Fundamental limitations of digital control

S. Lang†, F. Allgöwer‡

†University of Stuttgart, Institute for Systems Theory and Automatic Control,
Pfaffenwaldring 9, 70569 Stuttgart, Tel: +49 711 685 67754, E-Mail: lang@uni-stuttgart.de
‡University of Stuttgart, Institute for Systems Theory and Automatic Control,
Pfaffenwaldring 9, 70569 Stuttgart, Tel: +49 711 685 67733, E-Mail: allgower@uni-stuttgart.de

Digital communication and controller implementation have become ubiquitous in many modern control applications. However, digital control also imposes new challenges on the controller design as it imposes further requirements on the controller. One of the most important additional requirements is finite control range, i.e., in every finite time interval only finitely many control signals can be generated by the controller. This raises the question of what fundamental limitations controllers with finite control range have, and whether classical performance objectives can be achieved. In this talk, we investigate this question for the objective of quadratic performance and discuss that achieving quadratic performance requires controllers with infinite control range. This implies that controllers with finite control range, and therefore digital controllers, cannot achieve quadratic performance. Quadratic performance is a commonly used performance criterion which includes many important performance criteria, e.g., ℓ_2-performance or passivity, as a special case.

In order to prove that achieving quadratic performance requires controllers with infinite control range, we introduce the notion of (ϵ, δ)-quadratic performance. (ϵ, δ)-quadratic performance is a control performance criterion whose fulfillment is implied by the fulfillment of many classical performance criteria, e.g., quadratic performance or generalized H_2-performance. We exploit this property and show that achieving (ϵ, δ)-quadratic performance requires an infinite control range if the system possesses an unstable mode affecting the performance channel. Consequently, all performance criteria that implies fullfillment of (ϵ, δ)-quadratic performance require an infinite control range - including the criteria of quadratic performance and generalized H_2-performance. This result also holds if a bound on the input signal energy is given.

An immediate implication of this result is that it is not possible to design digital controllers that can guarantee quadratic performance.

The results also imply that dissipativity with quadratic supply rates and dissipativity-based feedback theorems, such as the small-gain or passivity theorem, cannot be used to guarantee stability of feedback interconnections when the individual systems within the interconnection should be controlled digitally.

In view of the significance of digital control, these limitations show the importance of defining new notions of control performance which can be achieved by digital controllers and which mathematically describe properties which are practically desired.

This talk is based on our publication [1].

Literatur

[1] Lang, S.; Allöwer, F.: Fundamental limitations of digital control: Quadratic performance and (Q, S, R)-dissipativity, Proc. of the 23rd IFAC World Congress, Busan, South Korea, 2026.