Analysis and Model Predictive Control of Mechanical Multibody Systems Subject to Non-Integrable Acceleration-Level Constraints

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Over the past decades, multibody systems have been studied extensively in the context of modeling and simulation. General formulations have been developed to describe the dynamics of mechanical systems subject to a wide variety of constraints. In addition to classical holonomic constraints, which can be expressed as algebraic equations on the position level, nonholonomic constraints play an important role in many applications. One of the most prominent examples is the no-lateral-slip condition for wheeled vehicles, which constitutes a kinematic constraint that cannot be integrated into an equivalent position-level constraint and is therefore representative of nonholonomic kinematic systems.

Non-integrable constraints may, however, also arise on the acceleration level. Such constraints typically do not originate directly from physical contact, but rather emerge as a mathematical description of underactuated mechanical systems that do not possess potential forces in the directions of the unactuated degrees of freedom. Examples include surface vessels and underactuated underwater vehicles as well as underactuated manipulators, see, e.g. [5, 3, 4]. Similar to systems subject to conventional nonholonomic constraints at the velocity level, this class of systems poses significant challenges for setpoint stabilization. In particular, controllability is lost when the system dynamics is linearized around a desired setpoint, and no smooth, time-invariant feedback law exists that asymptotically stabilizes the system [8]. This naturally raises the question of how suitable setpoint controllers can be designed for this class of systems.

The present talk addresses this question by investigating how recent results [7] on model predictive control (MPC) for systems subject to nonholonomic kinematic constraints can be extended to systems with non-integrable constraints on the acceleration level, also referred to as second-order nonholonomic constraints. A key ingredient of the geometry-conforming MPC approach proposed therein is the derivation of a homogeneous approximation of the system dynamics, which can subsequently be used to construct a tailored mixed-exponents stage cost following [2]. Initial work in this direction for the system class under consideration yielded promising results [6], but relied on mechanical insight into the particular system under consideration. The focus here is therefore on whether, in the more general setting, an appropriate set of coordinates can be selected systematically to derive a homogeneous approximation of the dynamics, analogous to the use of privileged coordinates for conventional nonholonomic kinematics [1]. The resulting methodological questions are illustrated using a representative mechanical system subject to second-order nonholonomic constraints, e.g., an omnidirectional mobile robot equipped with a mounted unactuated arm.

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