1) $BL = \int_{-1}^{1} \sqrt{1+(\cosh'(x))^2}\,dx = \int_{-1}^{1}\sqrt{1+(\frac{1}{2}e^{x}-\frac{1}{2}e^{-x})^2}\,dx = \int_{-1}^{1}\sqrt{1+\frac{1}{4}(e^{2x}+e^{-2x}-2)}\,dx$

$= \int_{-1}^{1}\sqrt{\frac{1}{4}e^{2x}+\frac{1}{2}+\frac{1}{4}e^{-2x}}\,dx = \frac{1}{2}\int_{-1}^{1}\sqrt{(e^{x}+e^{-x})^2}\,dx = \frac{1}{2}\int_{-1}^{1}e^{x}\,dx + \frac{1}{2}\int_{-1}^{1}e^{-x}\,dx$

$= \left(\frac{1}{2}e - \frac{1}{2e}\right) - \frac{1}{2e} + \frac{1}{2}e = e - \frac{1}{e}$

2) a) $\int_{0}^{\pi/2} \sin(2x) = \left[-\frac{1}{2}\cos(2x)\right]_{0}^{\pi/2} = \frac{1}{2}+\frac{1}{2} = 1$

b) Sehnen-Trapez-Regel: $\left(\frac{b-a}{2}\right)(f(a)+f(b))$

$\frac{\pi}{4}(\sin\pi + \sin 0) = 0$

Fass-Regel: $\frac{b-a}{6}\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right)$

$\frac{\pi}{12}\left(\sin 0 + 4\sin\frac{\pi}{2} + \sin\pi\right) = \frac{1}{3}\pi \approx 1{,}0472$

c) S-T-R.: $\text{Fehler} = \left|-\int_{a}^{b} f(x)\,dx + \frac{b-a}{2}(f(a)+f(b))\right|$

$= \frac{(b-a)^3}{12} f''(\vartheta) \le \frac{(b-a)^3}{12}\max_{\vartheta\in(a,b)}|f''(\vartheta)|$

$n\left(\frac{\pi}{2n}\right)^3\frac{4}{12} < 10^{-4} \Rightarrow n^2 > \left(\frac{\pi}{2}\right)^3\frac{1}{3\cdot 10^{-4}} \Rightarrow n \ge 114$

Fass: $\text{Fehler} \le \frac{(b-a)^5}{2880}\max_{\vartheta\in(a,b)}|f^{(4)}(\vartheta)|$

$n\left(\frac{\pi}{2n}\right)^5\frac{16}{2880} < 10^{-4} \Rightarrow n^4 > \left(\frac{\pi}{2}\right)^5\frac{16}{2880\cdot 10^{-4}} \Rightarrow n \ge 4$

3) a)

$$\int e^{-x}\sin x\,dx = \int e^{-x}(-\cos x)'\,dx = -\int e^{-x}\cos x\,dx - e^{-x}\cos x$$

$$= -\int e^{-x}\sin x\,dx - e^{-x}\sin x - e^{-x}\cos x = -\frac{1}{2}e^{-x}(\sin x + \cos x)$$

$$\Rightarrow \int_0^\infty e^{-x}\sin x = \lim_{T\to\infty}\underbrace{\left(-\frac{1}{2}e^{-T}(\sin x + \cos x)\right)}_{\to 0} + \frac{1}{2} = \frac{1}{2}$$

b) $$\int_0^1 \frac{e^x}{x}\,dx > \int_0^1 \frac{1}{x}\,dx \;\left(= \ln 1 - \lim_{T\to 0}\ln T\right) \to \infty$$

c) $\boxed{y=\sqrt{x}}$

$$\int \frac{dx}{\sqrt{x}(1+x)} = \int \frac{2\,dy}{(1+y^2)} = 2\arctan y = 2\arctan\sqrt{x}$$

$$\Rightarrow \int_0^\infty \frac{dx}{\sqrt{x}(1+x)} = \pi$$

d) $$\int_1^\infty \frac{dx}{1+\ln x} \geq \int_1^\infty \frac{1}{x}\,dx \to \infty \qquad (1+\ln x \leq x)$$

e) $$\int x e^{-x^2}\,dx = -\frac{1}{2}\int(-2x)e^{-x^2} = -\frac{1}{2}e^{-x^2}$$

$$\Rightarrow \int_0^\infty x e^{-x^2}\,dx = \lim_{T\to\infty}\left(-\frac{1}{2}e^{-T^2}\right) + \frac{1}{2} = \frac{1}{2}$$

f) $$\int \frac{dx}{1+x^2} = \arctan x \Rightarrow \int_{-\infty}^{+\infty}\frac{dx}{1+x^2} = \lim_{T\to+\infty}\arctan T - \lim_{T'\to-\infty}\arctan T'$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$$

4 a) $\int_1^\infty \left| \frac{\cos 5x}{x^{3/2}} \right| dx \leq \int_1^\infty \frac{1}{x^{3/2}} \, dx \Rightarrow$ Integral existiert

b) c) d)

$$\lim_{x \to 0} \frac{\cos x - 1}{x^2} = -\frac{1}{2} \quad \left(\cos x - 1 = 0 + x \sin 0 - \frac{x^2}{2} \cos 0 + \ldots \right)$$

$\boxed{\text{oder Regel von L'Hospital!!}}$ *

$\Rightarrow$ b) konvergiert (hebbare Singularität in 0)

c) konvergiert $\quad \frac{\cos(x) - 1}{x^2 \sqrt{x}} = \underbrace{\frac{\cos x - 1}{x^2}}_{\approx -\frac{1}{2} \, (x \to 0)} \overbrace{\frac{1}{\sqrt{x}}}^{\text{konv.}}$

d) divergiert $\quad \frac{\cos(x) - 1}{x^3} = \underbrace{\frac{\cos x - 1}{x^2}}_{\hat{=} -\frac{1}{2} \, (x \to 0)} \overbrace{\frac{1}{x}}^{\text{divergiert } (x \to 0)}$

---

* Regel von L'Hospital: Falls $f(x_0) = g(x_0) = 0$, dann

ist $\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$, wenn $\lim_{x \to x_0} \frac{f'(x)}{g'(x)}$ existiert.