

## Speech about Dipoma Thesis

### Solving Non-Deterministic Planning Problems with Pattern Database Heuristics

(Anwendung von Pattern-Database-Heuristiken zum Lösen  
nichtdeterministischer Planungsprobleme)

Author: Pascal Bercher

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  - $pre(a) \subseteq Var$  and
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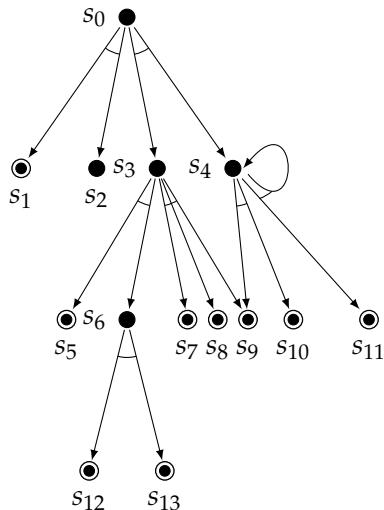
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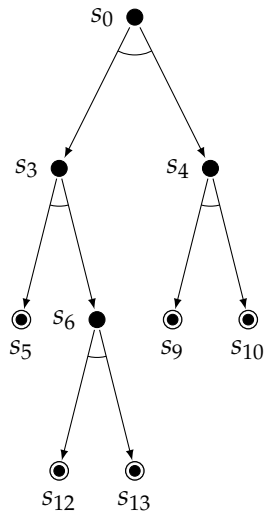
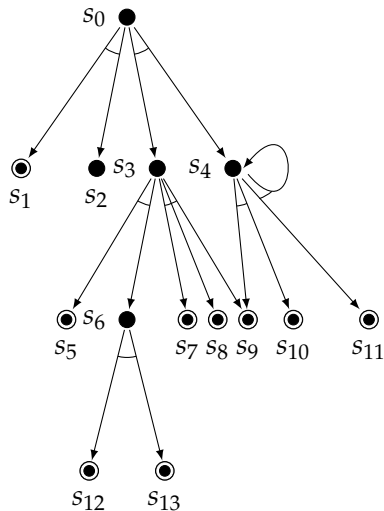
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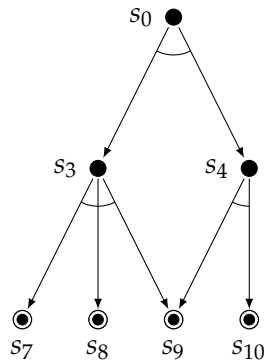
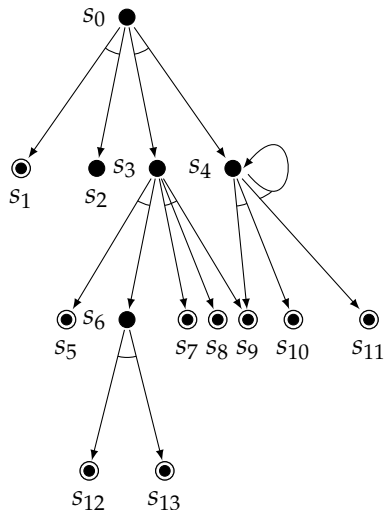
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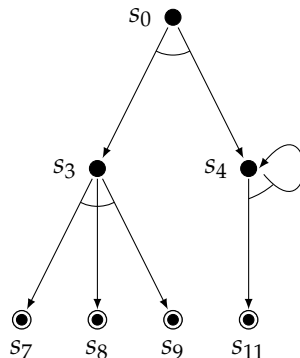
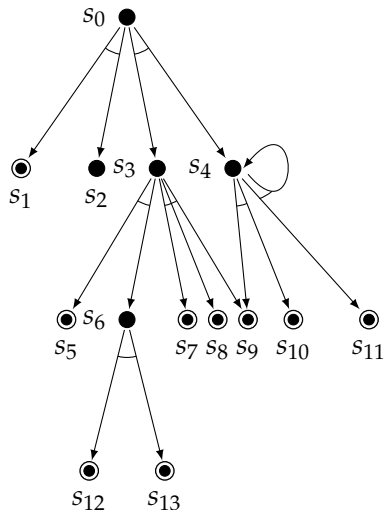


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## Forward step:

- Traverse graph using only marked connectors and find non-expanded leaf with maximal heuristic value (fail first).
- Expand this leaf.

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- Update cost value of nodes and update markings.
- Optimal cost value for  $v \in V$ :  $cost^*(v) := \min_{\mathcal{G}} height(\mathcal{G})$ ,  
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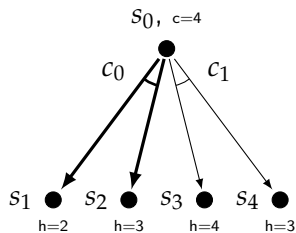
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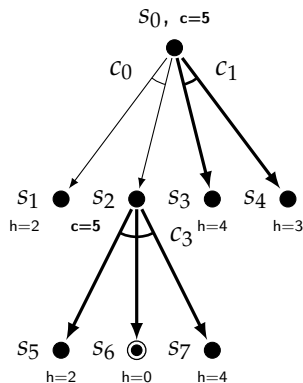
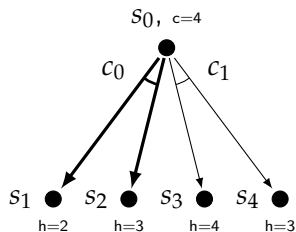
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- For each abstraction: Solve the problem completely.
- For each abstract state: Save its cost in Pattern Database.

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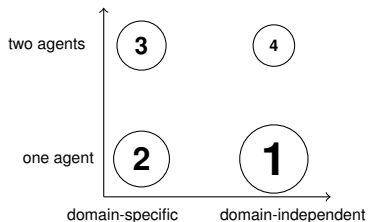
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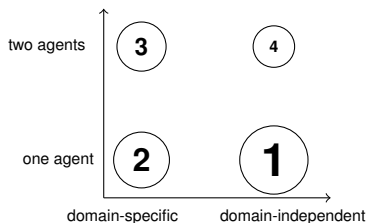
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The abstraction  $\mathcal{P}^i = (Var^i, A^i, s_0^i, G^i)$  is equal to the planning problem  $\mathcal{P}$ , restricted to the pattern  $P_i \subseteq Var$ .

It holds:

- $Var^i := P_i$ ,
- For  $s \in S$  is  $s^i := s \cap P_i$  and for  $var \subseteq Var$  is  $var^i := var \cap P_i$ , respectively.
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For a state  $s^i$  and  $s \in S$ , its heuristic  $h^i$  is

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Calculation process:

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Set of patterns  $P := \{P_1 \dots, P_m\}$ .

It holds:  $h_1(s) := \max_{i \in \{1, \dots, m\}} h^i(s^i)$  is admissible.

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**Answer:**

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The set  $P := \{P_1, \dots, P_k\}$  with  $k \in \mathbb{N}$  is additive, if for all states  $s \in S$  holds  $\sum_{i=1}^k h^i(s^i) \leq \text{cost}^*(s)$ .

Theorem, known from classical planning by Edelkamp (2001):

## Theorem:

If for all  $a \in A$  and for all patterns  $P_i \in P$  holds:

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The set  $P := \{P_1, \dots, P_k\}$  with  $k \in \mathbb{N}$  is additive, if for all states  $s \in S$  holds  $\sum_{i=1}^k h^i(s^i) \leq \text{cost}^*(s)$ .

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(Still) manually. Automation via local search in principle possible. Cf. Haslum et al. (2005,2007), Edelkamp (2006).



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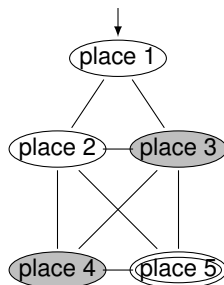
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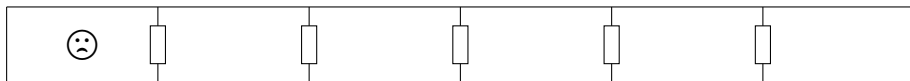
## Tireworld, problem description



## Actions:

- Use street (non-deterministically: flat tire).
- Take spare tire (if present).
- Change spare (if flat tire and spare present).

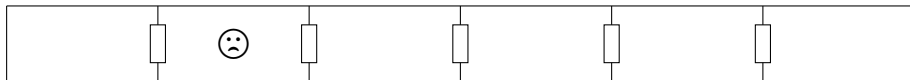
## Chain of Rooms, problem description



actions:

- Go into neighboring room, if door is open.
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- Open door.

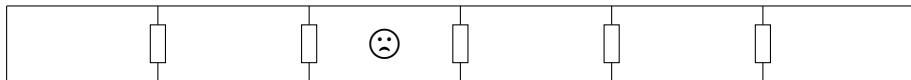
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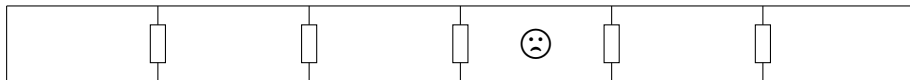


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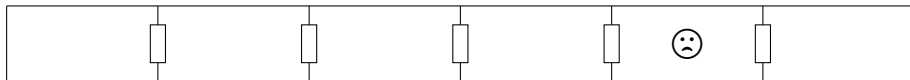
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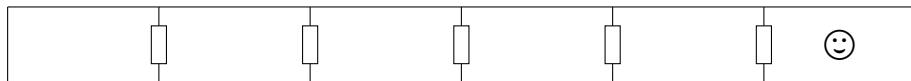
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## Coin Flip, problem description

### given:

- $n$  coins, all initially in a bag.
- Actions:
  - Flip coin, if not flipped before.
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### desired:

Strategy, that all coins show heads.

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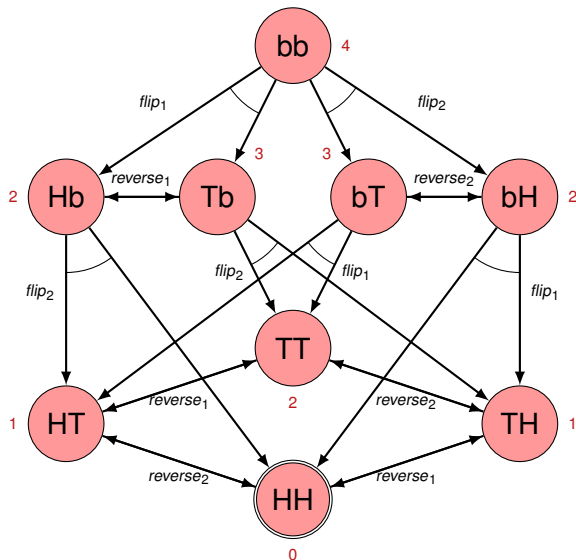
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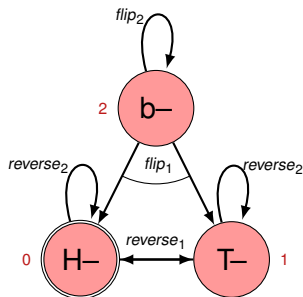
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# Coin Flip (two coins), example for additive patterns

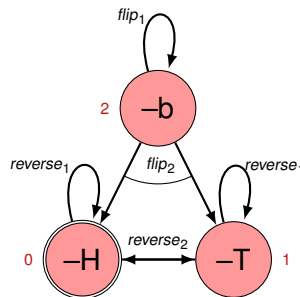






$$h^{P_1}(bb^{P_1}) = h^{P_1}(b-) = 2$$

## Coin Flip (two coins), example for additive patterns



$$h^{P_2}(bb^{P_2}) = h^{P_2}(-b) = 2 \quad \Rightarrow$$

$$h(bb) = h(bb^{P_1}) + h(bb^{P_2}) = h^{P_2}(b-) + h^{P_2}(-b) = 2 + 2 = 4$$

## Comparison between:

- AO\* algorithm and *no heuristik*,
- AO\* algorithm and *adversarial FF heuristic*,
- AO\* algorithm and *pattern database heuristic*,

## Comparison possible to:

- POND: Partially Observable Nondeterministic Planner (Bryce et al., 2006)
- MBP: Model Based Planner (Cimatti et al., 2003)
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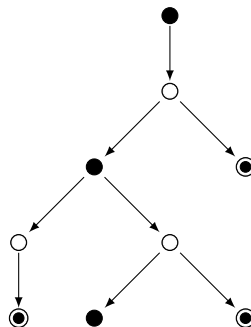
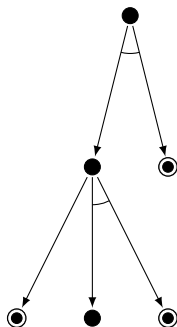
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(side note)

The planner uses explicit AND/OR nodes and edges instead of only one sort of nodes and connectors:



## Results: Tireworld

#places	uninformed			adv. FF heuristic			PDB heuristic				
	time	RAM	nodes	time	RAM	nodes	time pre	time search	time	RAM	nodes
4	0	0	85	0	0	62	0	0	0	0	46
6	0	0	84	0	0	47	2	0	2	8	37
8	0	1	153	0	0	43	1	0	1	6	43
10	0	1	293	0	0	96	2	0	2	7	67
20	0	4	504	0	1	126	2	0	2	8	128
40	93	2097	115541	6	117	6451	3	6	9	243	12827
60	2	196	9088	1	24	1086	2	0	2	41	1565
80	1	163	5924	0	26	915	1	0	2	30	746
100	17	745	20249	1	42	1101	1	0	2	50	1154
120	–	–	–	17	464	10739	1	1	3	81	1632
140	23	1584	33418	5	90	1849	2	0	2	66	1202
160	–	–	–	8	205	3914	2	0	3	72	1175
180	11	552	9599	2	62	1055	2	2	4	136	2239
200	41	1622	22175	5	130	1746	2	1	3	93	1081

– := out of memory (4 GB)

Ten patterns: Each pattern contains the goal place, the spare, the flat tire and seven random places.

### Results: Chain of Rooms

#rooms	uninformed			adv. FF heuristic			PDB heuristic				
	time	RAM	nodes	time	RAM	nodes	time pre	time search	time	RAM	nodes
4	0	0	38	0	0	30	0	0	0	0	30
6	0	0	91	0	0	62	0	0	0	0	57
8	0	1	170	0	0	109	0	0	0	0	81
10	0	1	272	0	1	169	0	0	0	1	147
20	0	6	1142	0	3	643	1	0	1	3	464
40	1	40	4682	8	20	2494	3	0	4	18	2043
60	5	121	10621	40	59	5543	9	1	10	52	4827
80	17	278	18962	122	131	9793	16	3	20	121	8804
100	43	533	29702	300	251	15244	30	9	39	230	13983
120	92	909	42841	609	443	21893	43	19	62	394	20367
140	162	1390	58382	1164	686	29743	63	37	101	617	27944
160	257	2090	76322	2054	994	38794	94	63	157	902	36723
180	468	2986	96661	3369	1457	49043	127	98	225	1283	46707
200	—	—	—	5855	1961	60493	164	154	319	1763	57884

- := out of memory (4 GB)

Patterns: Four neighboring rooms per pattern.

## Results: Coin Flip

#coins	uninformed			adv. FF heuristic			PDB heuristic				
	time	RAM	nodes	time	RAM	nodes	time pre	time search	time	RAM	nodes
4	0	1	349	0	1	313	0	0	0	0	69
6	0	11	4809	0	8	3374	0	0	0	0	161
8	18	145	58377	3	46	18641	0	0	0	1	293
10	1102	1542	588813	77	483	186221	0	0	0	1	465
20	–	–	–	–	–	–	0	0	0	7	1925
40	–	–	–	–	–	–	1	1	2	49	7845
60	–	–	–	–	–	–	3	6	9	161	17765
80	–	–	–	–	–	–	6	18	24	384	31685
100	–	–	–	–	–	–	12	45	57	714	49605
120	–	–	–	–	–	–	18	96	114	1176	71525
140	–	–	–	–	–	–	28	163	192	1784	97445
160	–	–	–	–	–	–	39	289	329	2589	127365
180	–	–	–	–	–	–	58	–	–	–	–
200	–	–	–	–	–	–	74	–	–	–	–

– := out of memory (4 GB)

Patterns: Two coins per pattern.



## Summary:

- Additivity criterion also applies to non-deterministic planning.
- Pattern database heuristics (therefore) can achieve good results (depending on decomposition).

## Outlook:

- Finding strong *cyclic* plans.
- Automated pattern selection.
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**Thanks for your attention!**



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