

Systematic SQNR Loss in Continuous-Time Extended-Counting Incremental Delta-Sigma ADCs

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Abstract—Continuous-time (CT) incremental delta-sigma (I-DS) analog-to-digital converters (ADCs) have been proposed for Nyquist-rate conversion with a CT input. The use of extended counting (EC) in I-DS ADCs is a well-established approach to increase resolution, and its extension to CT modulators has been explored. However, the resulting performance improvements fall short of the theoretical expectations, unlike in discrete-time (DT) EC I-DS ADCs, which motivates further investigation. This paper provides a detailed analysis and reveals a previously unreported systematic signal-to-quantization-noise ratio (SQNR) loss in CT EC I-DS ADCs compared to their DT counterparts.

Index Terms—Incremental Delta Sigma, Continuous-Time, Extended Counting, Extended Range

I. INTRODUCTION

Delta-Sigma ($\Delta\Sigma$) ADCs are one of the most favorable ADC architectures for high-resolution applications. However, they retain memory from one sample to the next, which makes them unsuitable for true Nyquist ADC applications such as multiplexing or single-shot conversion. The I-DS ADC is a suitable alternative, as it periodically resets all filters and allows true sample-to-sample Nyquist conversion [1].

I-DS ADCs show worse SQNR performance compared to their free-running counterparts, which is why architectures for improved resolution have been proposed. A well-known technique is a two-step conversion referred to in the literature as extended counting (EC) or extended range (ER) [2]. Thereby, the quantization error after M conversion cycles is reproduced at the output of the last integrator of the I-DS modulator (I-DSM), which can be further quantized in a second conversion step and subtracted from the overall digital output to further increase the overall I-DS ADC SQNR.

Delta-sigma modulators (DSMs) can be realized both with CT and DT loop filters, whereas the CT implementation comes with the advantage of implicit signal filtering, reduced dynamic requirements of the loop filter, and a CT (e.g. resistive) input with easier drivability. CT I-DS ADCs have been proposed [3], and their extension to CT EC ADCs is also known.

However, recently published CT EC designs [4]–[7] fail to obtain a significant performance improvement from the two-step operation, which is in contrast to their DT counterparts [8]–[11]. This motivates a closer investigation, and surprisingly one can find a systematic performance drop between DT and CT EC ADCs, as exemplarily illustrated in Fig. 1.

In consequence, this paper presents a detailed analysis of the operation of CT EC I-DS ADCs, highlights the differences from the DT architectures, and derives the reason for the fundamental SQNR loss in CT EC I-DS ADCs.

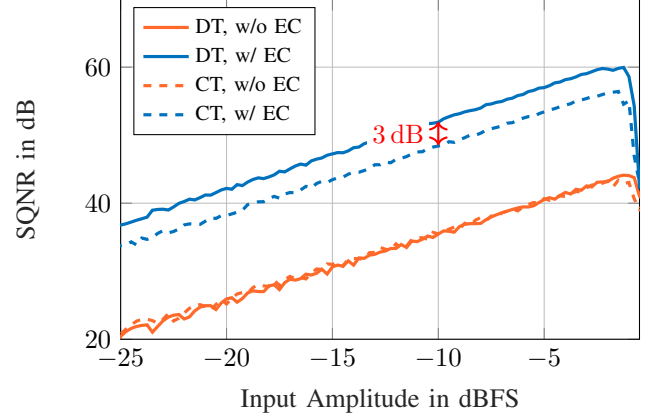


Fig. 1. Simulated SQNR of a second-order I-DS ADC with a CoI2 digital reconstruction filter and both a DT and CT loop filter, with and without EC. An OSR of 8 is used together with a 3-bit quantizer in the I-DSM and for the EC operation. A systematic 3 dB difference is observed.

II. DERIVATION OF THE SQNR LOSS IN CT EC I- $\Delta\Sigma$ ADCs

The state of the art (SoA) mainly implements first- to third-order architectures for I-DS ADCs because of the severely increasing noise-penalty factor for higher-order implementations [12]. Moreover, to allow EC operation, the digital reconstruction filter has to match the analog loop filter. For the commonly employed digital chain-of-integrators (CoI_D) reconstruction filter of N^{th} order, this necessitates a cascade-of-integrators with distributed feedforward (CIFF) loop filter. Thus, we limit our investigations to these architectures in the following, and an exemplary second-order EC I-DS ADC is shown in Fig. 2. EC architectures rely on the conversion of the quantization error present at the last integrator output at the final M^{th} clock cycle. Thus, the signal content at this node has to be minimized toward the end of a conversion. If not, the ISG in Fig. 2, which is used to amplify the residue before the requantization, cannot reach its theoretical maximum without overloading the EC quantizer (QTZ). The CIFF loop filter, including the low-distortion path from the input to the quantizer, ensures this by minimizing the signal content within the loop filter [13] [1].

A. DT Equivalent Filters

In the following, the DT equivalent description of the CT system is analyzed. This is done by applying a zero-order-hold (ZoH) transform to the CT system. For the noise transfer-function (NTF), an impulse-invariant transform (IIT) is always valid, as both the feedback signal and the

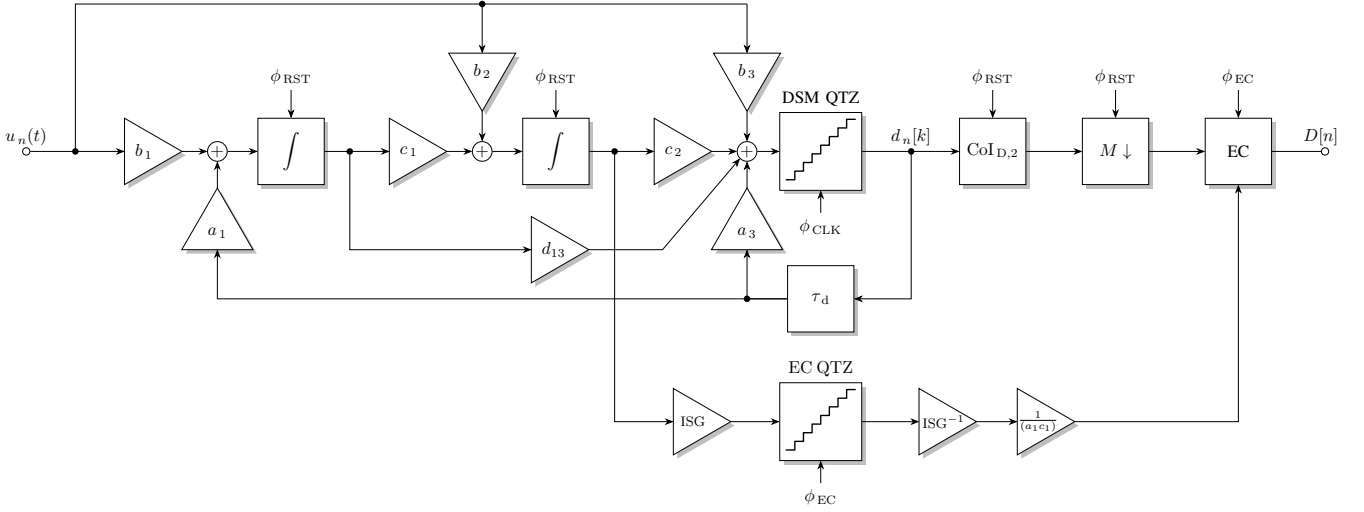


Fig. 2. Architecture of a second-order CT I-DS ADC with a residual-gain amplifier that amplifies the quantization error by ISG based on the fullscale of the requantization ADC and the resolution of the DSM quantizer. To reduce the signal content in the loop-filter, a low-distortion input feed-in path is added, weighted by b_3 .

output from the loop filter are sampled. The IIT can be replaced by a ZoH transform if a non-return-to-zero (NRZ) feedback digital-to-analog converter (DAC) pulse is assumed for simplicity. For the signal input, the argument of [13] can be applied: right after every reset, the I-DS modulator output shows the step response of the (equivalent) DT signal transfer-function (STF) weighted by the (constantly assumed) input signal. Thus, by assuming a constant input signal over one clock period and an output signal that is sampled with T_s , the ZoH approximation also holds for the STF.

With the low-distortion input feed-in path weighted by the coefficient b_{N+1} (b_3 in Fig. 2), the STF of the DSM yields $\text{STF}(z) = L(z) + b_{(N+1)}/L(z) + 1$, where $L(z)$ is the DT equivalent transfer function of the loop filter of an N^{th} -order DSM. The NTF is similarly given by $\text{NTF}(z) = 1/L(z) + 1$. The output of the last integrator $x_N[k]$ can be expressed in the $\mathcal{Z}$ -domain by

$$X_N(z) = \text{CoI}_{A,N}(z) \cdot [U(z) \cdot (1 - \text{STF}(z)) - E(z) \cdot \text{NTF}(z)], \quad (1)$$

where $U(z)$ is the input signal of the modulator DSM and $\text{CoI}_{A,N}(z)$ is the equivalent DT transfer function of an N^{th} -order analog chain-of-integrators (CoI_A). As can be seen from (1), the output of the last integrator does not contain any signal content as long as the equivalent DT STF is unity. It should be noted that this statement holds true even with non-zero excess loop-delay (ELD), since the DT equivalent STF can be restored by adding additional input feed-in paths (b_2 in Fig. 2), while the DT equivalent NTF can be restored by the direct feedback path (a_3 in Fig. 2), as shown in [14].

B. Second-order CIFF Example

Next, we exercise the derivation on the second-order example in Fig. 2; however, the same analysis can be carried out for other orders, as will be discussed in Section III. It

is furthermore assumed that the degrees of freedom in the second-order architecture are limited to the first integrator scaling coefficient c_1 and the feed-forward scaling coefficient d_{13} in Fig. 2, while $a_3 = b_2 = 0$ and all other coefficients are unity. Then, the transfer function of the DT CoI_A is

$$\text{CoI}_{A,2,\text{DT}}(z) = \frac{c_{1,\text{DT}} \cdot z^{-2}}{(1 - z^{-1})^2} \quad (2)$$

and the equivalent DT transfer function of the CT CoI_A becomes

$$\text{CoI}_{A,2,\text{CT}}(z) = \frac{c_{1,\text{CT}} \cdot 0.5(z^{-1} + z^{-2})}{(1 - z^{-1})^2}, \quad (3)$$

which follows from the ZoH approximation of the CT integrator transfer function $c_{1,\text{CT}}/s^2$. Based on (1), the transfer function of the quantization error to the integrator output can be rewritten, together with (2) and (3), as

$$\frac{X_{2,\text{DT}}(z)}{E(z)} = \frac{-c_{1,\text{DT}}}{z^2 + z\alpha + \beta} \quad (4)$$

$$\alpha = d_{13,\text{DT}} - 2 \text{ and } \beta = c_{1,\text{DT}} - d_{13,\text{DT}} + 1$$

for the discrete-time architecture and as

$$\frac{X_{2,\text{CT}}(z)}{E(z)} = \frac{-c_{1,\text{CT}} \cdot 0.5(z + 1)}{z^2 + z\alpha + \beta} \quad (5)$$

$$\alpha = \frac{c_{1,\text{CT}}}{2} + d_{13,\text{CT}} - 2 \text{ and } \beta = \frac{c_{1,\text{CT}}}{2} - d_{13,\text{CT}} + 1$$

for the continuous-time one.

For simplicity, an ideal scaling of the I-DSM is considered next, which yields an unscaled $\text{NTF}(z) = (1 - z^{-1})^2$. This is achieved with $c_{1,\text{CT/DT}} = 1$, $d_{13,\text{DT}} = 2$, and $d_{13,\text{CT}} = 1.5$ [15]. With that, the transfer function from the quantizer to the last integrator output becomes

$$\frac{X_{2,\text{DT}}(z)}{E(z)} = -z^{-2} \quad (6)$$

for the DT, and

$$\frac{X_{2,\text{CT}}(z)}{E(z)} = -0.5(z^{-2} + z^{-1}) \quad (7)$$

for the CT implementation.

C. Probability Density of the Residue in CT and DT I-DSM

Next, we investigate the variance of the residue present at the last integrator output directly before the reset is triggered, which is the value taken for requantization in the EC QTZ. This is done both for the DT and CT I-DSM to assess the achievable SQNR improvement of EC. With the uniformly distributed quantization error $e[k]$ passing through the transfer functions in (6) and (7), the last integrator output will behave as a random variable $\mathbf{X}_N$ with a distinct distribution. For the DT second-order I-DSM, (6) shows that the distribution of $\mathbf{X}_2$ is still uniform, as it only experiences a delay. To determine the distribution of $\mathbf{X}_2$ in the CT case, (7) can be rewritten in the time domain by applying an inverse $\mathcal{Z}$ -transform:

$$x_2[k] = -0.5 \cdot (e[k-1] + e[k-2]). \quad (8)$$

The quantization error e is assumed to be uniformly distributed with $\mathbf{E} \sim \mathcal{U}(-\Delta/2, \Delta/2)$, which is valid if the quantizer is not overloaded. Δ denotes the analog value represented by one least-significant bit (LSB) of the quantizer. Since two consecutive quantization noise samples are statistically independent and identically distributed, $\mathbf{E}$ is considered i.i.d.. Thus, in the example of a second-order DSM, the distribution at the output of the second integrator can be derived by a convolution of two uniform probability density functions (PDFs) of $\mathbf{E}$:

$$\text{PDF}_{\mathbf{X}_2}(x_2) = 2 \cdot \text{PDF}_{e[k-1]+e[k-2]}(-2 \cdot x_2), \quad (9)$$

with

$$\text{PDF}_{e[k-1]+e[k-2]} = \text{PDF}\{e[k-1]\} * \text{PDF}\{e[k-2]\}. \quad (10)$$

This is illustrated in Fig. 3, where (9) together with (10) results in a triangular distribution in the CT case in contrast to the uniform distribution of $\mathbf{X}_2$ in the DT case.

D. Implications of the Residue PDF in CT EC I-DSMs

The different error PDFs have implications for the maximum achievable resolution improvement obtainable from the residue requantization inherent to all EC I-DSMs. It can be seen from Fig. 3 that not all quantization levels of the EC

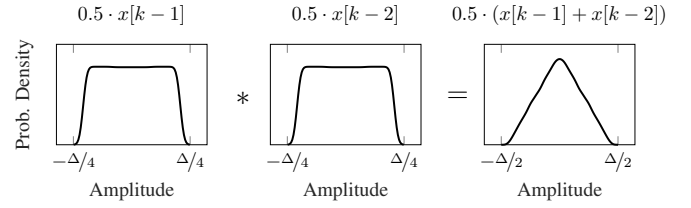


Fig. 3. Visualization of (9). The distribution of the output of the second integrator is defined by the two uniform distributions of two consecutive quantization noise samples.

QTZ are used with the same probability in the CT case, which is why CT architectures suffer from a certain loss in SQNR.

First, the additionally introduced quantization noise of the residual quantizer, denoted by EC QTZ in Fig. 2, is determined. Since the quantization noise is uniformly distributed in the range $[-\Delta_{\text{EC}}/2, \Delta_{\text{EC}}/2]$, the noise power calculates to $\Delta_{\text{EC}}^2/12$. It therefore depends on the fullscale voltage V_{FS} and the number of bits B_{EC} . Because the quantization error has to be amplified by an appropriate ISG based on the resolution B_{DSM} and the fullscale range of the DSM QTZ, the power of the amplified residue calculates to $\text{ISG}^2 \cdot \sigma_{\mathbf{X}_N}^2$. Then, the SQNR improvement due to the EC operation is

$$\Delta\text{SQNR}_{\text{EC}} = \text{ISG}^2 \cdot \frac{\sigma_{\mathbf{X}_N}^2}{\Delta_{\text{EC}}^2/12}. \quad (11)$$

By considering (7) and Fig. 3, it has been shown that the output of the last integrator of a second-order CT I-DSM is triangularly distributed ($\mathbf{X}_2 \sim \text{tri}(-\Delta_{\text{DSM}}/2, \Delta_{\text{DSM}}/2, 0)$). The variance of a symmetrical triangular distribution in the bounds $[-\Delta/2, \Delta/2]$ is known to be $\Delta^2/24$. If we assume equal ISGs and resolution for the DSM and EC QTZ, the achievable performance improvements due to employing EC in the DT and CT implementations compare by

$$\frac{\Delta\text{SQNR}|_{\text{CT}}}{\Delta\text{SQNR}|_{\text{DT}}} = \frac{\sigma_{\mathbf{X}_2}^2|_{\text{CT}}}{\sigma_{\mathbf{X}_2}^2|_{\text{DT}}} = \frac{1}{2} = -3 \text{ dB} \quad (12)$$

As can now be seen, the second-order CT EC I-DS ADC has a systematic loss in SQNR performance of 3 dB compared to the DT counterpart, as was observed in the simulation in Fig. 1, and to the authors' knowledge, this has not yet been reported in the literature.

III. FIRST- AND THIRD-ORDER ARCHITECTURES

The shown analysis can be extended to both the first- and third-order architectures. For the first-order system, due to the equivalent DT CoI_D transfer function yielding $\text{CoI}_{D,1,\text{CT}}(z) = c_{1,\text{CT}} \cdot z^{-1}/(1 - z^{-1})$, the ZoH approximation of the CT loop filter is exactly the same. Thus no systematic difference in performance will be observed between the CT and DT systems.

For a third-order loop filter, extended from the one shown in Fig. 2, the same approach as in (4) and (5) can be used. Using again an unscaled $\text{NTF}(z) = (1 - z^{-1})^3$, this is

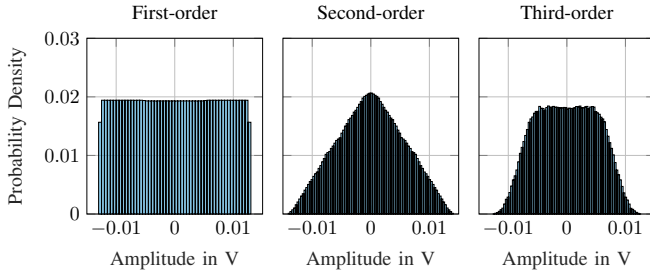


Fig. 4. Comparison of simulated histograms for the analysed I- $\Delta\Sigma$ modulators from first to third order. A 5 bit resolution has been assumed both for the DSM and EC quantizer, with a fullscale voltage of 0.8 V.

achieved by using $d_{14,DT} = 3$ and $d_{24,DT} = 3$ for a DT DSM, and $d_{14,CT} = 11/6$ and $d_{24,CT} = 2$ for the CT DSM. This yields the following transfer functions from the quantizer to the third integrator output:

$$\frac{X_{3,DT}(z)}{E(z)} = -z^{-3}, \quad (13)$$

and

$$\frac{X_{3,CT}(z)}{E(z)} = -1/6 \cdot z^{-3} - 2/3 \cdot z^{-2} - 1/6 \cdot z^{-1}. \quad (14)$$

As before, the quantization error PDF does not change with the simply delaying DT transfer function, while the CT transfer function again yields a modified distribution. For comparison, Fig. 4 illustrates the distribution of the residue seen at the output of the last integrator in the first-, second- and third-order I-DSM. The variance of the residue distribution in the third-order CT I-DSM is derived by combining (14) and the variance of the uniform distributions:

$$\begin{aligned} \sigma_{\mathbf{x}_3}^2 &= 2 \cdot (1/6)^2 \cdot \Delta_{EC/12}^2 + (2/3)^2 \cdot \Delta_{EC/12}^2 \\ &= \Delta_{EC/24}^2. \end{aligned} \quad (15)$$

Thus the achievable SQNR improvement in the CT EC ADC is again reduced by $\sigma_{\mathbf{x}_3|CT}^2 / \sigma_{\mathbf{x}_3|DT}^2 = -3$ dB compared to the DT architecture.

Even though Fig. 4 might suggest better EC performance for the third-order case, since the quantizer levels around the mid-code distribute more uniformly compared to the triangular distribution of the second-order case, it does not utilize the outer quantizer levels as often as the second-order modulator, which finally explains the same performance drop.

IV. CONCLUSION

When comparing the ideal performance of CT I-DS ADCs employing extended counting with that of their DT counterparts, one observes a systematic mismatch in ideally achievable SQNR. Even though comparatively small, the effect is surprising, as commonly DT-CT transformed DSMs show identical performance, and it has not been described in

the literature so far. By deriving the SQNR improvement of EC CT I-DS modulators, this paper identifies the underlying root cause, which is found in the probability density of the analog residue used for requantization by extended counting. The derivation is shown exemplarily for a second-order modulator and is later extended to first- and third-order modulators, highlighting their differences in the CT EC SQNR loss.

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