Exercise 4

Plastic Deformation by Oscillating Force and Observation of Strain Hardening on a Cantilever

Learn how to …

… add ideal elastic - plastic material

… simulate an oscillating force

… distribute strain hardening on the beam

1. Given

Beam under 3-point bending with an oscillating centric applied force F as shown in Fig. 1

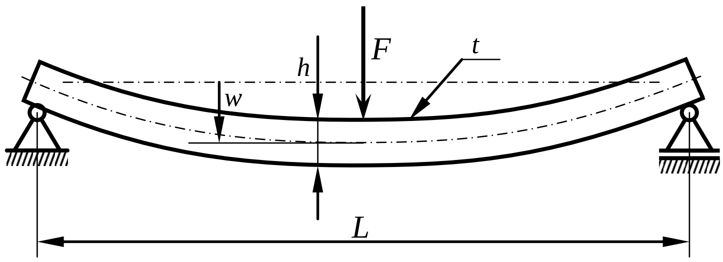


Fig. 1: Beam under 3-point bending.

Relevant geometrical and material data for our problem are given in Table 1:

F	See Fig. 2	Applied force
L	= 2,000 mm	Length of the beam
h	= 60 mm	Height of the beam cross section
t	= 20 mm	Thickness of the beam cross section
E	= 72400 N/mm^2	Young's modulus
v	= 0.33	Poisson's ration
E_t	= 4140 N/mm^2	Tangent modulus
σ_{yield}	= 414 N/mm^2	Allowable stress: yield stress of Al 2014-T6

Table 1: Geometry and material data.

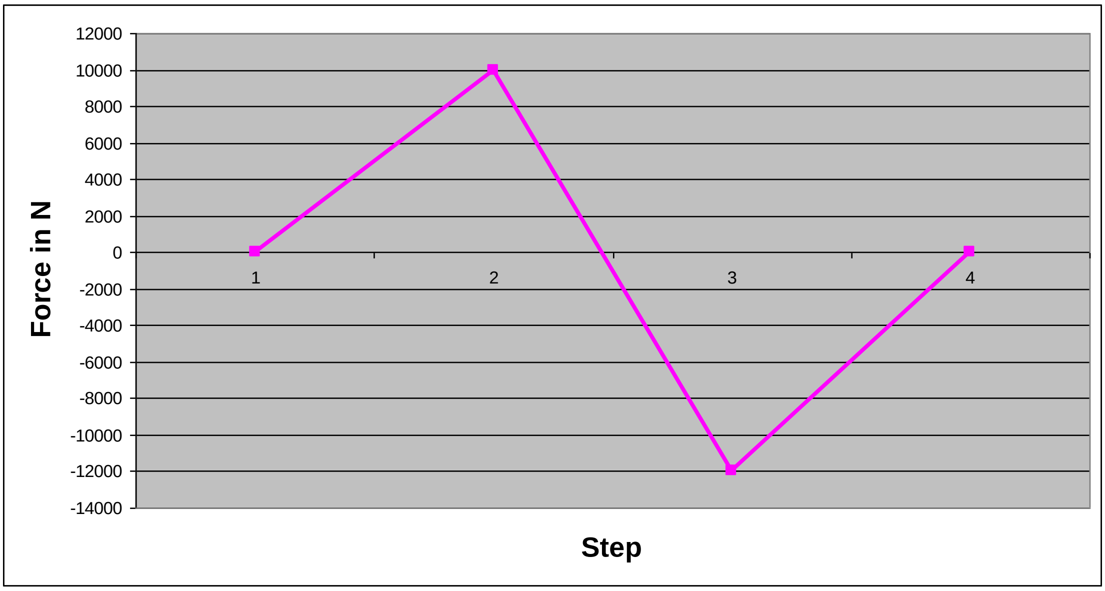


Fig. 2: Oscillating force

2. Questions

1. What is the elastic strain on the cantilever?
2. What is the plastic strain on the cantilever?
3. What is the normal stress along X direction on the cantilever?
4. After unloading the cantilever, why there is still elastic strains exist?
5. After unloading the cantilever, where the stress concentrates and why?
6. Where and why does strain hardening occur?

3. Background

The resulting stress-strain curve or diagram gives a direct indication of the material Properties

Hooke's Law (Robert Hooke, 1678) : Describes the elastic behaviour (up to yielding point) of materials for a bar subjected to uniaxial extension.(1)

$$\sigma = E\varepsilon \qquad \text{Eq.1}$$

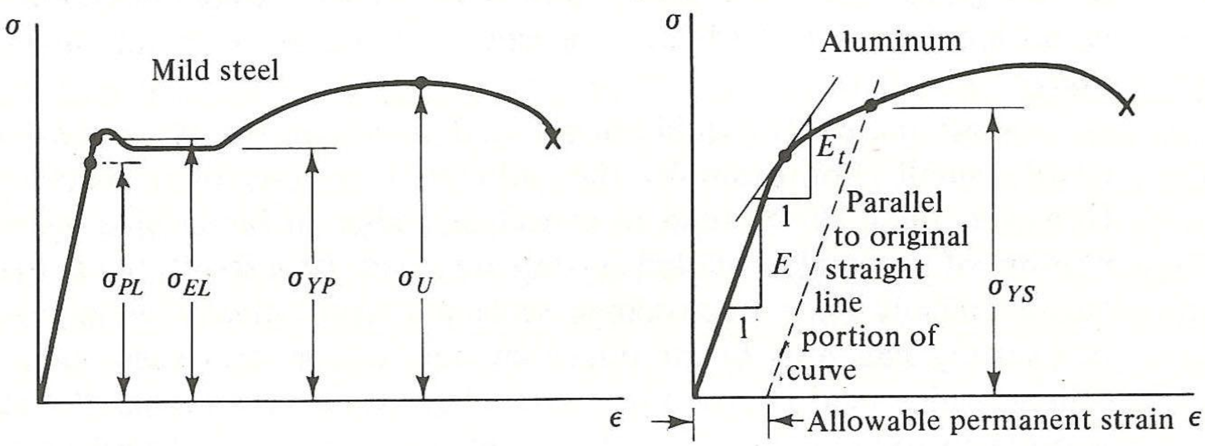


Fig. 4: The meaning of stress-strain curve at different positions(1)

σ_{PL} → **Proportional Limit** - Stress above which stress is not longer proportional to strain.
σ_{EL} → **Elastic Limit** - The maximum stress that can be applied without resulting in permanent deformation when unloaded.
σ_{YP} → **Yield Point** - Stress at which there are large increases in strain with little or no increase in stress. Among common structural materials, only steel exhibits this type of response.
σ_{YS} → **Yield Strength** - The maximum stress that can be applied without exceeding a specified value of permanent strain (typically .2% = .002 in/in).
σ_{U} →**Ultimate Strength** - The maximum stress the material can withstand (based on the original area).

E →**Modulus of Elasticity** - Slope of the initial linear portion of the stress-strain diagram. The modulus of elasticity may also be characterized as the "stiffness" or ability of a material to resist deformation within the linear range.
E_t →**Tangent Modulus** - Slope of the stress-strain curve above the proportional limit. There is no single value for the tangent modulus; it varies with strain.
v →**Poisson's ratio** – The ratio of lateral or transverse strain to the longitudinal strain. Poisson's ratio for most materials ranges from 0.25 to 0.35. (1)

$$\nu = -\frac{\varepsilon_t}{\varepsilon_a}$$

Eq.2

Isotropic – Isotropic materials have elastic properties that are independent of direction. Most common structural materials are isotropic.
Anisotropic – Materials whose properties depend upon direction. An important class of anisotropic materials is fiber-reinforced composites.(1)

Ductile Material – Materials that are capable of undergoing large strains (*at normal temperature*) before failure. An advantage of ductile materials is that visible distortions may occur if the loads before too large. Ductile materials are also capable of absorbing large amounts of energy prior to failure. Ductile materials include mild steel, aluminium and some of its alloys, copper, magnesium, nickel, brass, bronze and many others.
Brittle Material – Materials that exhibit very little inelastic deformation. In other words, materials that fail in tension at relatively low values of strain are considered brittle. Brittle materials include concrete, stone, cast iron, glass and plaster. (1)

Equation 3 is the usual representation of the **non-linear behavior**:

$$\varepsilon = \frac{\sigma}{E_0} + 0.002\left(\frac{\sigma}{\sigma_{0.2}}\right)^n$$

Eq.3

where ε is the strain, σ is the stress, $\sigma_{0.2}$ is the 0.2% proof stress, E_0 is the initial elastic modulus and n is the **Ramberg-Osgood parameter** (strain hardening exponent) which is a measure of the non-linearity of the curve. (2)

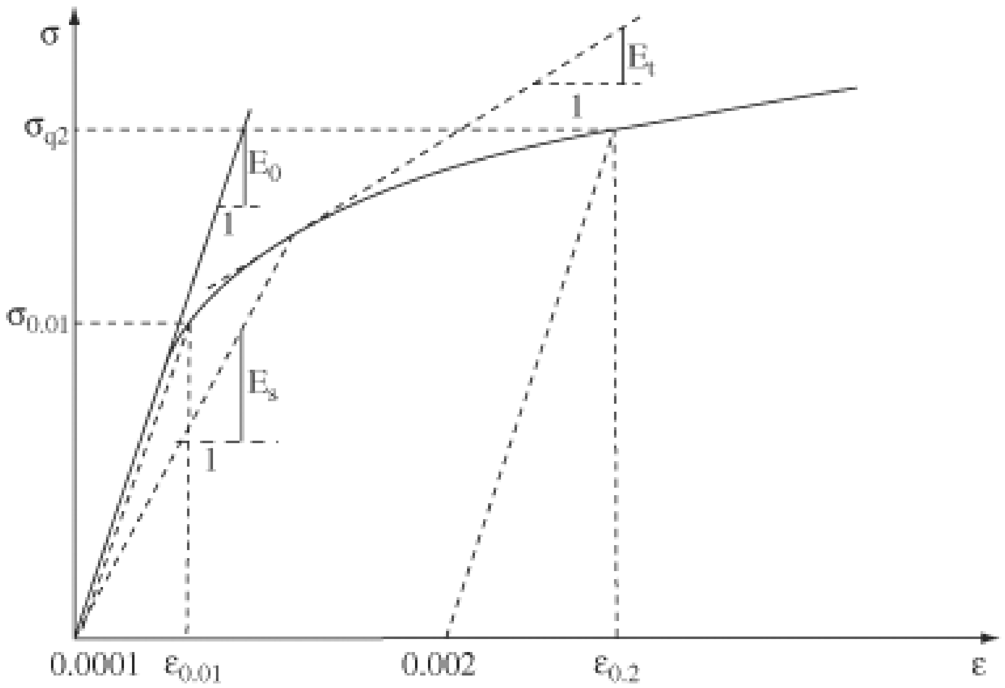


Fig. 5: Typical non-linear stress - strain curve of an aluminium sample (2)

The first term of Equation 3 represents the linear behaviour and the second represents the non-linear behaviour. The component value of non-linear strain in $\sigma_{0.01}$ is 0.0001 (0.01%) and decreases with the reduction of the stress. For low stress values, the non-linear component is not significant when compared to the linear component.(2)

The tangent modulus (E_t), Equation 4, is defined as the slope of the stress-strain curve at each value of stress. It is defined by Equation 3 as the inverse of the first derivative in respect to strain. (2)

$$E_t = \frac{E_0 \sigma_{0.2}}{\sigma_{0.2} + 0.002 n E_0 \left(\frac{\sigma}{\sigma_{0.2}}\right)^{n-1}} \qquad \text{Eq.4}$$

Strain hardening (also called work-hardening or cold-working) is the process of making a metal harder and stronger through plastic deformation. When a metal is plastically deformed, dislocations move and additional dislocations are generated. The more dislocations within a material, the more they will interact and become pinned or tangled. This will result in a decrease in the mobility of the dislocations and a strengthening of the material. This type of strengthening is commonly called cold-working. It is called cold-working because the plastic deformation must occurs at a temperature low enough that atoms cannot rearrange themselves. When a metal is worked at higher temperatures (hot-working) the dislocations can rearrange and little strengthening is achieved. (3)

Strain hardening can be easily demonstrated with piece of wire or a paper clip. Bend a straight section back and forth several times. Notice that it is more difficult to bend the metal at the same place. In the strain hardened area dislocations have formed and become tangled, increasing the strength of the material. Continued bending will eventually cause the wire to break at the bend due to fatigue cracking. (After a large number of bending cycles, dislocations form structures called Persistent Slip Bands (PSB). PSBs are basically tiny areas where the dislocations have piled up and moved the material surface out leave steps in the surface that act as stress risers or crack initiation points.) (3)

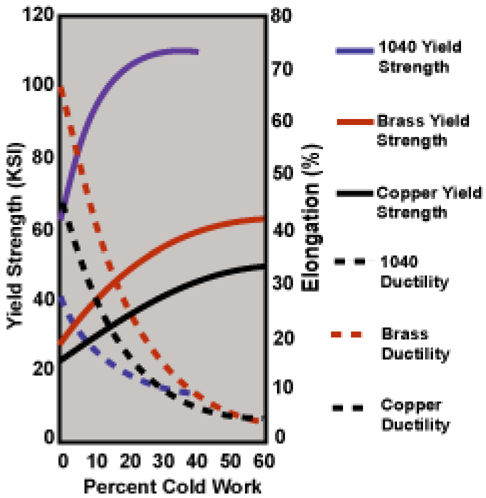


Fig. 6: The effect of strain hardening on yield strength and ductility (3)

It should be understood, however, that increasing the strength by cold-working will also result in a reduction in ductility. The graph to the right shows the yield strength and the percent elongation as a function of percent cold-work for a few example materials. Notice that for each material, a small amount of cold-working results in a significant reduction in ductility. (3)

4. Strain Hardening Simulation on ANSYS Workbench

Please follow the instructions below:

- Input the provided FE-Exercise4.inp File
- Enter the material properties on the “Engineering Data” window.
  - ⇒ Additionally to Exercise 1; add “**Plasticity**” by selecting the “**Bilinear Isotropic Hardening”**, see Figure 7
  - ⇒ The corresponding values are given on **Table 1**

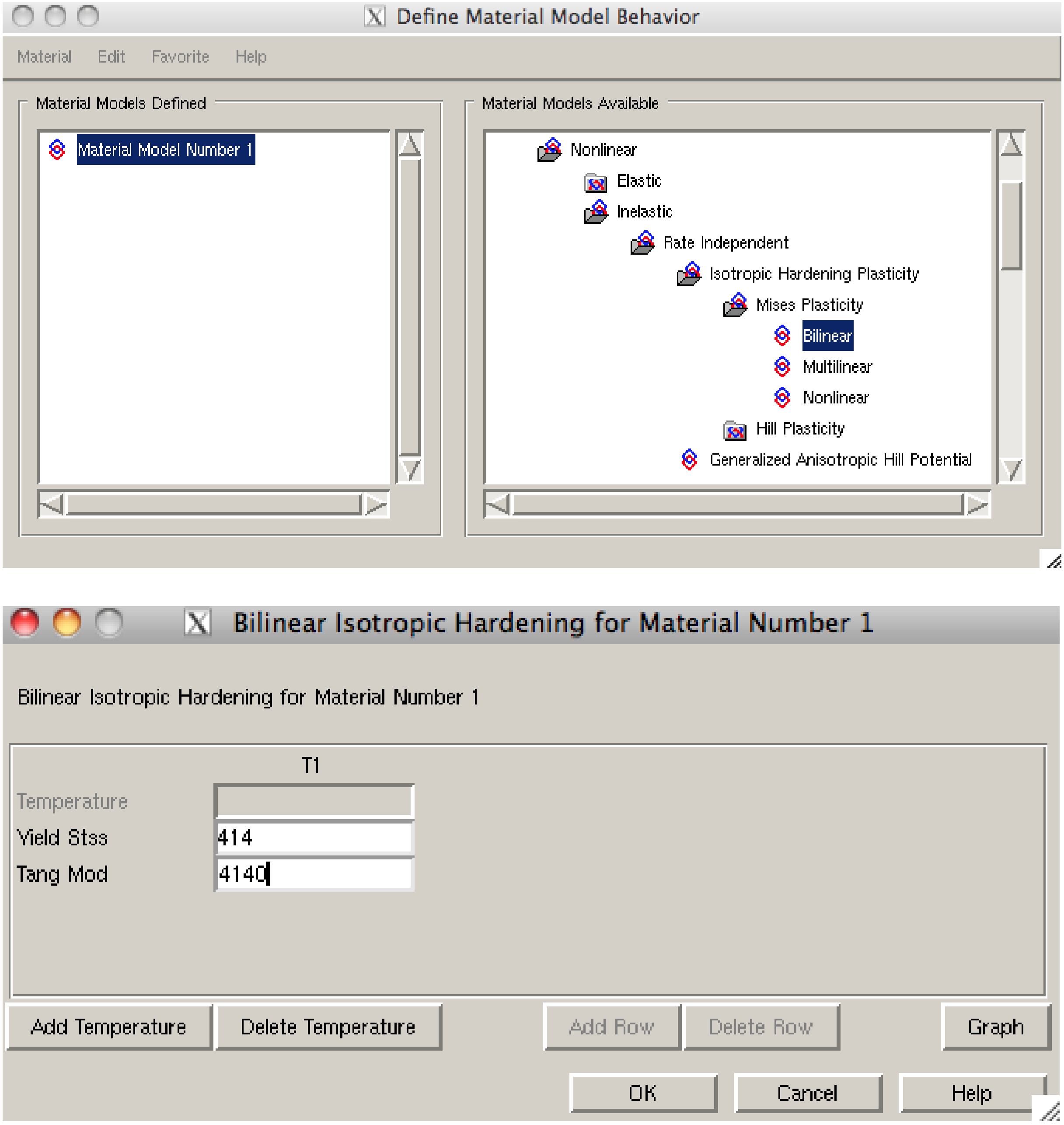
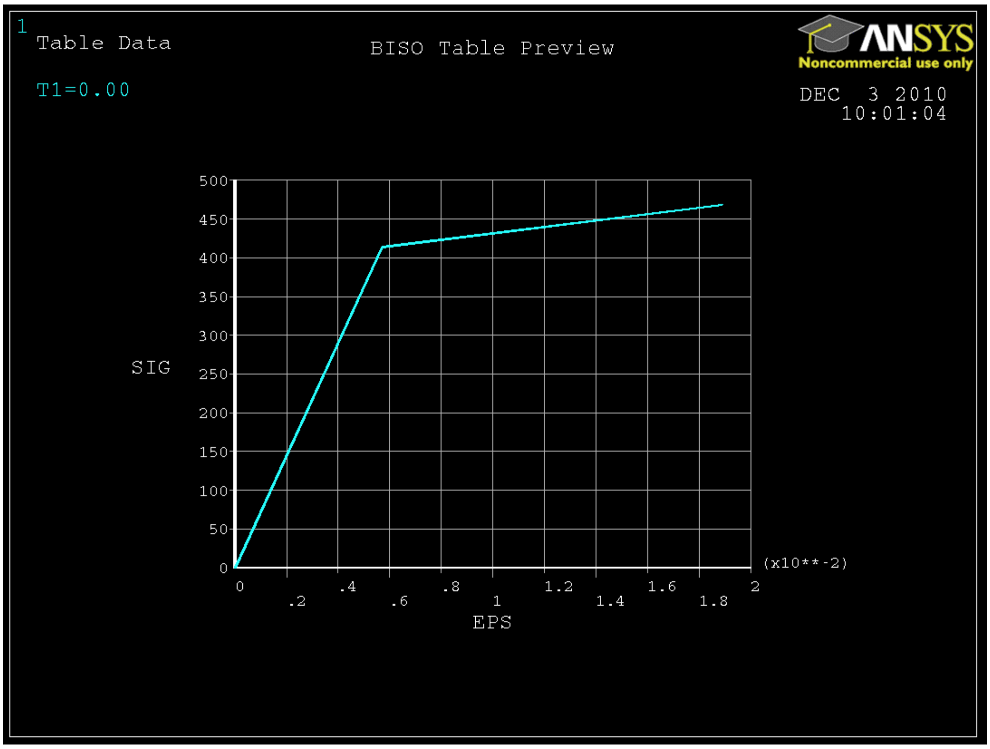


Fig.7: Properties of Al 2014 – T6

- Plot the corresponding stress-strain curve unsing the “Graph”-Button



- Apply load and boundary conditions
  - ⇒ Oscillating force profile is given on **Fig.2**:
    Step 1: Fy = 10000
    Step 2: Fy = -12000
    Step 3: Fy = 0
  - ⇒ To define and solve these steps, use the commands **lswrite** and **lssolve**

- Complete the provided FE-Exercise4.inp File

- Solve
  - ⇒ Total Deformation
  - ⇒ Normal Stress
  - ⇒ Normal Elastic Strain
  - ⇒ Normal Plastic Strain
    - Change the scale to 1.0 (True Scale)

- Answer Questions 1 to 6

5. References

(1) http://www.optics.arizona.edu/optomech/references/OPTI_222/OPTI_222_W4.pdf
(2) Study of the non-linear stress-strain behavior in Ti-Nb-Zr alloys; S. Schneider, S. G. Schneider, H. Marques da Silva, C. de Moura Neto, Materials Research. vol.8 no.4, Oct./Dec. 2005
(3) http://www.ndt-ed.org/EducationResources/CommunityCollege/Materials/Structure/strengthening.htm
(4) http://en.wikipedia.org/wiki/Strain_hardening_exponent