1 Theory, assumed state of knowledge

Lectures: AWIMUC, CE II; Multiplex, OFDM

2 What is shown?

The background shows the block diagramm of an OFDM transmission, which builds up the basis for this demonstration. It is designed for a transmission with two possible modulation alphabets: BPSK (2-PSK) or QPSK (4-PSK). The following signals can be seen in 10 separate small windows (from upper left to lower left):

- **Window 1**:

  BPSK: transmission symbol vector $\underline{x}(k), \quad x_i(k) \in A_x = \{1, -1\}$

  QPSK: transmission symbol alphabet $A_x = \{1+j, -1+j, -1-j, 1-j\}$

- **Window 2:** real part of $\underline{Y}(k)$

  $\underline{Y}(k)$ results from $\underline{x}(k)$ by insertion of zeros (*zero padding*) for non-used subcarriers (block "x-Y mapping" in the picture) and can be seen as vector in the frequency domain, which has to be transmitted. The non-used subcarriers can be found in the spectrum of the transmission signal on the left and right edge of the frequency band used for transmission. This supports the bandlimitation which is needed in real applocations, the remaining si (sinc) functions hve to be small enough at the edges.

- **Window 3**: real part of $\underline{y}_{ext}(k)$

  $\underline{y}_{ext}(k)$ results from $\underline{Y}(k)$ by applying an inverse discrete Fourier transform (IDFT) and insertion of a cyclic repetition (*cyclic prefix*, *periodic extension* in the picture) at the beginning. $\underline{y}_{ext}(k)$ can be regarded as vector in time domain which must be transmitted. The cyclic repetiton can be seen as yellow part. The duration of this part is the *guard time* T_G. It should be at least as long as the duration of the longest channel impulse response $h(t)$ (see window 5).

- **Window 4**: real part of $s_T(i\Delta t)$ and $s_T(t)$ respectively

  $s_T(i\Delta t) = y_{ext,i}(k)$ are the sampling values of the transmission signal in the equivalent low pass domain for the kth symbol interval. They are identical with the components of the vector $\underline{y}_{ext}(k)$ from window 3. The colours are adopted from window 2. The continuous-time transmission signal $s_T(t)$ in the equivalent low pass domain, which can be seen as red curve in window 4, belongs to sample values $s_T(i\Delta t)$. To emphasize the connection between the previous symbol interval $(k-1)$ and the future symbol interval $(k+1)$, two samples are added at the left and at the right. The left one (green) is the last of the previous symbol interval and the right one is the first of the following symbol interval (yellow).

- **Window 5**: $h_T(i\Delta t)$, sampling values of the channel impulse response in the equivalent low pass domain

  The corresponding continuous-time impulse response is $h_T(t) = \sum\limits_{i=0}^{N_h-1} h_T(i\Delta t)\, \delta_T(t - i\Delta t)$.

  Assumed is a discrete multipath propagation in the Δt raster with N_h paths. The values $h_T(i\Delta t)$ can be changed, also their number. When starting the demonstration, we have $N_h = 2$ and $\underline{h}_T = [0.75\ 0.5]$. The components of the vector $\underline{h}_T$ are identical with the values of $h_T(i\Delta t)$. $\delta_T(t)$ is the corresponding dirac impulse in the equivalent low pass domain, i. e. $\delta_T(t) = 2 \cdot 2f_g\, si(\pi 2 f_g t)$.

- **Window 6**: Absolut value of the transfer function $H_T(f)$ of the equivalent low pass channel

  The f-axis of $|H_T(f)|$ is in the FFT window format: On the left of the center are positive frequencies, on the right negative ones. This implies for the subcarriers: on the left 0, Δf, $2\Delta f$, ... and on the right ..., $-2\Delta f$, $-\Delta f$. The position of all subcarriers is also shown. It has to be noted, that the subcarriers in the center – as is common for OFDM – are not used for transmission (see "zero padding", window 2).

- **Window 7**: real part of $g_T(i\Delta t)$ and $g_T(t)$ respectively

  The displayed received signals in the equivalent low pass domain correspond to the signals in window 4, now at the output of the channel. Since after the channel the position of the T_S and T_G intervals is not known a priori, the colour green is used. If the guard time T_G is longer than the length of the channel impulse response $h(t)$, a full period – corresponding to the stationary part – results in the received signal. This single period has to be further processed with the discrete Fourier transform (DFT).

- **Window 8**: real part of $\underline{\tilde{y}}_{ext}(k)$

  $\underline{\tilde{y}}_{ext}(k)$ is the estimated value of the vector $\underline{y}_{ext}(k)$ (see window 3). The components are identical with the samples of the received signal ($g_T(i\Delta t)$ in window 7). The position of the window containing the samples used by the DFT (respectively the FFT) is also shown. The remaining part with duration T_G is – according to the OFDM principle – not used for reception. The adjustment of the right position of the window is task of the synchronisation, which can be changed manually with button "FFT window".

- **Window 9**: $\underline{\tilde{Y}}(k)$ (BPSK) and $\underline{\tilde{x}}_0(k)$ (QPSK) respectively

  Whereas for BPSK only the (relevant) real part is shown, the received values of all symbols for QPSK are displayed in a cummulative way. $\underline{\tilde{Y}}(k)$ is the estimated value of $\underline{Y}(k)$ (see window 2) and $\underline{\tilde{x}}_0(k)$ is the estimated value of $\underline{x}(k)$ after removing the unused components of $\underline{\tilde{Y}}(k)$ (Y-x in the picture, next to last block). More about the topic of equalization see below.

- **Window 10**: detected symbol vector $\underline{\hat{x}}(k)$ (BPSK) and estimated value $\underline{\tilde{x}}(k)$ for (QPSK) respectively

  For QPSK the estimated values for $\underline{\tilde{x}}(k)$ are displayed cumulative – in the same way as in window 9 $\underline{\tilde{x}}_0(k)$. This means that the values for all vector components and symbol intervals are displayed.

Parameter adjustments:

- channel impulse response $h(t)$: start of the routine OFDM4ModifyChannel.m
- positon of the FFT sequence (synchonisation): button "FFT window" (left/right mouse button)
- guard time T_G in Δt-steps: button "Guard time" (left/right mouse button)
- used subcarriers: button "Sel subcarriers"

The selected subcarriers can be changed with the button "Sel subcarriers" . One click with the right mouse button opens a selection window, in which the individual symbols in the transmit symbol vector can be set to fixed values: 0 or 1 for BPSK and 0 or $1+j$ for QPSK. The value 0 means that the corresponding subcarrier is not used. The selection can be shifted cyclically with the left mouse button.

If the guard time is too small for OFDM, *interblock interference* occurs, i. e. there is a mutual influence of the signals in successive symbol intervals. The interblock interference is the generalization of the *intersymbol interference*, which can occur in case of a transmission of a single (scalar) sequence $x(k)$ of transmit symbols. To get an impression of the interblock interference, the matrix $R(k)$ of the *time discrete channel on symbol basis* can be displayed for the current channel and the current guard time. For this purpose the routine OFDM3ShowRmatrix.m must be started. Then the sequence $R(-2), R(1), R(0), R(1), R(2)$ can be seen. The $R(k)$ window can be removed with the menu "Delete Scopes".

It has to be mentioned, that for the OFDM transmission in this demonstration no channel coding is used, see remarks below.

3 What is demonstrated?

To understand this demo, a basic knowledge of OFDM is needed. The aim is then to deepen this knowledge with an example. The number of subcarriers used ($M = 6$) as well as the length of the FFT vector ($N = 8$) are – compared to real OFDM systems – small. For example the WLAN standard IEEE 802.11a uses $M = 52$ subcarriers out of $N = 64$. Therefore the zero padding part consists of 12 subcarriers. This means that 6 unused subcarriers on the left and on the right edge of the frequency band used for transmission ensure that neighbouring frequency bands are not disturbed. For digital television according to the DVBT standard the number of subcarriers is larger: up to $M = 6817$ are used from a total of $N = 8192$.

Despite of all those differences in the number of the subcarriers, the OFDM principle is always the same: For the digital transmission eigenfunctions of the channel are used. Because the channel is modeled as a linear time-invariant system (LTI system), the eigenfunctions are complex exponentials ($exp(j2\pi f_i t)$). More specific, a cut-out of these eigenfunctions is taken as basic waveforms $u_i(t)$ for the OFDM transmission:

$$u_i(t) = rect(\frac{t}{T_S + T_G}) \cdot exp(j2\pi f_i t), \quad \Delta f = f_i - f_{i+1} = \frac{1}{T_S}$$

T_S is the symbol duration. With an adequate guard time T_G a basic property of eigenfunctions can be used: The orthogonality of the $u_i(t)$ can be maintained if a proper section of duration T_S in the received signal is taken (a *steady state* part). Then, a simple model for the transmission of symbol vectors $\underline{x}(k)$ results:

$$\underline{\tilde{x}}_0(k) = R(0)\underline{x}(k) + \underline{\tilde{n}}(k).$$

$\underline{\tilde{x}}_0(k)$ is the estimation for the transmit symbol vector $\underline{x}(k)$ at the receiving side, $R(0)$ the already mentioned channel matrix on symbol basis for $k = 0$, and $\underline{\tilde{n}}(k)$ is a sample function of a (coloured) vector noise process. $R(k)$ is zero for $k \neq 0$ and for $R(0)$ the following results:

$$R(0) = \begin{bmatrix} |H_T(f_1)|^2 & & & \\ & |H_T(f_2)|^2 & & \\ & & \cdots & \\ & & & |H_T(f_N)|^2 \end{bmatrix}.$$

$H_T(f_i)$ is here the transfer function of the channel in the equivalent low pass domain at the subcarrier frequency f_i. For OFDM in real applications the correlation filter for the channel – which is contained in $R(k)$ – is normally left out. Then, instead of $R(0)$ in our model, a matrix has to taken, which has only $H_T(f_i)$ entries on its main diagonal. Both models – i. e. with $H_T(f_i)$ or $|H_T(f_i)|^2$ on the main diagonal – are theoretically equivalent. For common transmit symbol alphabets a further processing of the $\underline{\tilde{x}}_0(k)$ vectors is necessary in any case. Therefore in real OFDM the missing multiplication with $H_T^*(f_i)$ in the components of $\underline{\tilde{x}}_0(k)$ is included in this further processing. “Further processing” means in real OFDM systems – as well as in this demo – that a *zero forcing equalizer* (ZF equalizer) is used. It divides the components of $\underline{\tilde{x}}_0(k)$ by $H_T(f_i)$, so the $H_T(f_i)$ values must be known in the receiver as good as possible. For this purpose pilot subcarriers (with pilot symbols) are usually transmitted. They are known in the receiver and are used for *channel estimation*. We do not consider this topic here, it could be a separate demo. Here we take the known $H_T(f)$ directly for use in the receiver.

The disadvantage of the ZF equalizer becomes obvious, if the $H_T(f_i)$ values are small. An amplified influence of the noise occurs. This can be demonstrated well, when using a special channel impulse response for QPSK: If there are $H_T(f_i)$ with value 0, it is not allowed to divide, so in the demo receiver a threshold is applied.

Beside deepening the basic understanding of OFDM with this demo, it is also possible to find out what happens in case of an insufficient guard time. If the weights of $h_T(t)$ are equal to [1 1], it can be clearly observed in window 6 ($H_T(f)$) that the subcarrier in the middle is transmitted with $H_T(f_4) = 0$. The $R(k)$ matrix (start OFDM3ShowRmatrix.m) shows the corresponding 0 as a white entry. The matrices for $k \neq 0$ are zero matrices. If the guard time is set to 0 (key “Guard Time”), one can observe that crosstalk between the vector components occurs. The matrices on the left and on the right of $R(0)$ in particular are no longer zero. Making $h_T(t)$ longer by insertion of zeros between both paths (e. g. [1 0 1]), the crosstalk increases. When using [1 0 0 0 1] it can be clearly observed that there are two effects:

- *intersymbol interference* on the subcarriers: corresponding entries on the main diagonals of R(-1), R(0), R(1)
- *intersubchannel interference*: all other entries in the matrices.

Entries in $R(k)$ for $k \neq 0$ are described general by the term *interblock interference*. When using general vector-valued transmisson techniques, interblock interference and intersubchannel interference can occur, which is a generel phenomenon caused by multipath propagation. A necessary countermeasure is to use adequate *vector detection* techniques. This means in case of an uncoded transmission to apply a *vector equalization*.

Remarks

As already mentioned, *channel coding* is not used in this demonstration. For the basic principle of OFDM this doesn’t play any role. But in every real OFDM transmission we will find it. This is denoted by the term COFDM (coded OFDM). Channel coding is needed to cope with the errors occuring on subcarriers with small values of the transfer function, i.e. $H_T(f_i)$, which are again caused by multipath propagation on the radio channel. The *frequency-selective* behaviour of a multipath propagation channel can be seen in the example with the [1 1] paths for $h_T(t)$. The symbols faded out by the channel have no effect if the channel code can correct the resulting errors. For this purpose the *code rate* r_c has to be small enough. In practice $r_c = \frac{1}{2}$ is often applied. If interblock/intersubchannel interference occurs, the conditions for OFDM are not fulfilled: The guard time is too small, as in the example with [1 0 0 0 1] and $T_G = 0$. To cope also with interblock/intersubchannel interference, appropriate vector detection schemes must be applied. While optimum schemes usually lead to unrealistic complexity in the receiver, so-called *turbo schemes* perform well with moderate complexity. Here an appropriate vector equalizer works together with a *softin-softout vector decoder* in an iteration loop. Taking into account the add on in complexity in the receiver, a guard time zero in the OFDM transmission can lead to better bit error rate performance than for a pure OFDM transmission with properly chosen guard time.