 \leq c_i \leq 1$

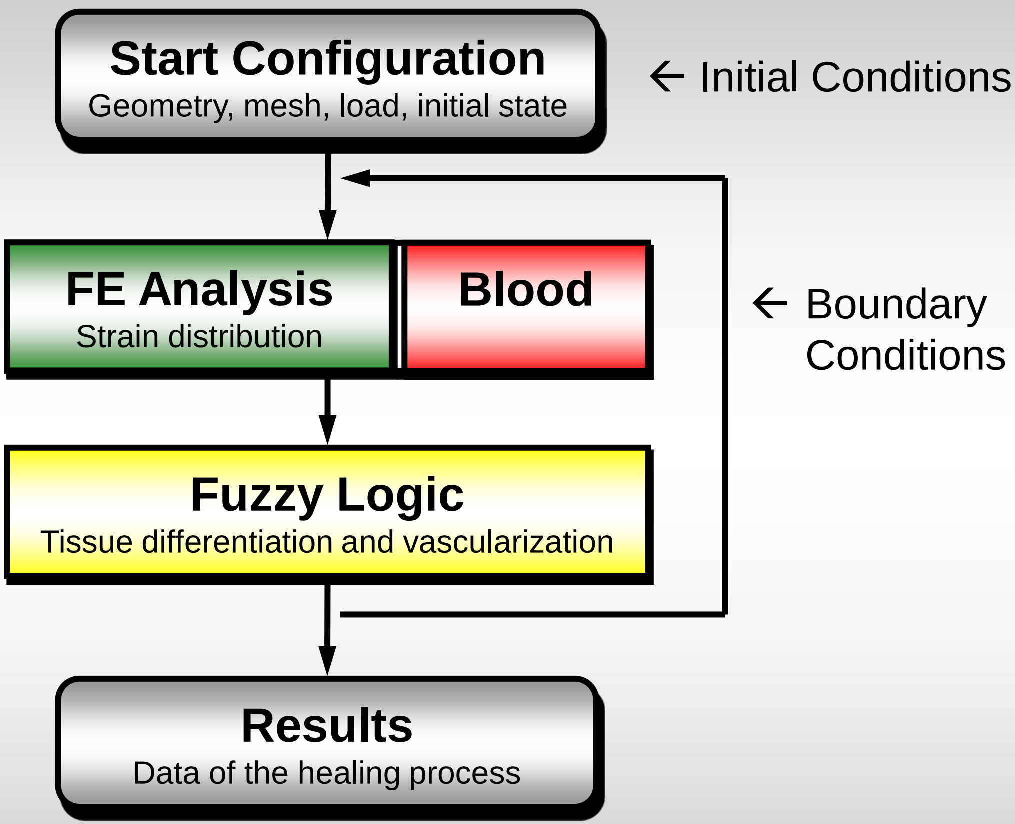
$$\underline{C}(\underline{x},t) := \begin{bmatrix} \text{Blood perfusion} \\ \text{Cartilage concentration} \\ \text{Bone concentration} \end{bmatrix} = \begin{bmatrix} c_{\mathrm{perf}}(\underline{x},t) \\ c_{\mathrm{cart}}(\underline{x},t) \\ c_{\mathrm{bone}}(\underline{x},t) \end{bmatrix}$$

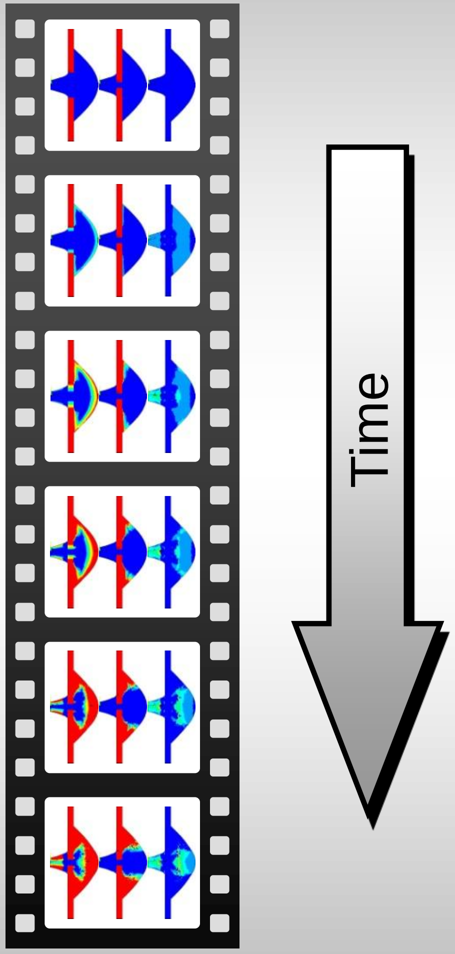
Rule of Mixture

$$E_{\mathrm{el}} = \sum_{\mathrm{tiss}} E_{\mathrm{tiss}}\, c^3_{\mathrm{el,tiss}}.$$

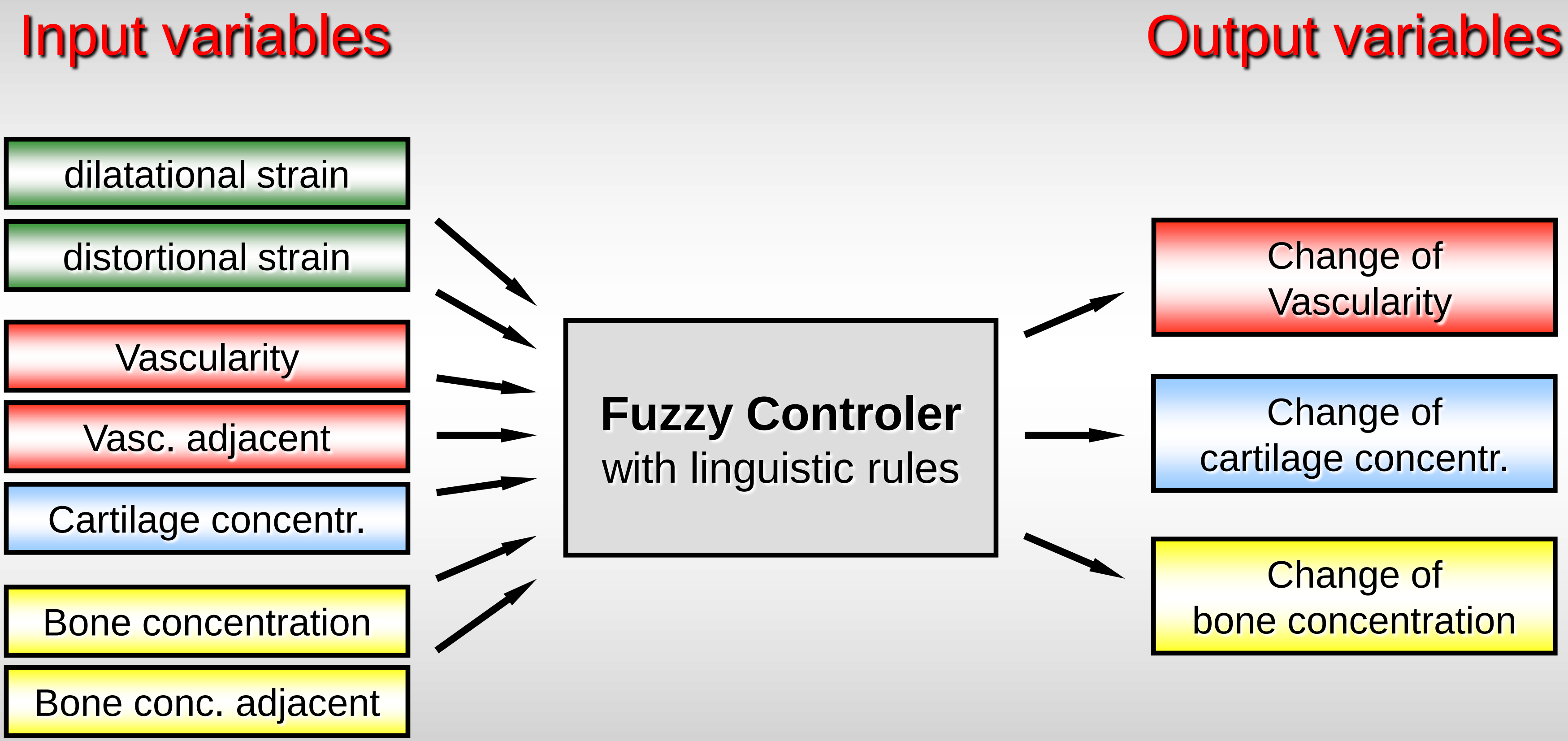
$$\nu_{\mathrm{el}} = \sum_{\mathrm{tiss}} \nu_{\mathrm{tiss}}\, c_{\mathrm{el,tiss}}.$$

Methods: Iterative Algorithm

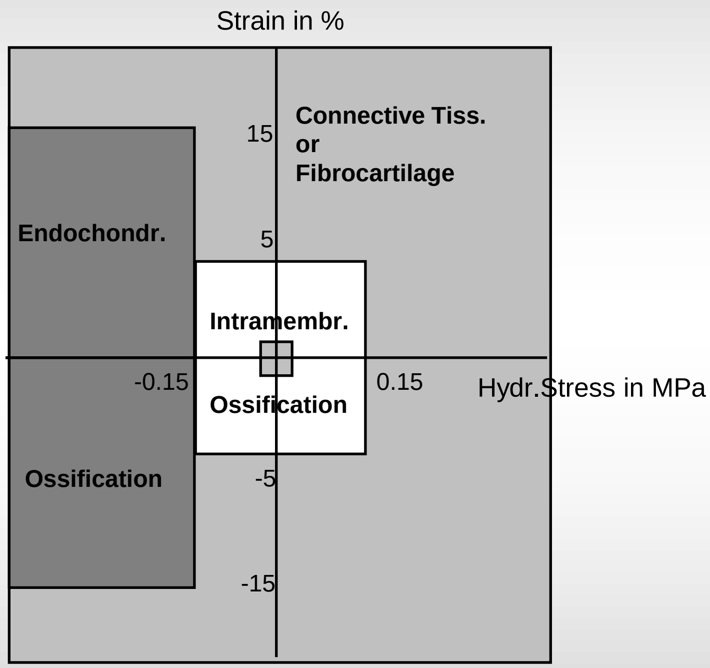




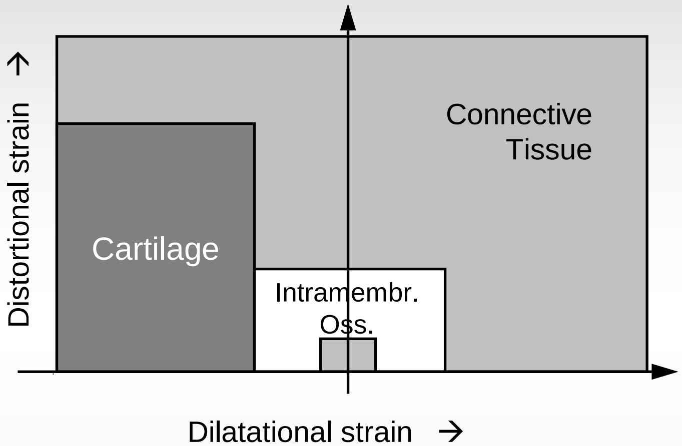
Methods: Tissue Healing with Fuzzy Logic



Tissue Transformation Function



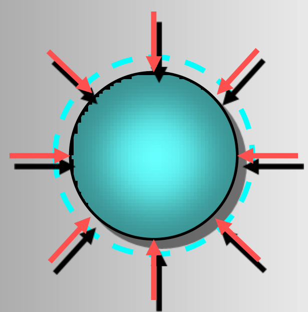
Claes and Heigele 99



Simon et al. 2011

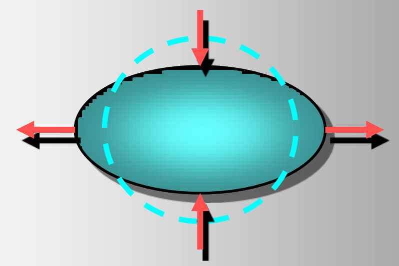
Methods: Mechanical Stimuli

Two invariants of the strain tensor



pure dilatational strain

In other studies:
- hydrostatic pressure
- octahedral (normal) stress
- fluid pressure / flow



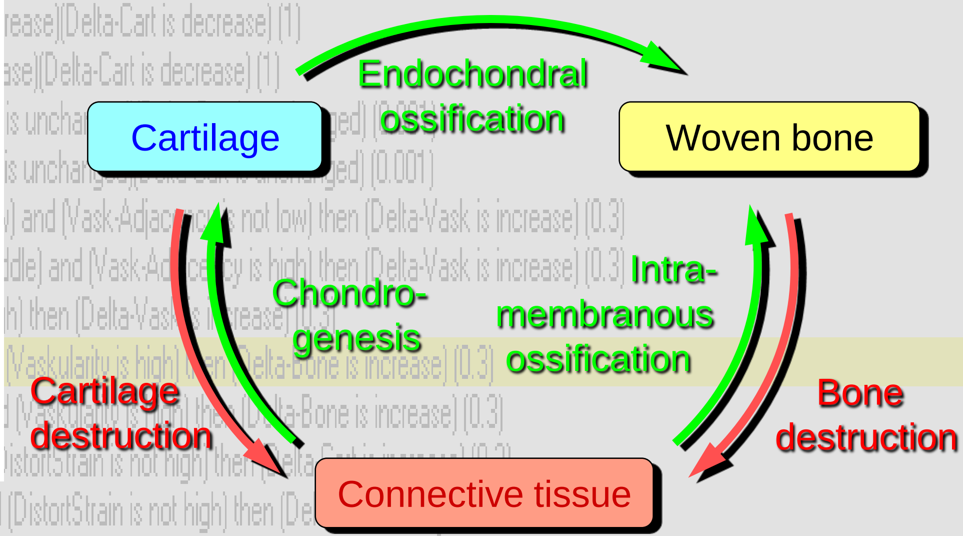
pure distortional strain

In other studies:
- octahedral shear stress
- von Mises equivalent stress
- von Mises equivalent strain

Methods: Fuzzy Rules

Processes:

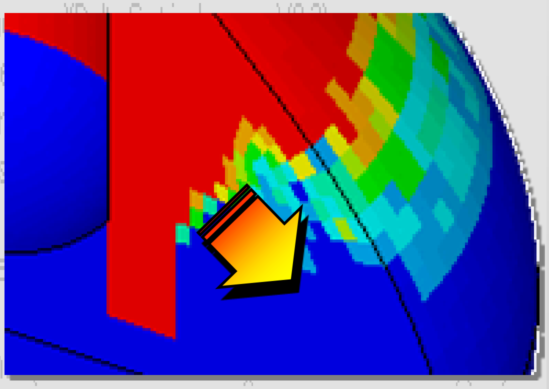
- Angiogenesis
- Intramembranous ossification
- Chondrogenesis
- Cartilage calcification
- Endochondral ossification
- Tissue destruction



Example: Rule #3, Intramembranous ossification:

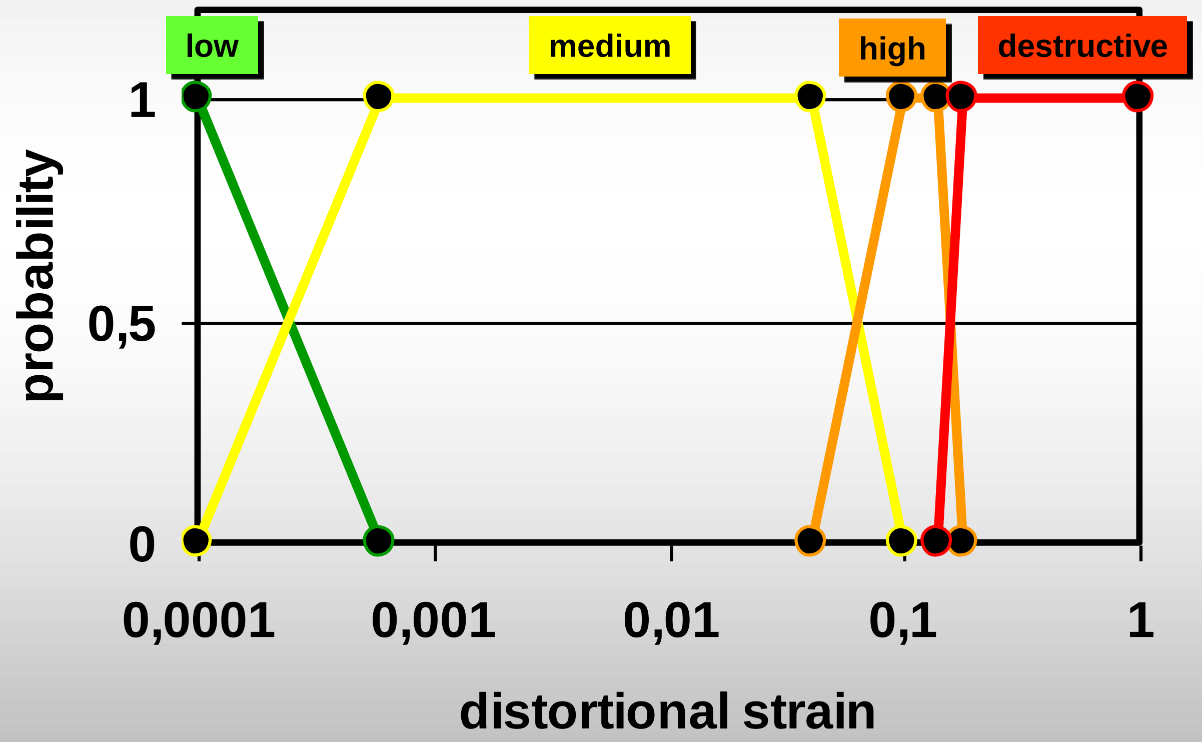
IF *(blood supply is high) AND (bone concentration adjacent is high) AND (dilatational strain is neg. low or pos. low) AND (distortional strain is low)*

THEN *(bone concentration has to increase)*

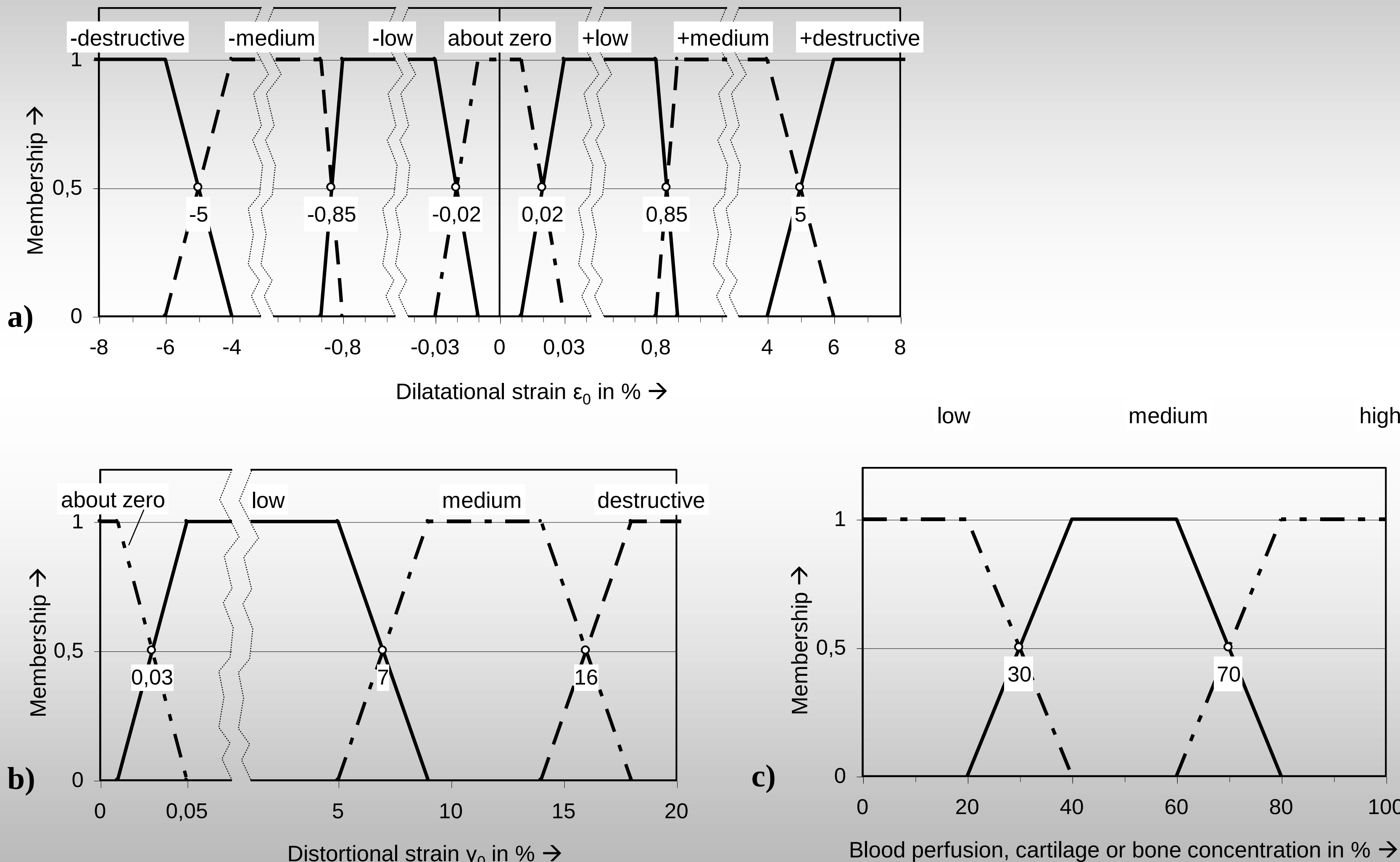


Methods: Membership Functions

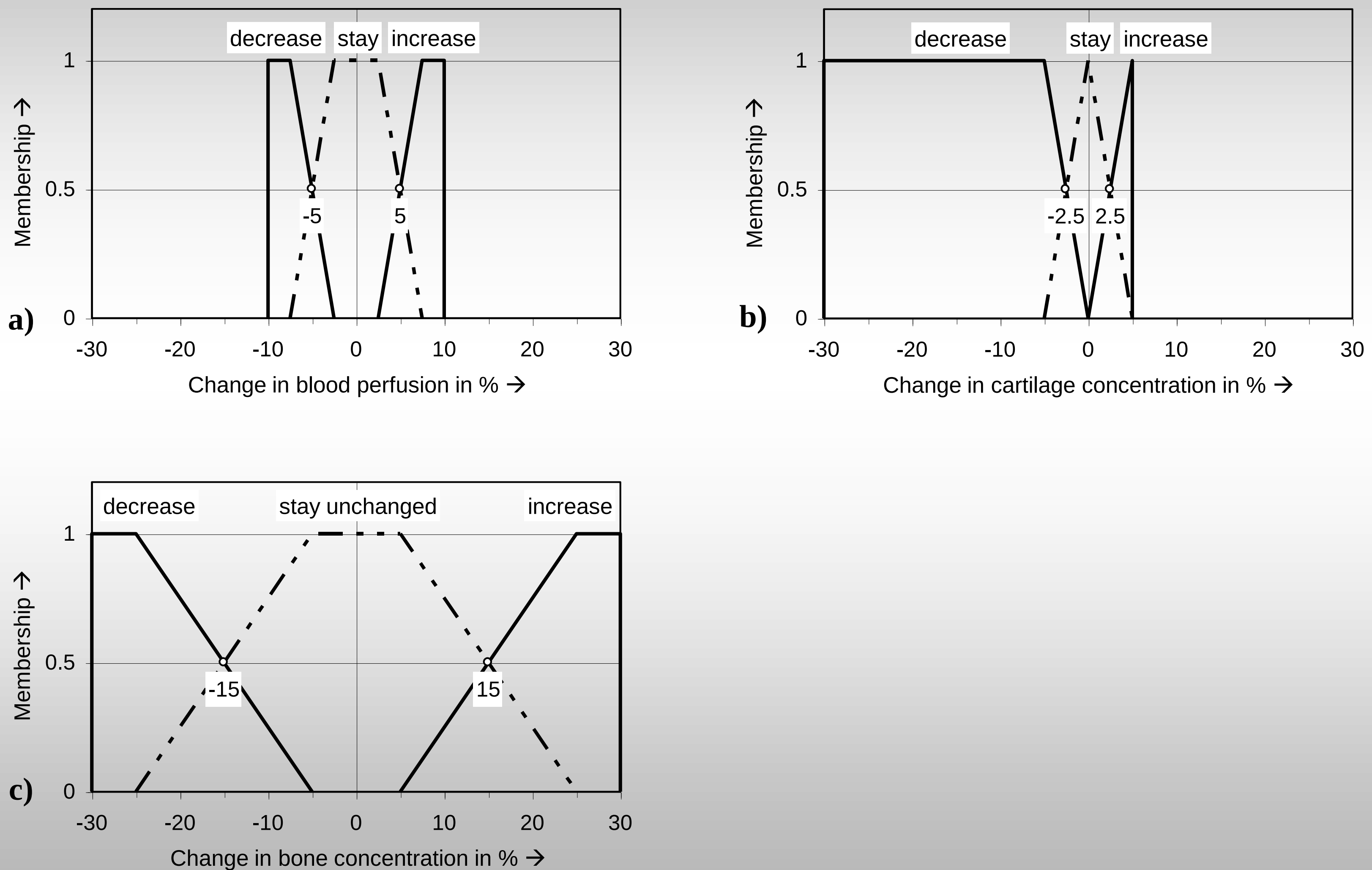
Example: One of 10 membership functions



Zugehörigkeitsfunktionen für Eingangsgrößen



Zugehörigkeitsfunktionen für Ausgangsgrößen

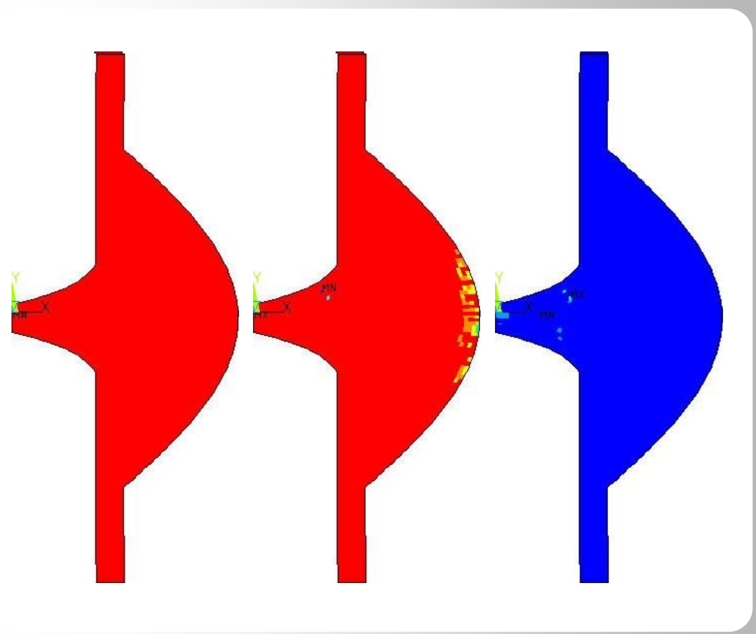


Results

Case A

Small initial movement 0.25 mm

Blood Bone Cartilage



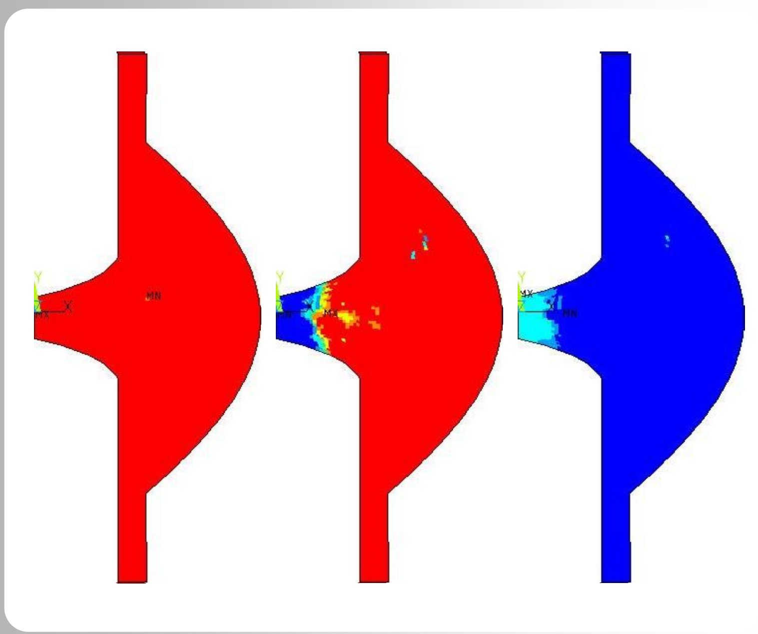
Day

63

Case B

Large initial movement 1.25 mm

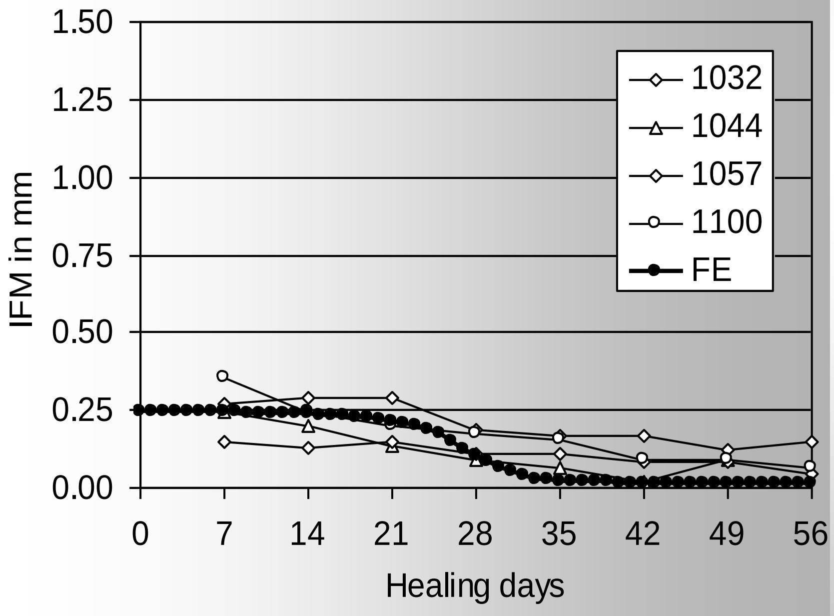
Blood Bone Cartilage



Results: Interfragmentary Movement

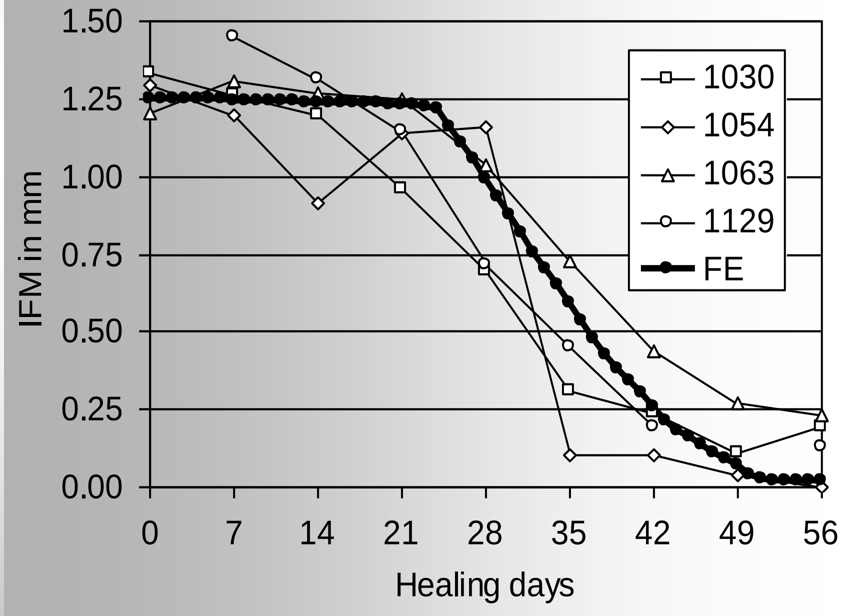
Case A

Small initial movement 0.25 mm



Case B

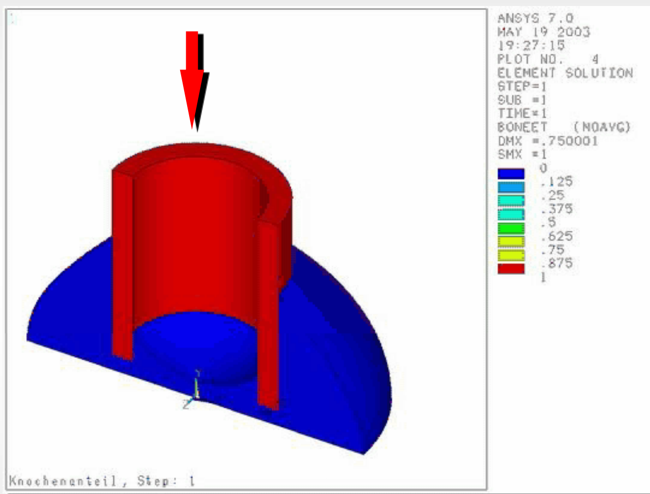
Large initial movement 1.25 mm



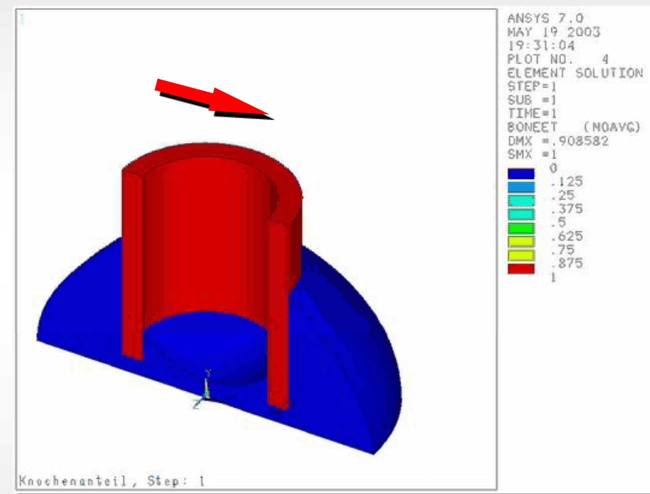
Animal experiments: Claes, et al. 97

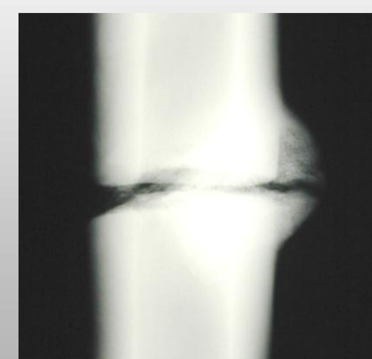
Simulation of 3D bone healing under axial and shear movement

Axial movement

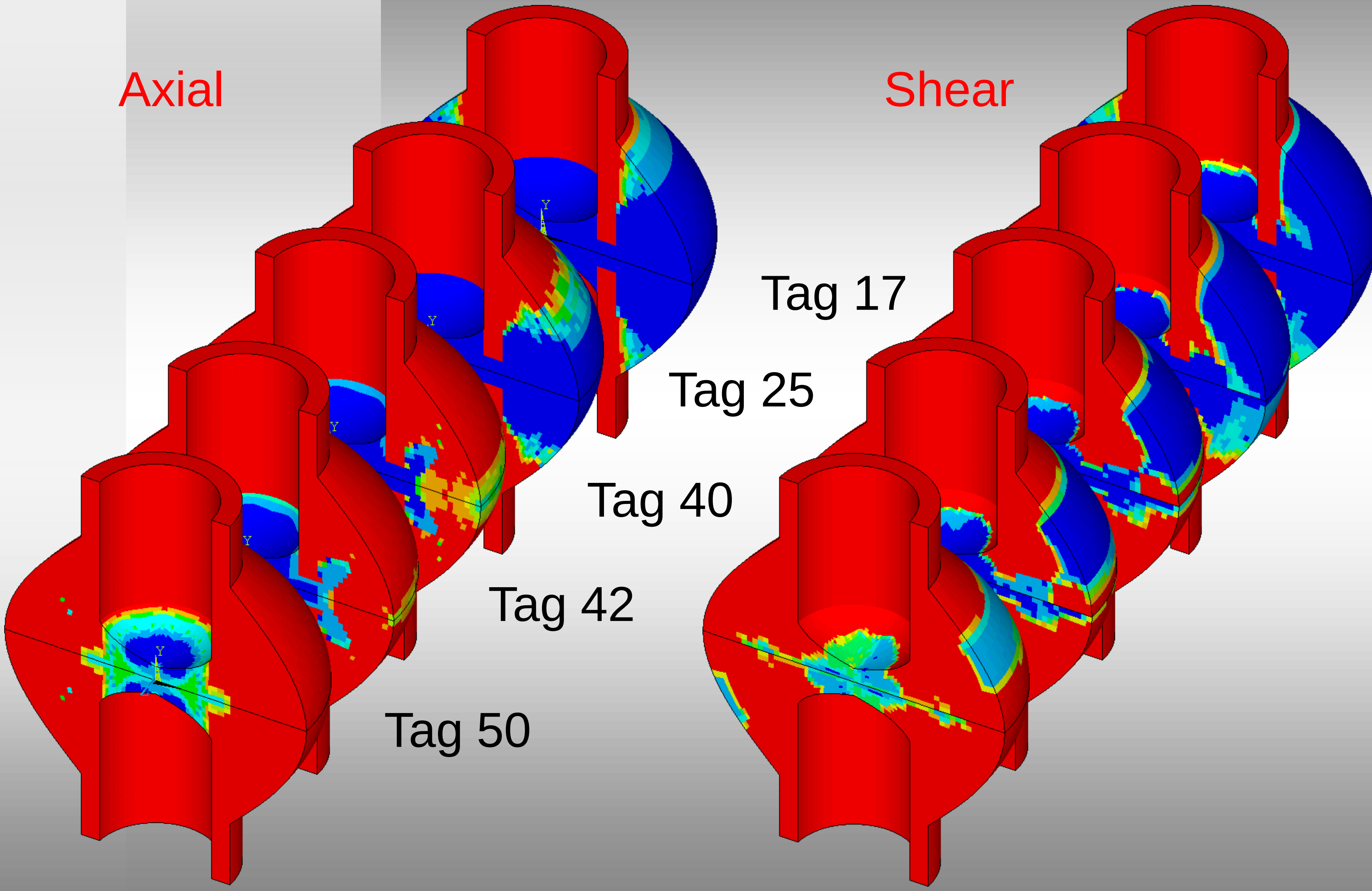


Shear movement

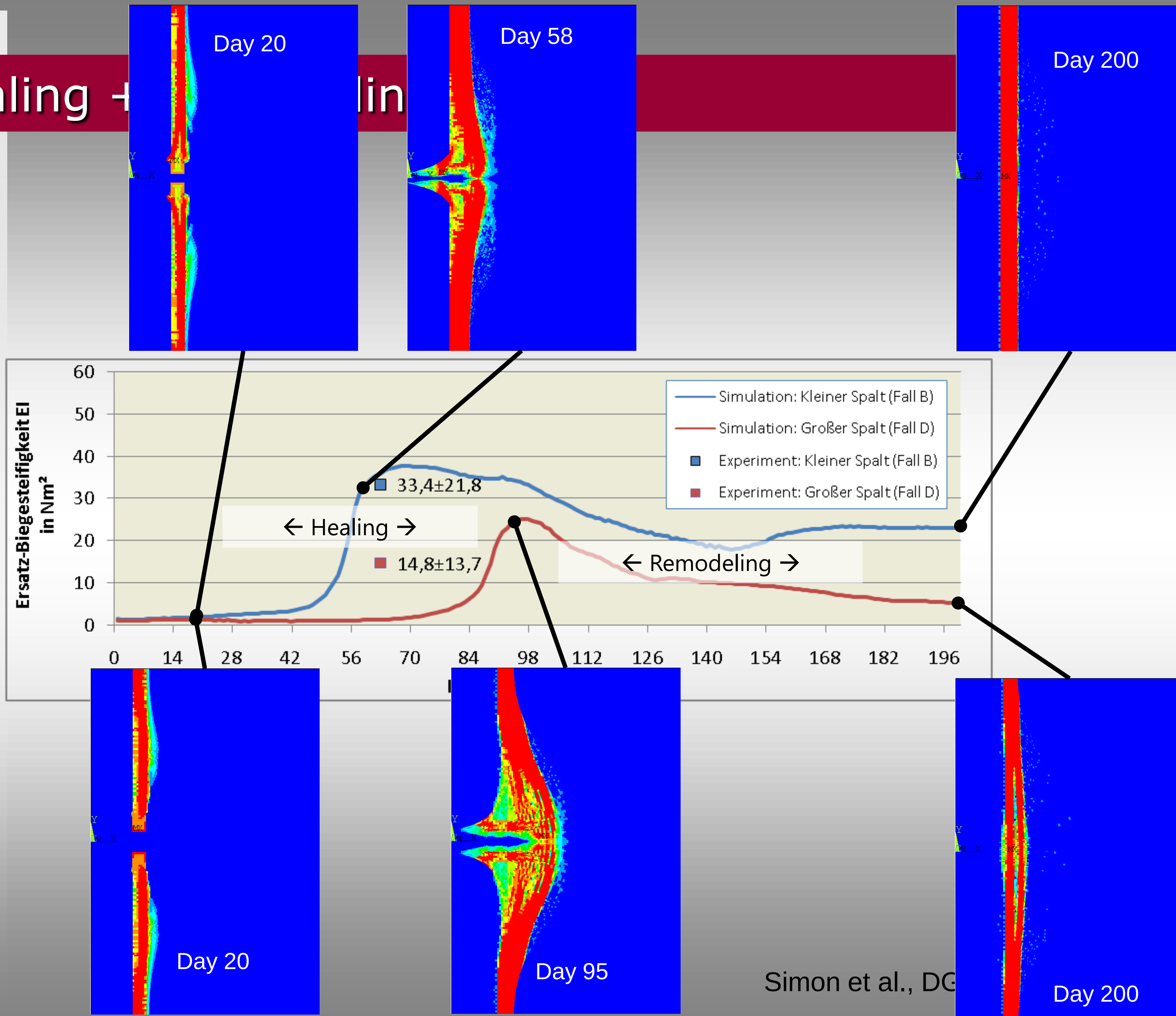




3D Stiffness Directions

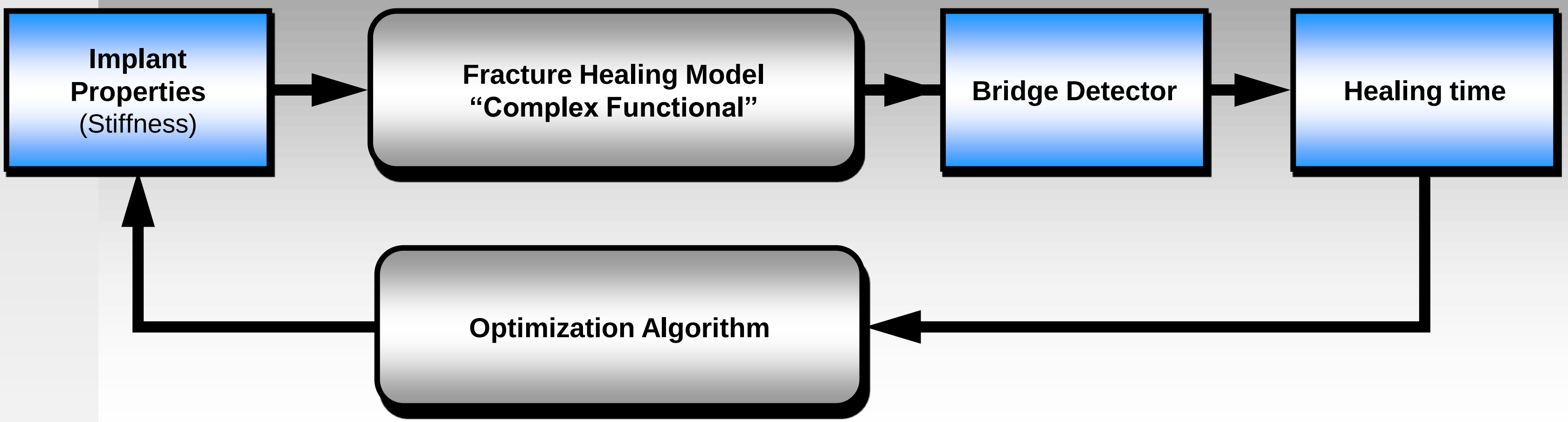


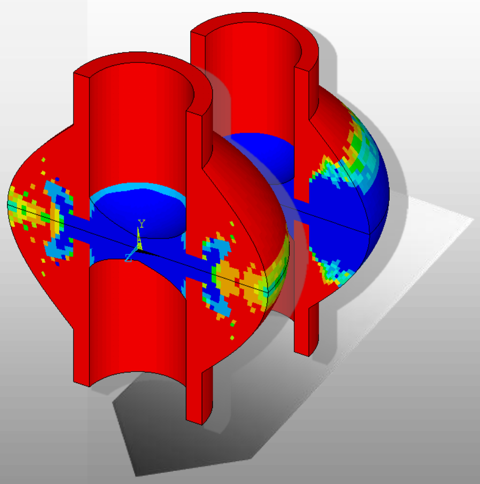
Healing + lin

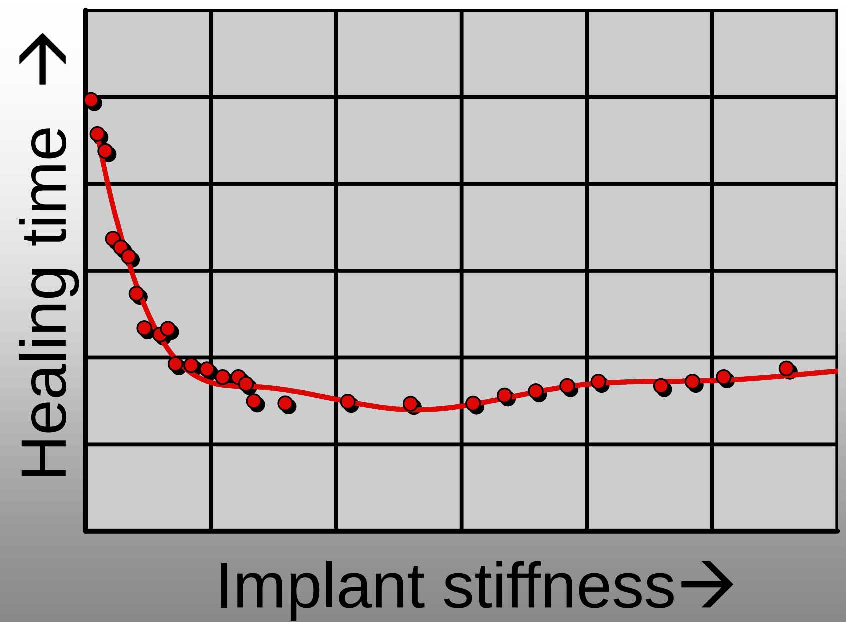


Simon et al., DG

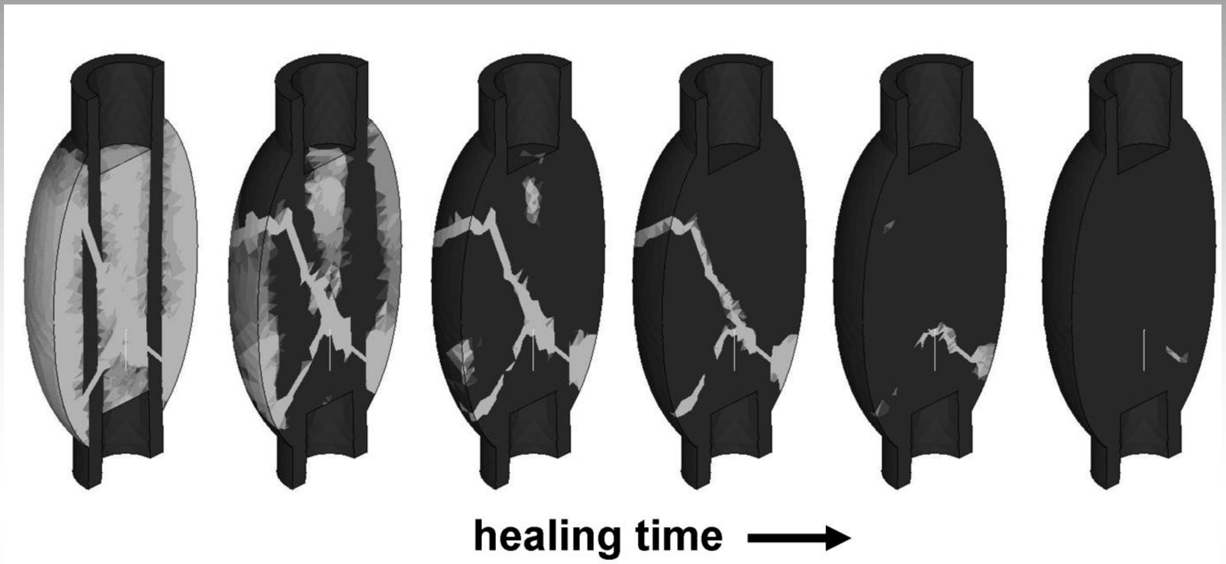
Optimal Implant Stiffness?

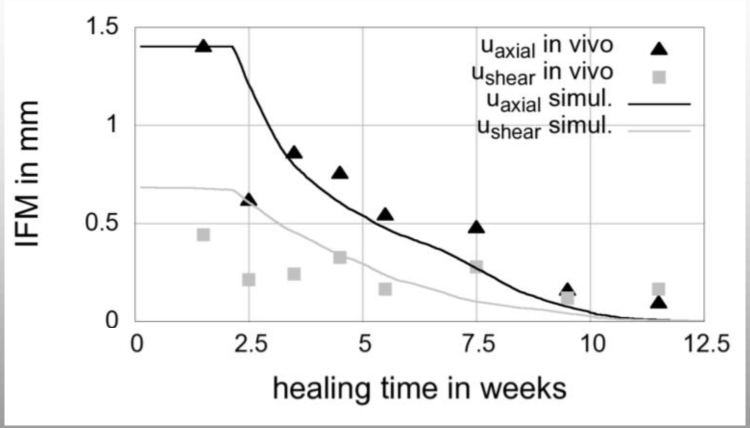






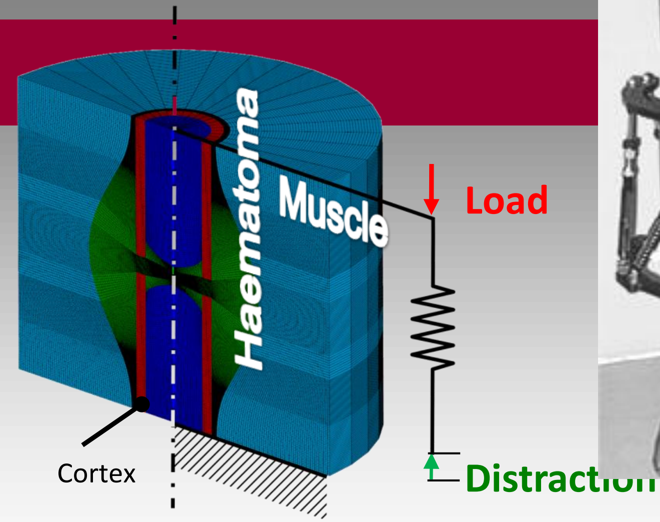
Complex Fracture Geometry

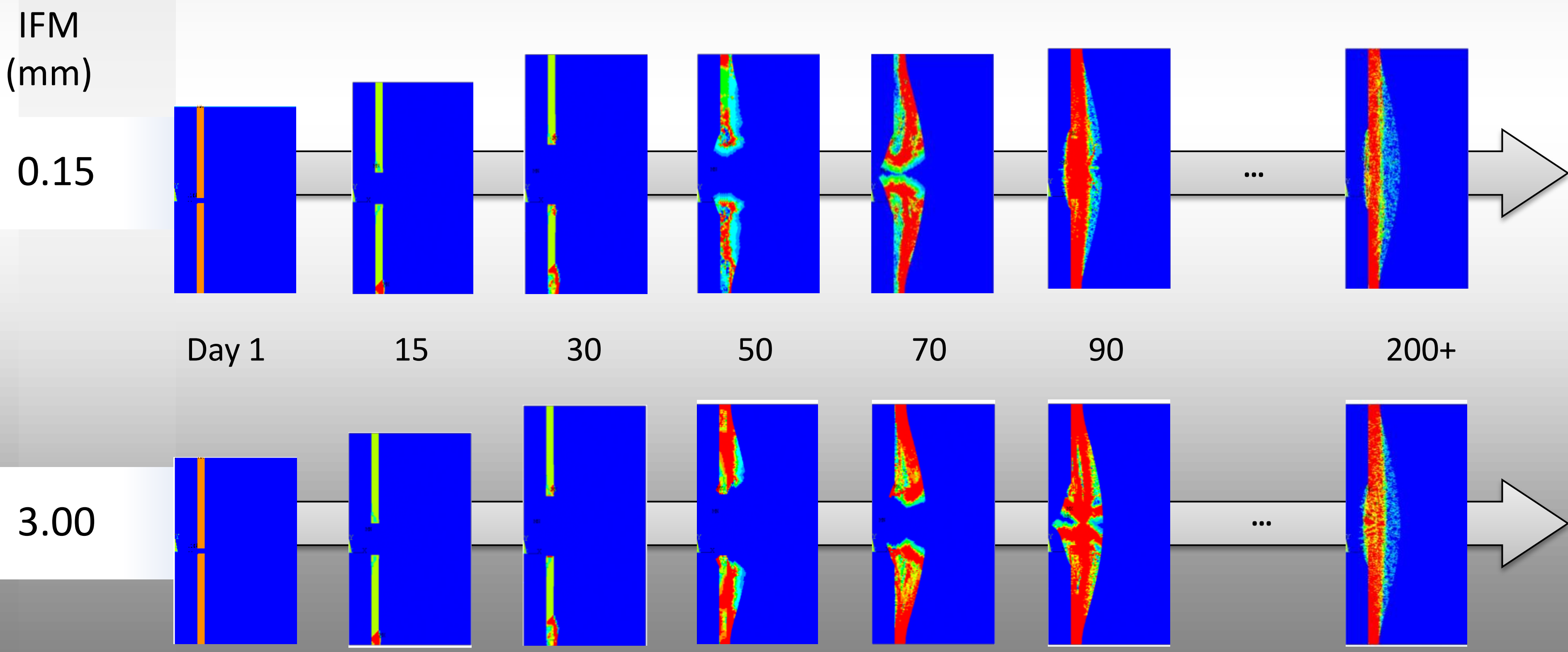




Wehner et al., Clin Biom 2010

Distraction Osteogenesis





1.3 Variables, Dimensions and Units

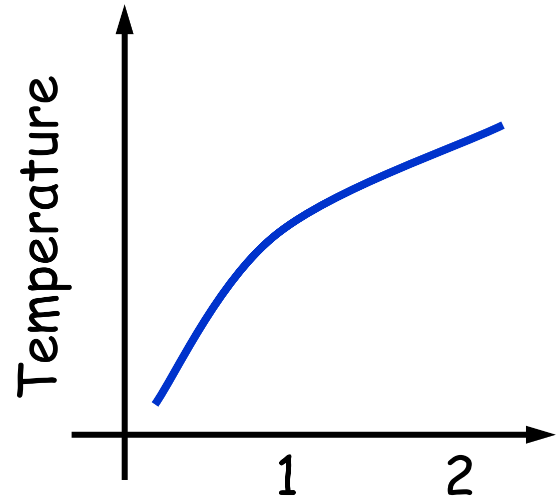
Standard: ISO 31, DIN 1313

Variable = Number · Unit

Length L = $2 \cdot m$ = $2\ m$

{Variable} = Number

[Variable] = Unit



~~Length L [m]~~

Length L / m

Length L in m →

Three mechanical SI-Units:

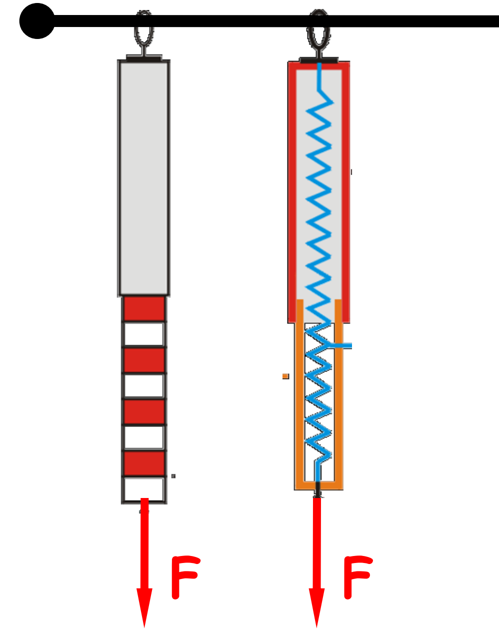
m (Meter)

kg (Kilogram)

s (Seconds)

2 STATICS OF RIGID BODIES

2.1 Force



- We all believe to know what a *force* is.
- But, *force* is an invention not a discovery!
- … it can not be measured directly.

Newton's 2nd Law [Axiom]:

Force = Mass times Acceleration or $F = m \cdot a$

Note to Remember:

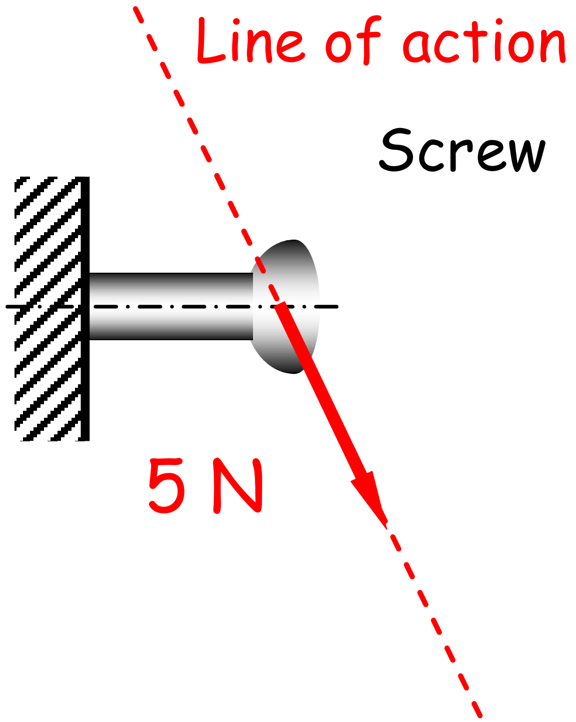
„A force is the cause of acceleration or deformation of a body"

Representation of Forces

... with arrows

Forces are Vectors with

- **Magnitude**
- **Direction**
- **Sense of Direction**



Units of Force

Newton

$N = kg \cdot m/s^2$

$F_G = m \cdot g = 0{,}1\ kg \cdot 9{,}81\ m/s^2$

$= 0{,}981\ kg\ m/s^2$

$\approx 1\ N$

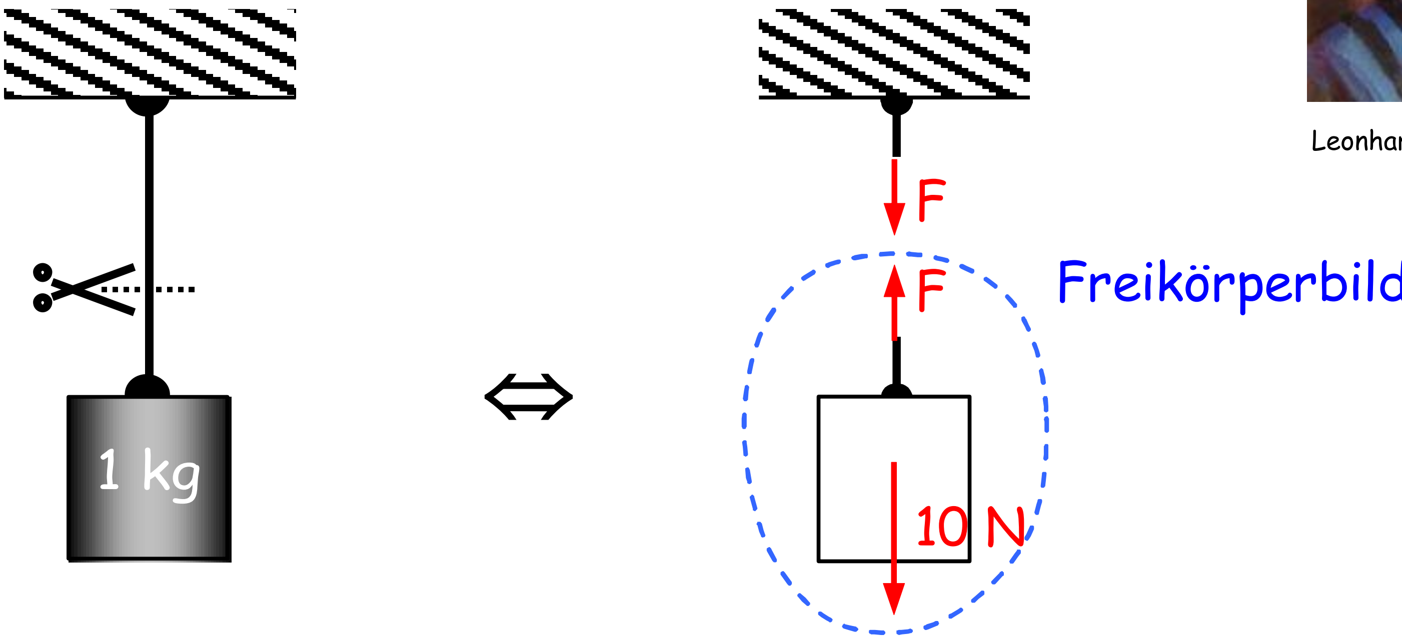
Note to Remember:

1 Newton $\approx$ Weight of a bar of chocolate (100 g)

2.2 Method of Sections (Euler) [Schnittprinzip]

Leonhard Euler, 1707 - 1783

Free-Body Diagramm (FBD) [Freikörper-Bild]

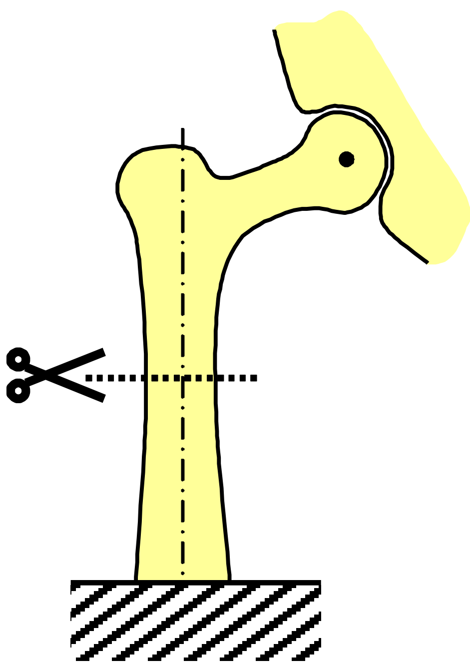


Note to Remember:

First, cut the system, then include forces and moments.

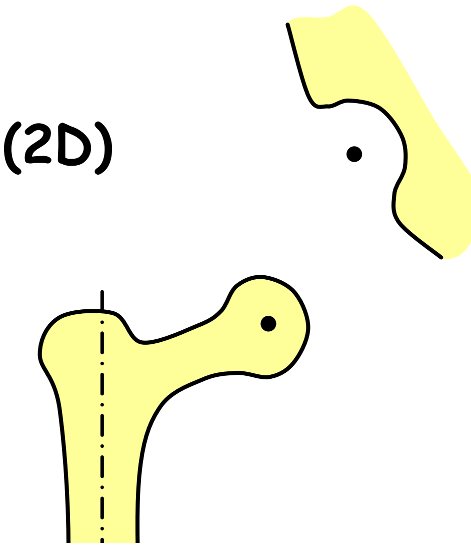
Free-body diagram = <u>completely</u> isolated part.

2.2 Method of Sections

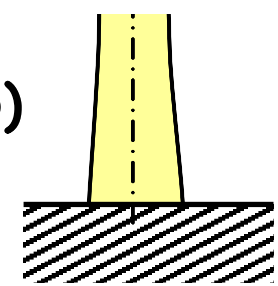


⇔

Cut through joint (2D)

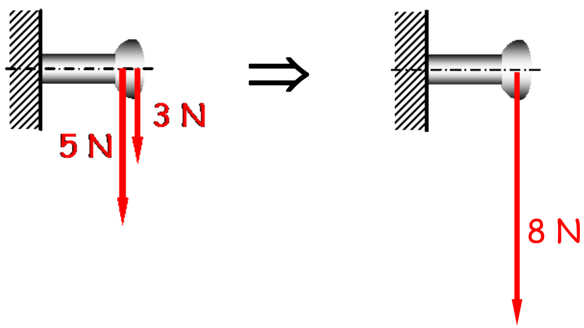


Cut through bone (2D)

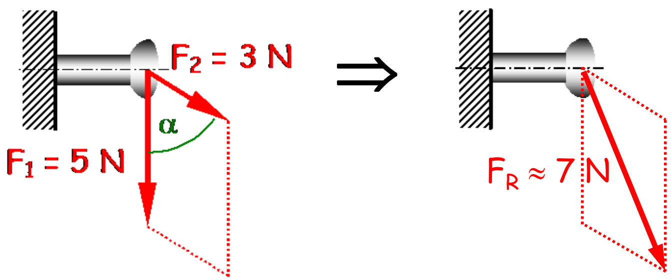


2.3 Combining and Decomposing Forces

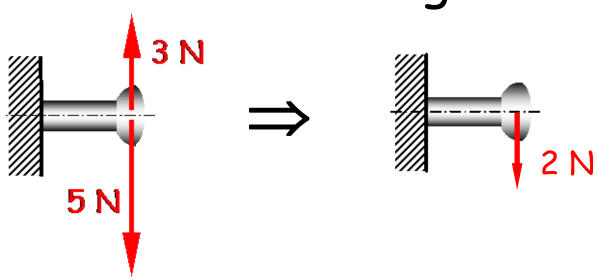
Summation of Magnitudes



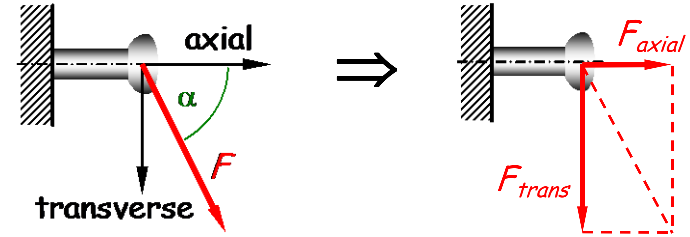
Vector Addition



Subtraction of Magnitudes

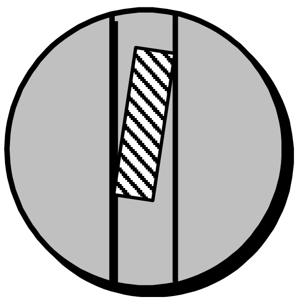


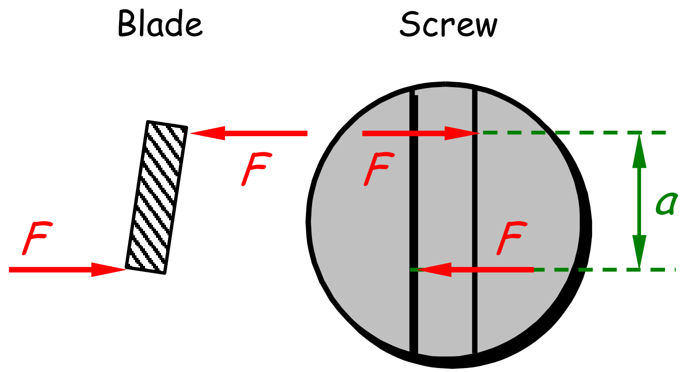
Decomposition into Components



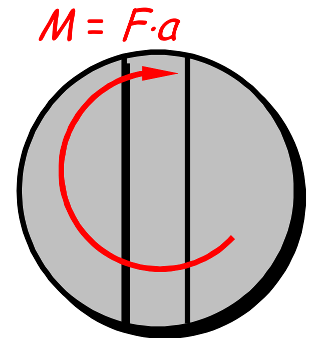
2.4 The Moment [Das Moment]

Slotted screw with screwdriver blade





Force Couples (F, a)



Moment M

Note to remember:

The moment $M = F \cdot a$ is equivalent to a force couple (F, a).

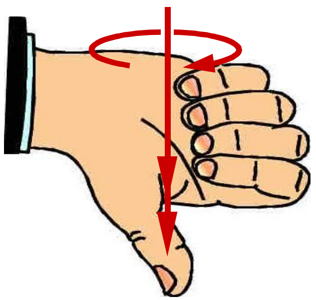
A moment is the cause for angular acceleration or angular deformation (Torsion, Bending) of a body.

Units for Moment

Newton-Meter

$N \cdot m = kg \cdot m^2/s^2$

Rechte-Hand-Regel:

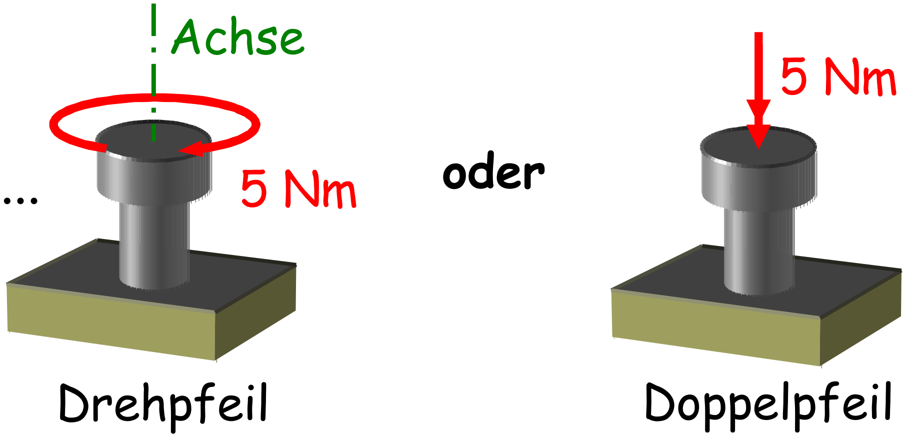


Representation of Moments

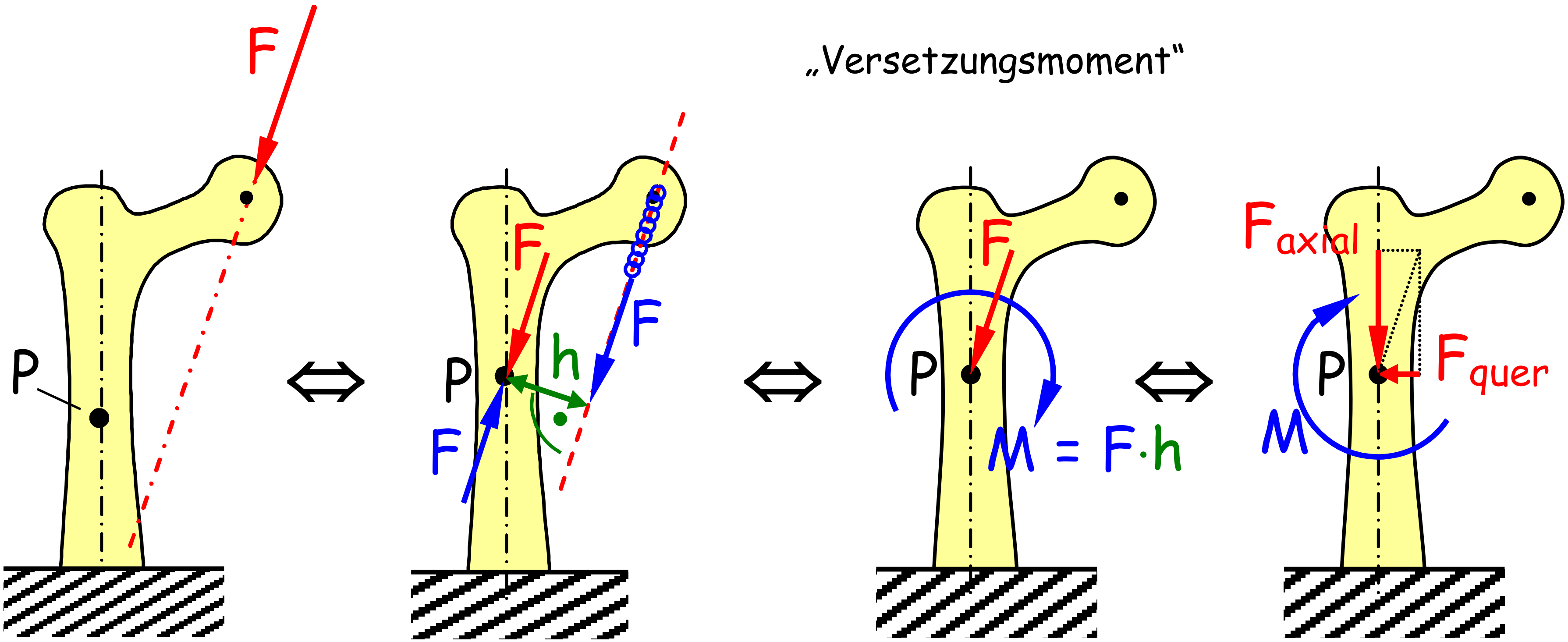
... with rotation arrows or double arrows

Moments are Vectors with ...

- **Magnitude**
- **Direction**
- **Sense of Direction**



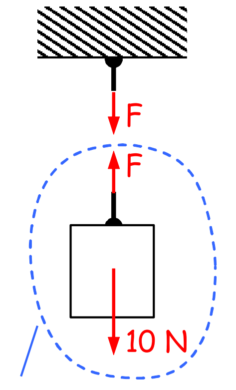
2.5 Moment of a Force about a Point [Versetzungsmoment]



Note to Remember:

Moment = Force times lever-arm

2.7 Static Equilibrium



Important:

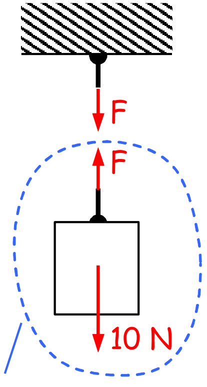
Free-body diagram (FBD) first, then equilibrium!

For 2D Problems max. 3 equations for each FBD:

The sum of all forces in *x*-direction equals zero:	$F_{1,x} + F_{2,x} + \ldots \overset{!}{=} 0$
The sum of all forces in *y*-direction equals zero:	$F_{1,y} + F_{2,y} + \ldots \overset{!}{=} 0$
The sum of Moments with respect to P equals zero:	$M_{1,z}^{P} + M_{2,z}^{P} + \ldots \overset{!}{=} 0$

(For 3D Problems max. 6 equations for each FBD)

2.7 Static Equilibrium



Important:

Free-body diagram (FBD) first, then equilibrium!

3 equations of equilibrium for each FBD in 2D:

$$\text{Sum of all forces in x-direction:} \quad F_{1,x} + F_{2,x} + \ldots \overset{!}{=} 0,$$

$$\text{Sum of all forces in y-direction:} \quad F_{1,y} + F_{2,y} + \ldots \overset{!}{=} 0,$$

$$\text{Sum of all moments w.resp.to P:} \quad M_{1,z}^{P} + M_{2,z}^{P} + \ldots \overset{!}{=} 0.$$

- Force EEs can be substituted by moment EEs
- 3 moment reference points should not lie on one line

6 equlibrium equations for one FBD in 3D:

Summe aller Kräfte in x - Richtung: $\sum_i F_{ix} \overset{!}{=} 0,$

Summe aller Kräfte in y - Richtung: $\sum_i F_{iy} \overset{!}{=} 0,$

Summe aller Kräfte in z - Richtung: $\sum_i F_{iz} \overset{!}{=} 0,$

Summe aller Momente um x - Achse bezüglich Punkt P: $\sum_i M_{ix}^{P} \overset{!}{=} 0.$

Summe aller Momente um y - Achse bezüglich Punkt Q: $\sum_i M_{iy}^{Q} \overset{!}{=} 0.$

Summe aller Momente um z - Achse bezüglich Punkt R: $\sum_i M_{iz}^{R} \overset{!}{=} 0.$

- Force EEs can be substituted by moment EEs
- Max. 2 moment axis parallel to each other
- Determinant of coef. matrix not zero

2.8 Recipe for Solving Problems in Statics

Step 1: Model building. Generate a simplified replacement model (diagram with geometry, forces, constraints).

Step 2: Cutting, Free-body diagram. Cut system and develop free-body diagrams. Include forces and moments at cut, as well as weight.

Step 3: Equilibrium equations. Write the force- and moment equilibrium equations (only for free-body diagrams).

Step 4: Solve the equations. One can only solve for as many unknowns as equations, at most.

Step 5: Display results, explain, confirm with experimental comparisons. Are the results reasonable?