Bone Remodeling

We will use COMSOL to implement a simple bone remodeling simulation, more precisely a simplified version of the model described by Mullender & Huiskes, 1995. The evolution of “relative bone density” $m$ shall be governed by

$$m(x,t) = \tau \cdot (S(x,t) - k)$$

with the strain energy density (SED) $S$, a reference SED $k$ and a remodeling rate constant $\tau$. In this simplified model, the remodeling stimulus is thus simply the deviation of the SED at the current location from a reference value, but not the weighted influence of surrounding “osteocytes”.

Mechanical loads, boundary conditions and other parameter values should generally correspond to those described in the paper. Only for the parameter $k$ we recommend a slightly different value of $k = 0.05$ MPa.

Tasks

1. Implement the model using COMSOL (2D, time-dependent).
2. Compare your results to those presented by Mullender & Huiskes 1995.
3. Investigate the influence of the parameters.

Suggestions

- Your model should consist of two “physics” components: “Solid Mechanics” to compute the SED and a “Coefficient Form PDE” to describe the evolution of $m$.
- Initial conditions: Instead of a homogenous density distribution, we recommend using a simple initial structure, e.g. a “trabecular grid”, instead.
- It is critically important to ensure $0 < m \leq 1$. Hint: The $\lfloor f \rfloor$-operator may prove useful.
- For a more stable solution, choose “discontinuous Lagrange” with a “constant” Ansatz for the discretization of $m$ (meaning that $m$ is considered to be piecewise-constant).