1 Theory, assumed state of knowledge

Lecture (CE II/Script): Chaper 2; section 2.3, add-on

2 What is shown?

This demo deals with wav-files, and it is charcterized by three steps:

- **Step 1:**
  A given audio signal $s(t)$ without noise (i.e. a wav-file) was taken and a noise sample function $n_1(t)$ was added. The sum $g_1(t)$ was saved as a new wav-file. So $g_1(t)$ is given by

  $$g_1(t) = s(t) + n_1(t).$$

  This signal $g_1(t)$ can be interpreted as a received signal resulting from a transmission of $s(t)$ over a noisy transmission channel. The signal-to-noise ratio (SNR) was set to -16 dB, so it is not possible to hear the audio signal $s(t)$ in the sum $g_1(t)$.

- **Step 2:**
  Another noise sample function $n_2(t)$ was added to the same signal $s(t)$:

  $$g_2(t) = s(t) + n_2(t)$$

  Basically this is the same as in step 1, but $g_2(t)$ can be interpreted as reception of a repeated transmission of $s(t)$ over the same channel at a later time. Adding $g_1(t)$ and $g_2(t)$ gives the sum $gs_2(t)$:

  $$gs_2(t) = 2s(t) + ns_2(t)$$
  $$ns_2(t) = n_1(t) + n_2(t)$$

  In $gs_2(t)$ the signal $s(t)$ is multiplied by the factor 2. The two noise sample functions do not add algebraically. With the assumption that $n_1(t)$ and $n_2(t)$ are *uncorrelated* – because of the different transmission times this is a realistic assumption – the *noise powers* are added. For a stationary noise process and sufficienty long duration of $s(t)$ (and hence $n_i(t)$) the noise power is doubled, while the signal power is increased by a factor of 4. A *gain in SNR* of 3 dB results.

- **Step N**:
  Continuation gives in step N:

  $$gs_N(t) = \sum_{i=1}^{N} g_i(t) = N\,s(t) + \sum_{i=1}^{N} n_i(t)$$

  The SNR gain $gs_N(t)$is now $10\,log(N)$ dB.

Two set of signals are given. Each set consists of $s(t)$, $gs_N(t)$ for some N. The first set is based on the wav-file "mrc00fruehl.wav" which is $s(t)$. "mrc01fruehl0001.wav" is $g_1(t)$ and for each further $gs_N(t)$ the number N is appended to the file name. The second set uses a rect signal as a basis.

3 What is demonstrated?

By repeated a transmission of a signal $s(t)$ over a noisy channel and a proper superposition at the receiving side the SNR can be improved arbitrarily. For the demo the audio signal SNR could be improved by about 30 dB with 1024 transmissions. A precondition is that the signal contributions in $gs_N(t)$ are added properly in amplitude, i.e. proper synchronization was assumed at the receiving side.

The principle explained here is a special case of a technique called *Maximum Ratio Combining* (MRC). MRC always maximizes the SNR, but in general the signal contributions $g_i(t)$ in the sum are weighted by factors depending on i. For the demo these weighting factors are the attenuation factors of the channel for those times when $g_i(t)$ was received. If a signal $g_i(t)$ is attenuated more than the others, its contribution to the sum should be smaller. The same principle is valid also for the *matched filter*. Here values of a received signal are weighted for each time instant and integrated (which corresponds to the sum). The weighting factors in this case are identical with values of the transmitted signal. In a similar way multiple receiving antennas can be used to *combine* (add) the individual antenna signals. If the noise contributions in the signals $g_i(t)$ are uncorrelated, also a gain $10\,log(N)$ dB results for N receiving antennas.