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Practical Tableau-based Reasoning in Description Logics

Reminder: Reasoning Problems in DLs

$\mathcal{ALC}$ Tableau with TBoxes

Two Essential Optimizations

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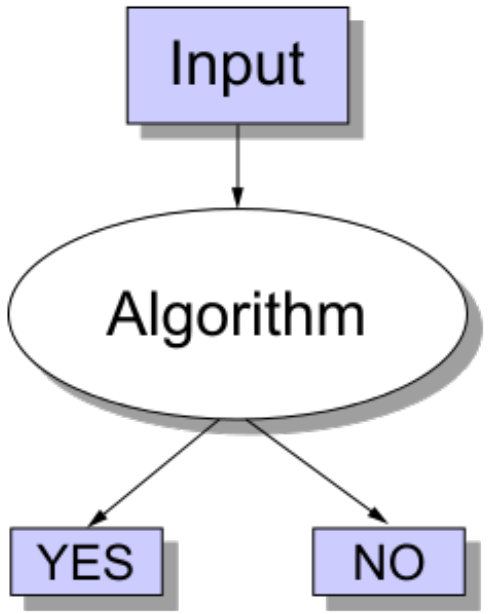
Decision Problem and Decision Procedure

Decision problem: a well-formulated yes/no question



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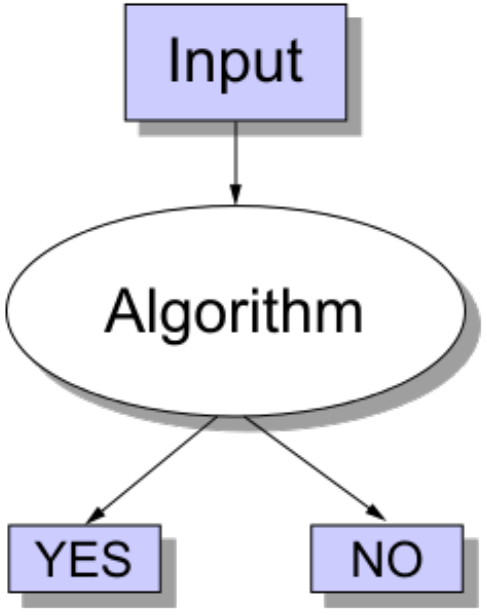

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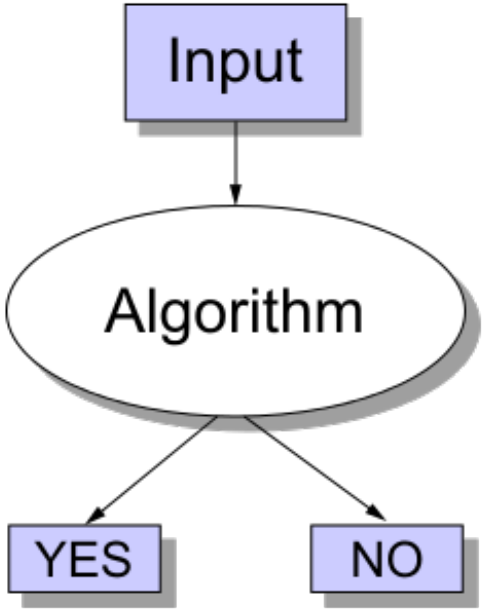
- Sound: if it answers yes, then the input is in J
- Complete: if the input is in J, then it answers yes
- Terminating: it returns an answer after a finite number of steps



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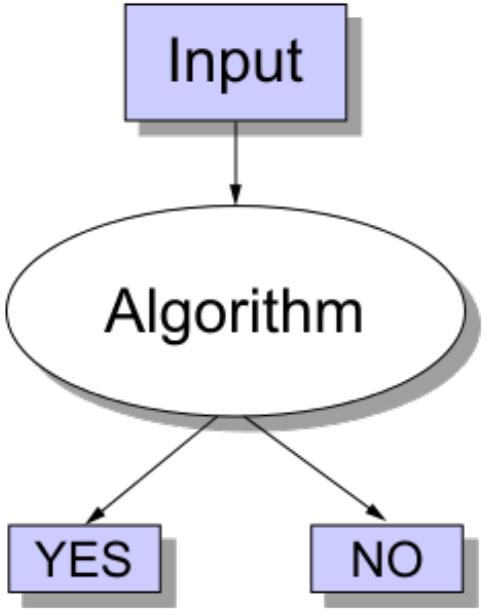

We only talk about decision problems in this course

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  (for all concept names A, B)
- realization $\mathcal{O} \models b : \mathsf{B}$?
  (for all concept names A, individual names b)

Question: do we need 4 different algorithms for these?

Consistency Suffices

Theorem: Let $\mathcal{O}$ be an ontology and a a fresh individual name:

1. C is satisfiable w.r.t. $\mathcal{O}$ iff $\mathcal{O} \cup \{a : C\}$ is consistent
2. $\mathcal{O}$ is coherent iff $\mathcal{O} \cup \{a : \mathsf{A}\}$ is consistent (for each concept name A)
3. $\mathcal{O} \models \mathsf{A} \sqsubseteq \mathsf{B}$ iff $\mathcal{O} \cup \{a : (\mathsf{A} \sqcap \neg \mathsf{B})\}$ is not consistent
4. $\mathcal{O} \models b : \mathsf{B}$ iff $\mathcal{O} \cup \{b : \neg \mathsf{B}\}$ is not consistent

Answer: a decision procedure to solve consistency decides all standard DL reasoning problems

Consistency for $\mathcal{ALC}$

The way we will do it:

- Concept satisfiability for $\mathcal{ALC}$ (no ABoxes or TBoxes)
- ABox consistency (very slight generalization)
- Ontology consistency (TBoxes enter the picture)

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We will work through examples

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This approach is different from syntactic proof calculi, e.g., natural deduction (Lecture 4)

$\mathcal{ALC}$ Concept Interpretations

A reminder from yesterday: what $\mathcal{ALC}$ interpretations look like

$C^{\mathcal{I}}$ can be represented as a labeled graph $\mathcal{G}_{C^{\mathcal{I}}} = \langle V, E \rangle$, where:

- V set of domain elements where $v_0 \in C^{\mathcal{I}}$
- $C \in L(v)$ if $v \in C^{\mathcal{I}}$
- (x, y) is a R-labeled edge if $(x, y) \in \mathsf{R}^{\mathcal{I}}$

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$\dot{\neg} C$ denotes $\mathrm{NNF}(\neg C)$

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Tableau algorithm:

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Graphs computed by the procedure are called completion graphs

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$\sqcap$-rule: if $C_1 \sqcap C_2 \in L(a)$ for some a and $\{C_1, C_2\} \not\subseteq L(a)$

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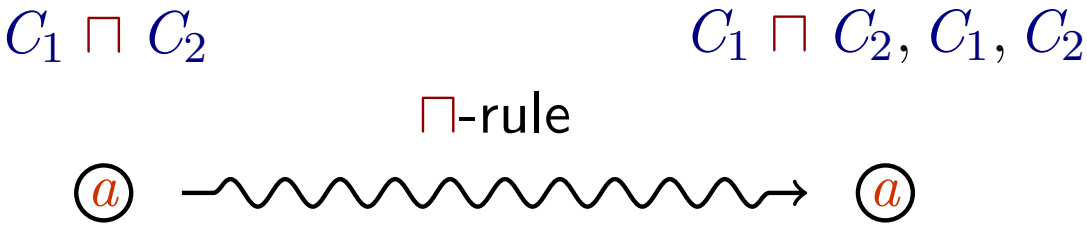

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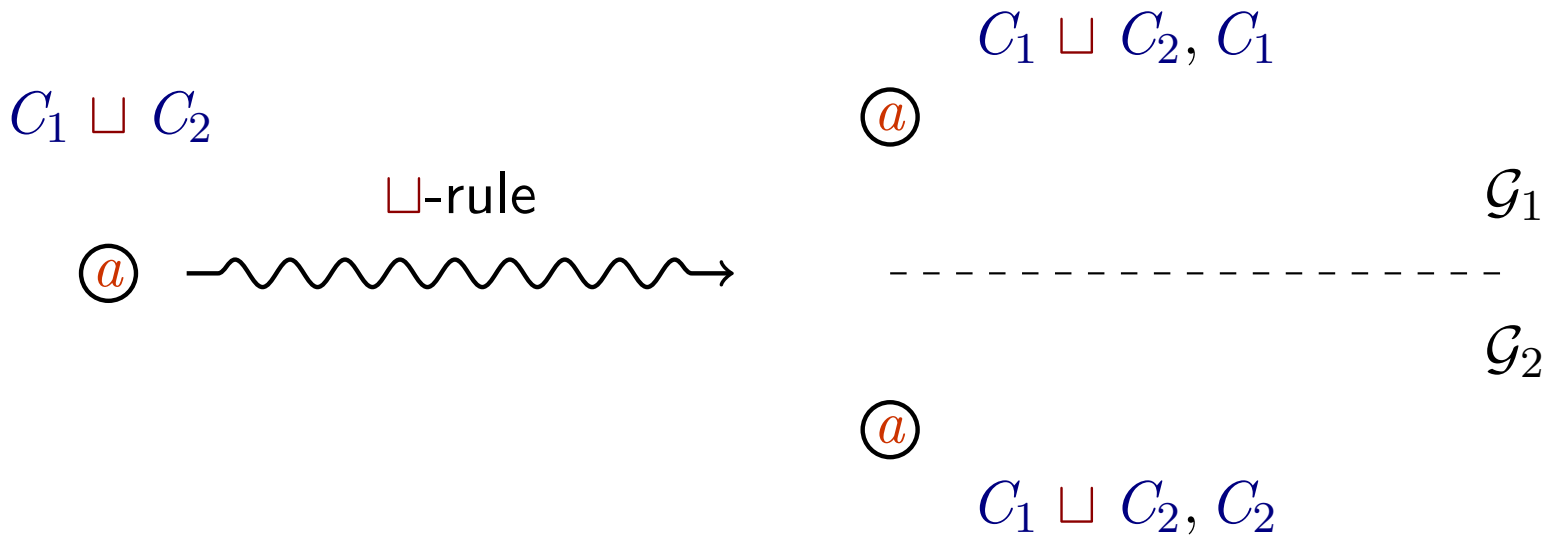

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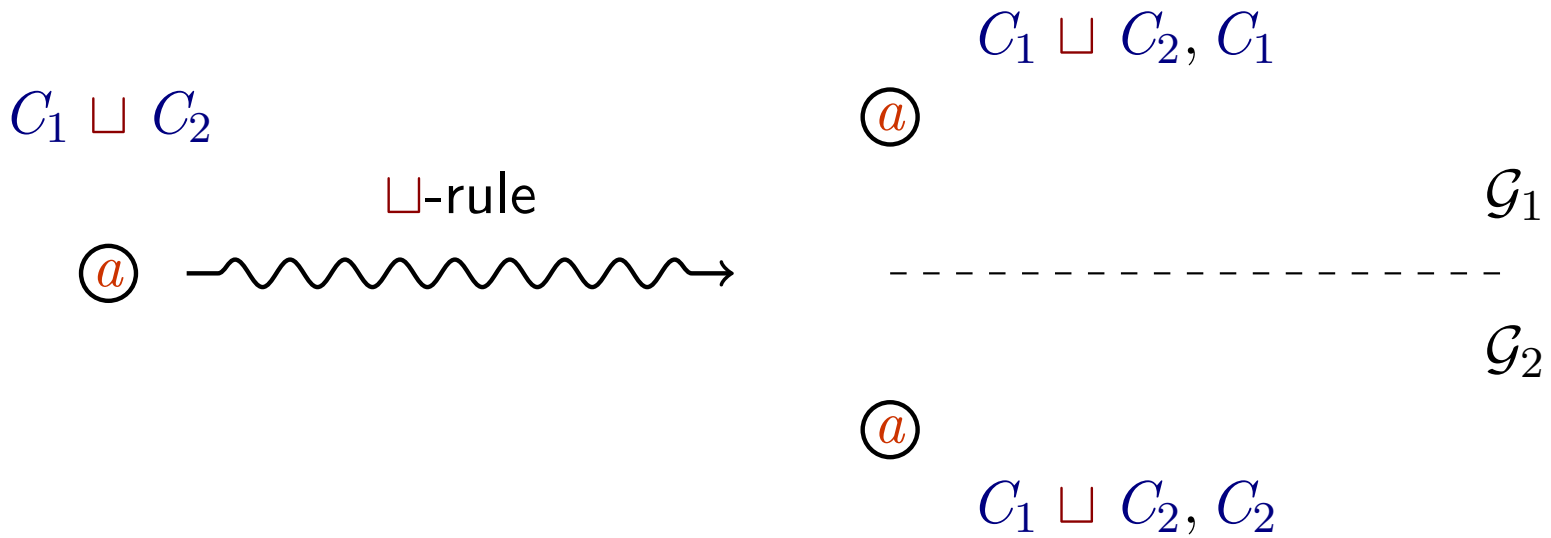

This is a don't know (disjunctive) non-determinism

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$\exists$-rule: if $\exists \mathsf{R}.C \in L(a)$ for some a and there is no b s.t. $C \in L(b)$ and an R-edge from a to b is in $\mathcal{G}$

then create a new node b,
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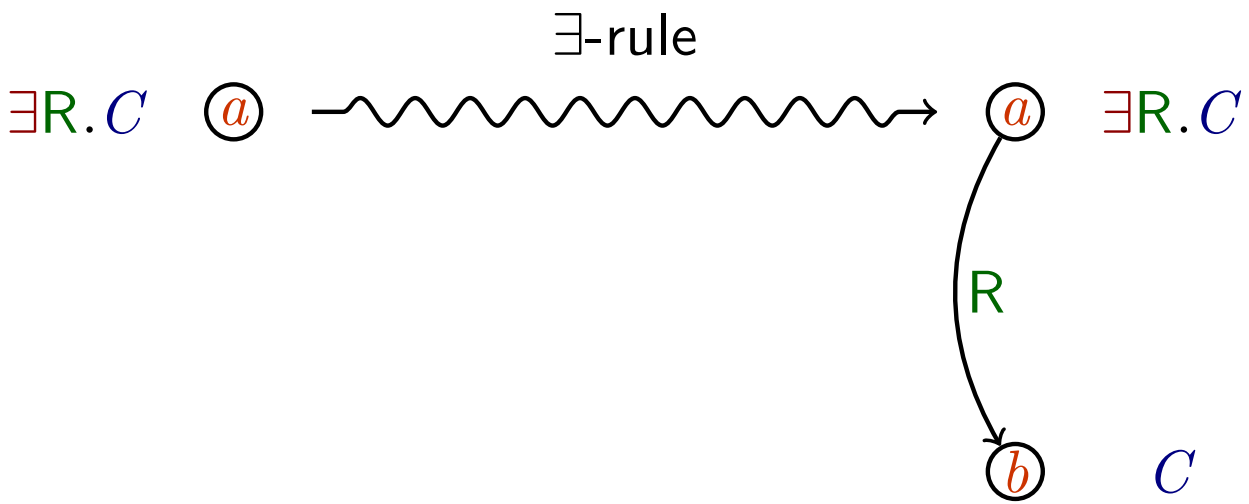

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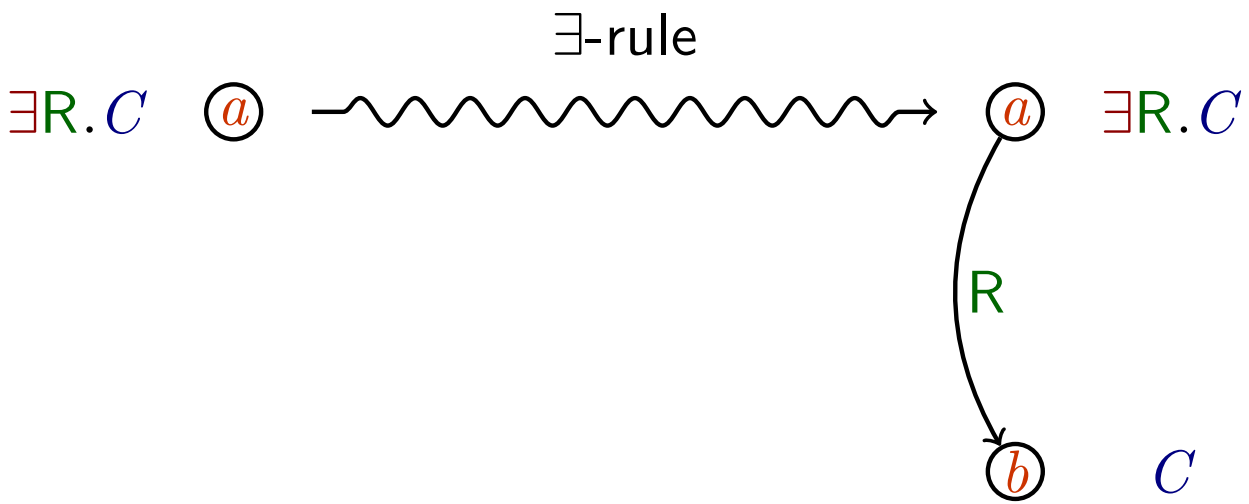

This is the only rule that generates new nodes

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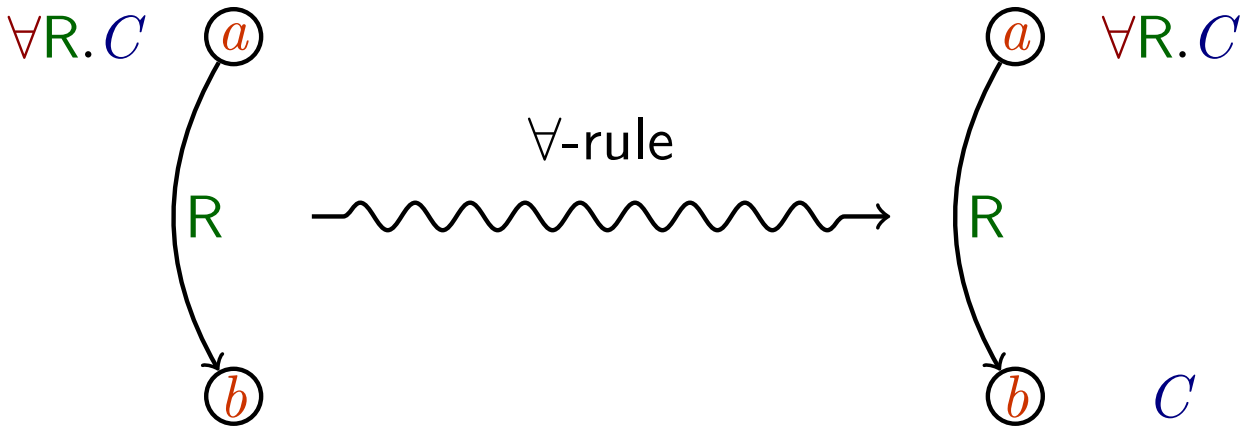

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Terminating

- Construction is monotonic, things are only added to graphs
- Number of nodes and size of labels are bounded

Tableau Algorithm: Example 1

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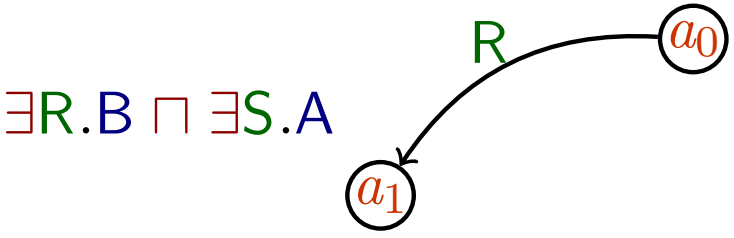

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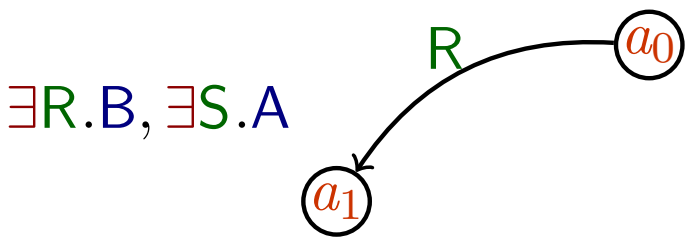

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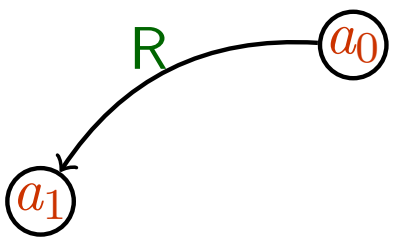

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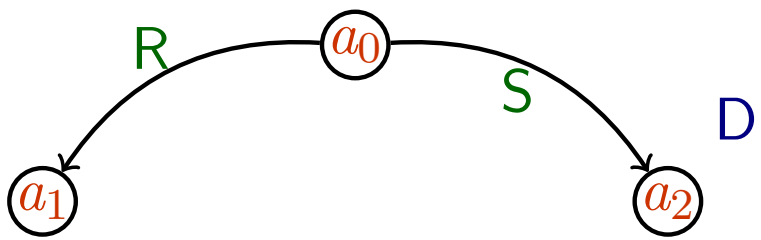

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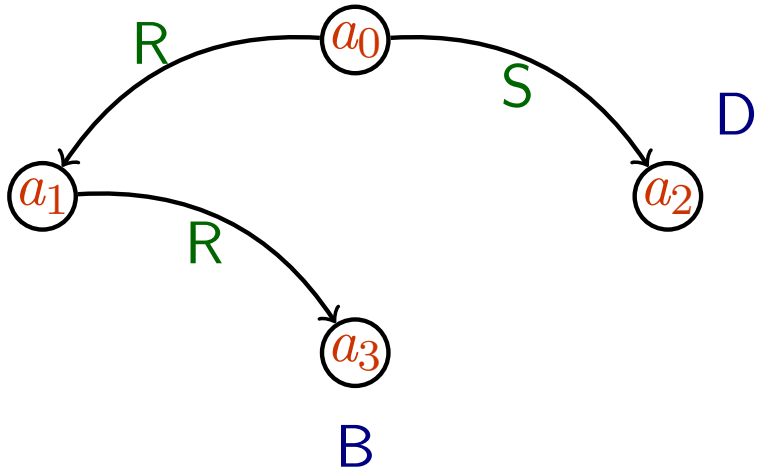

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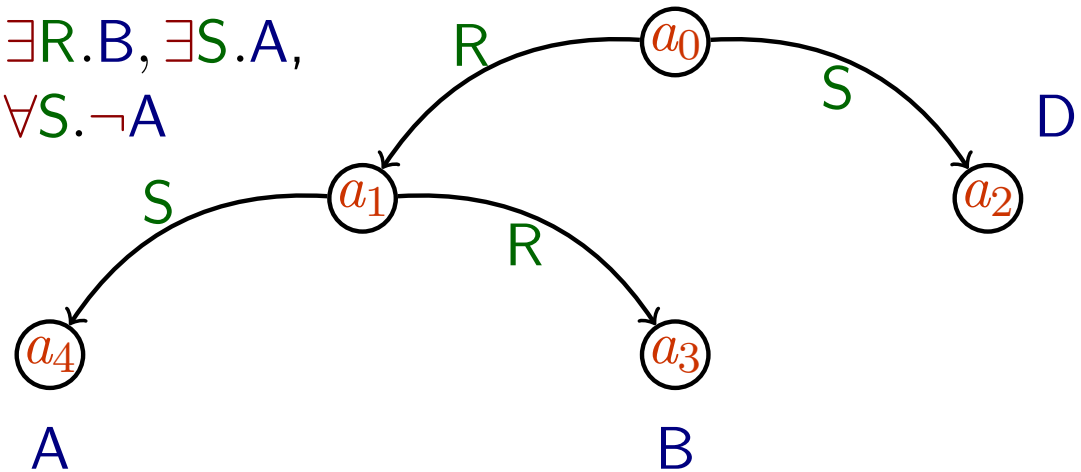

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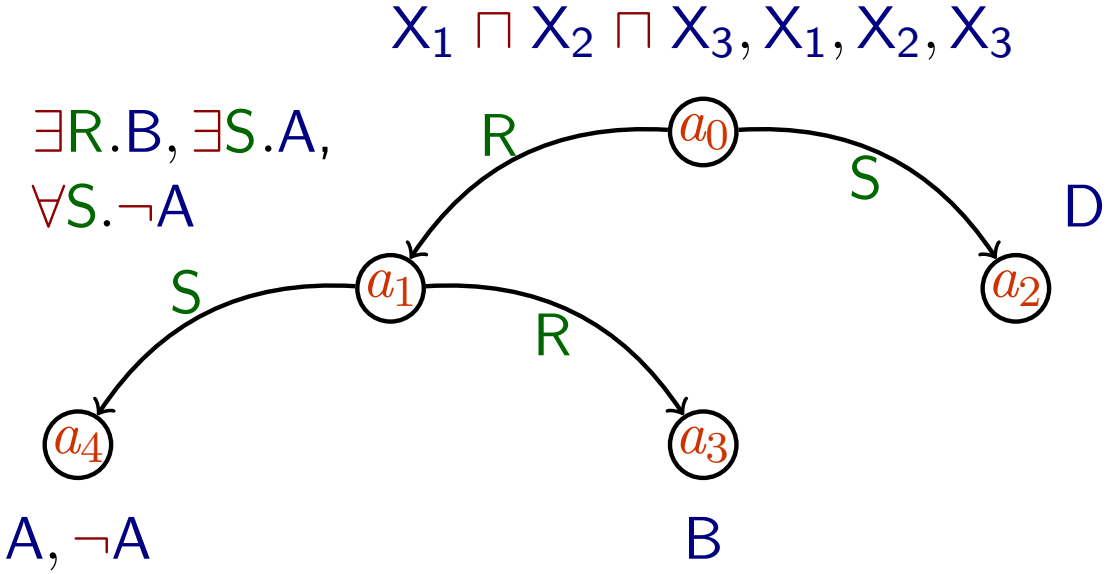

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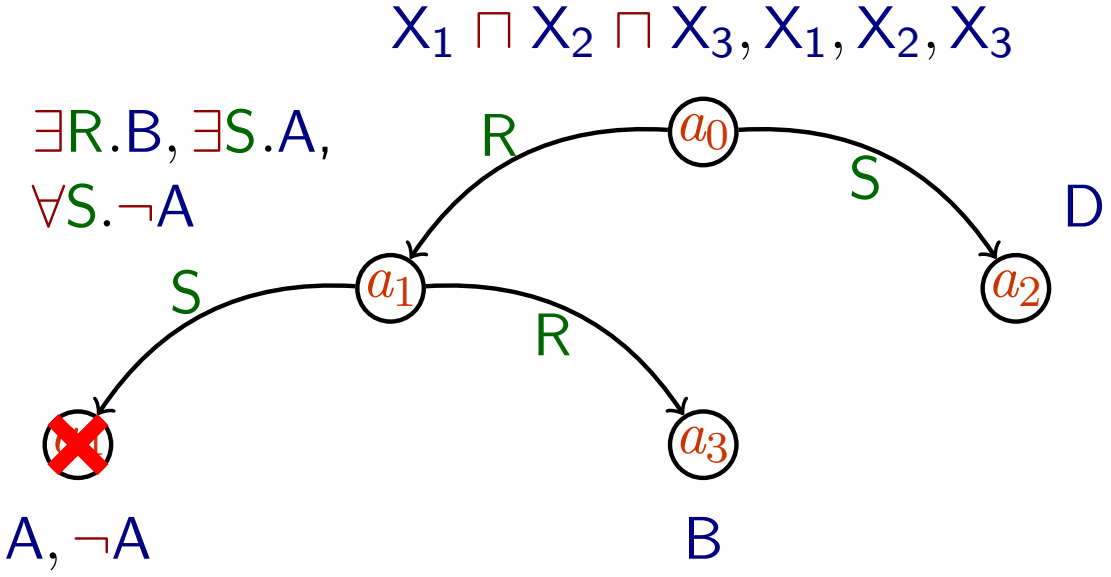

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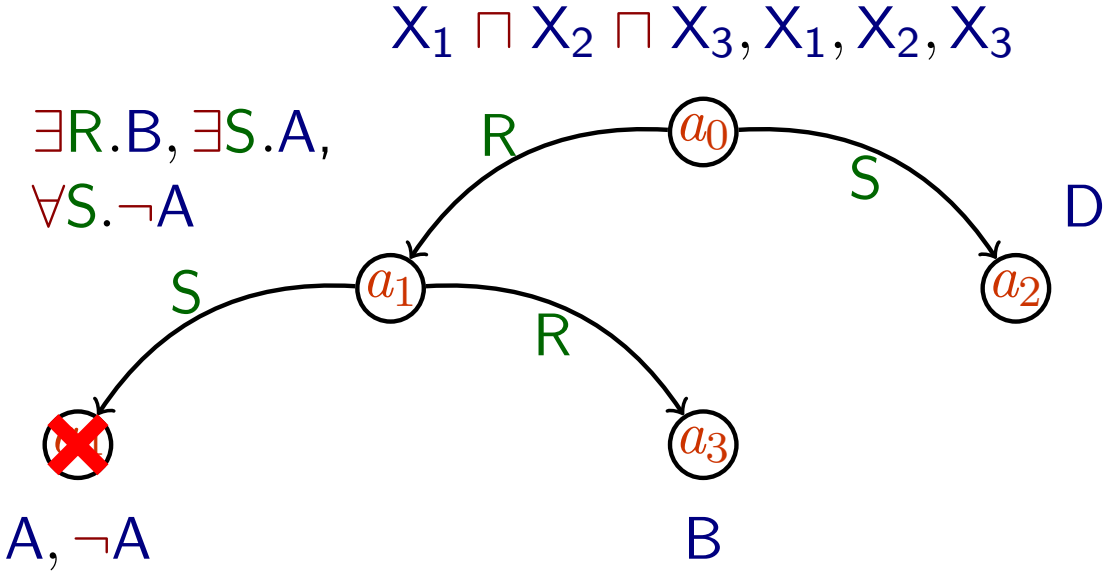

$\mathcal{G}_0$ contains a clash; no other graphs in $\mathcal{S} \Rightarrow X$ is unsatisfiable

Tableau Algorithm: Example 2

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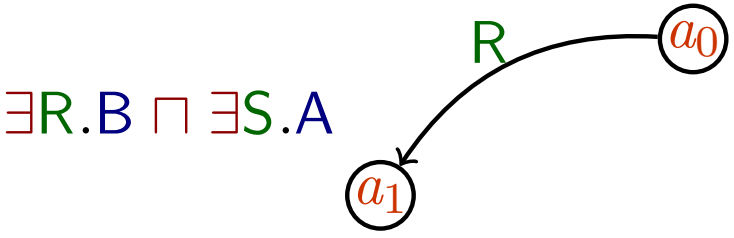

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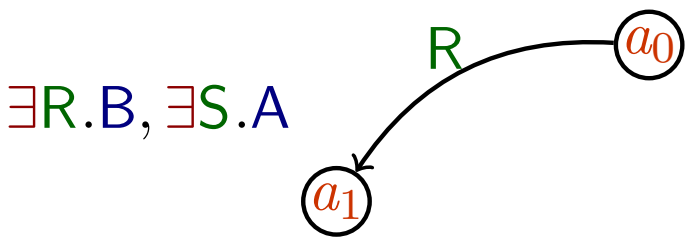

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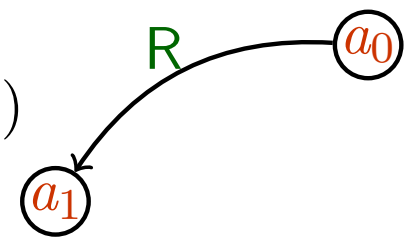

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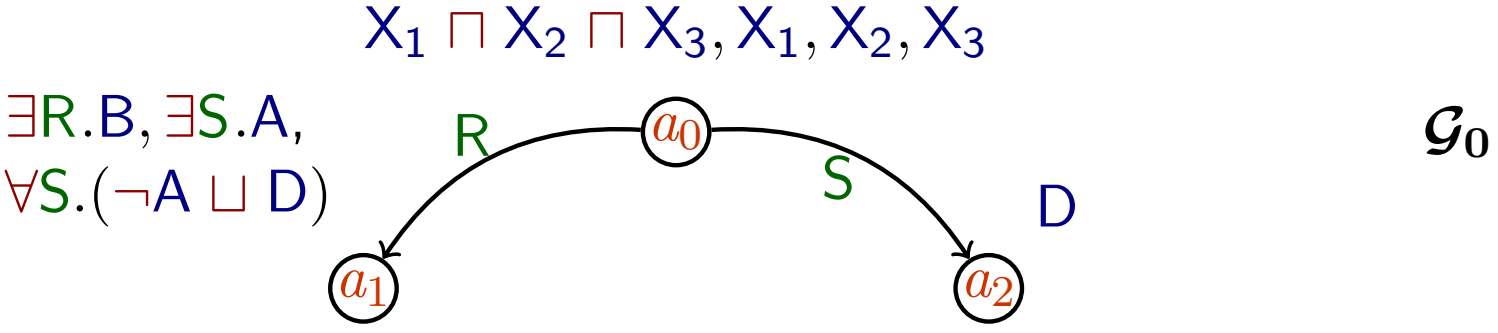

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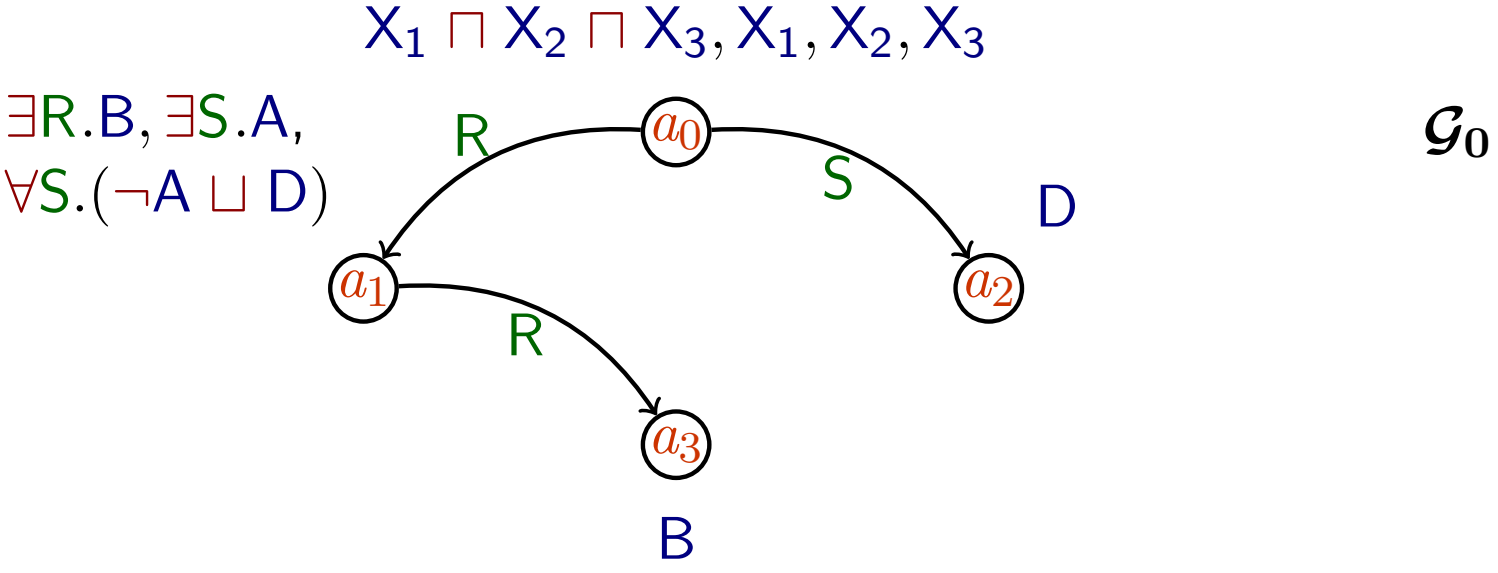

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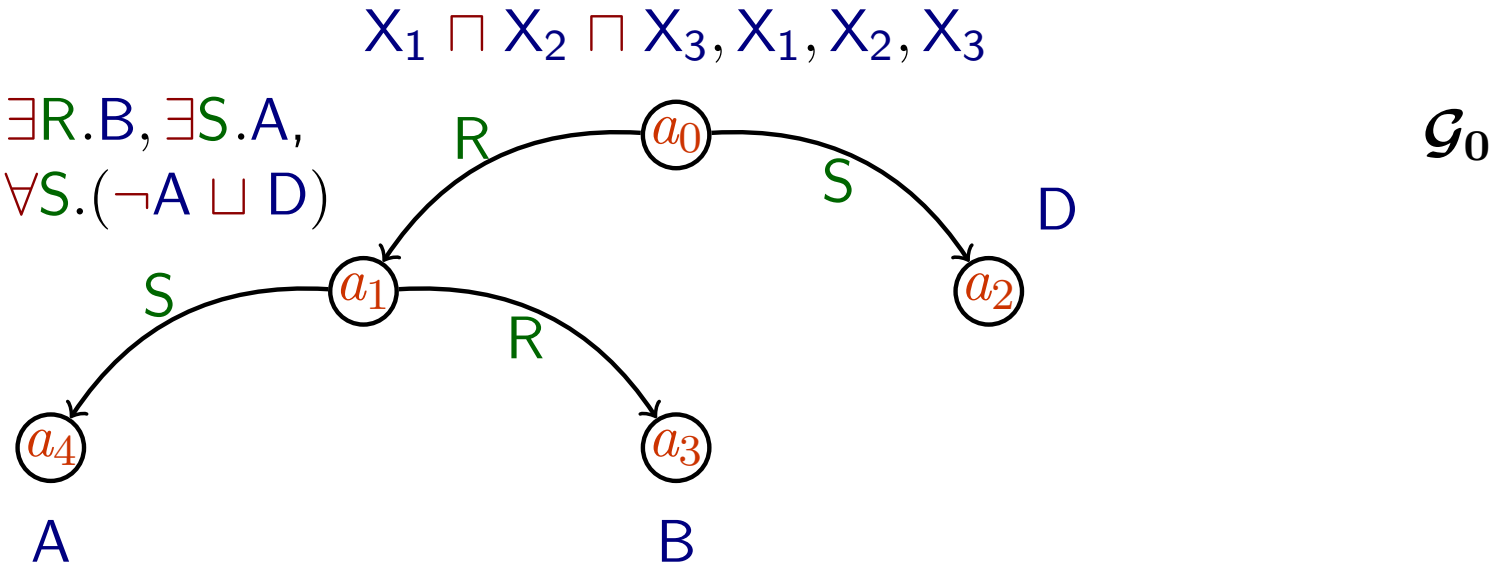

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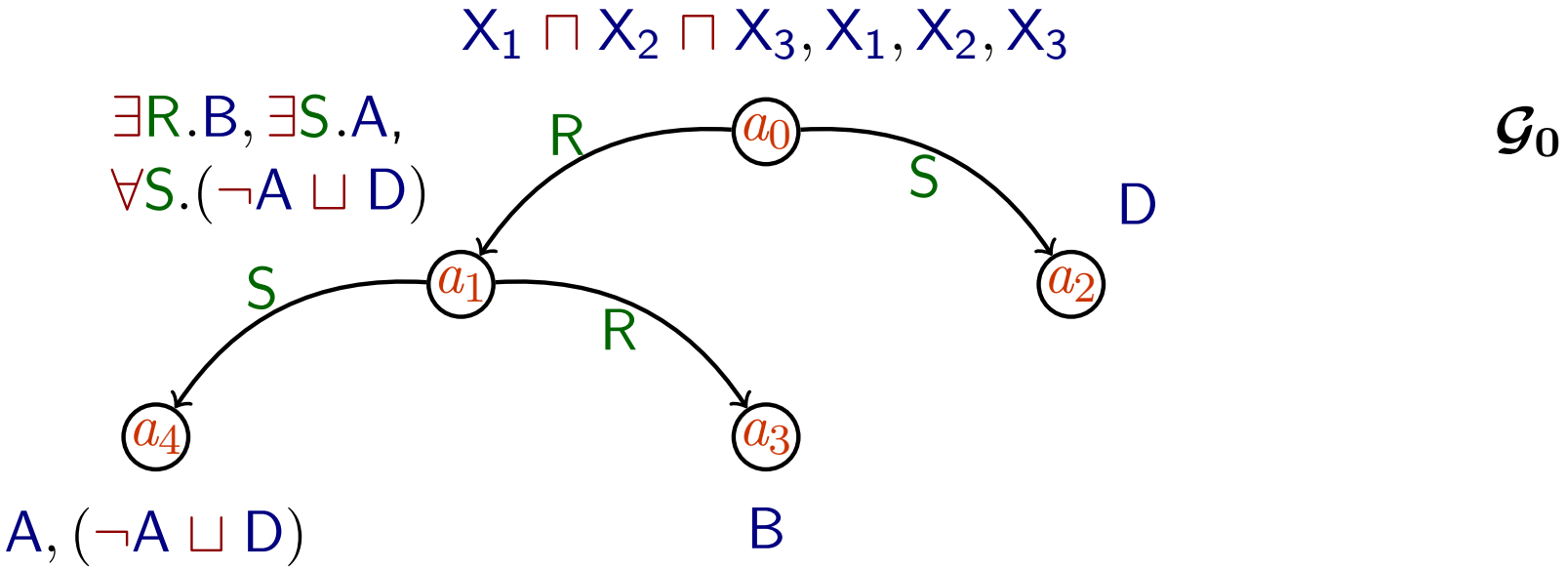

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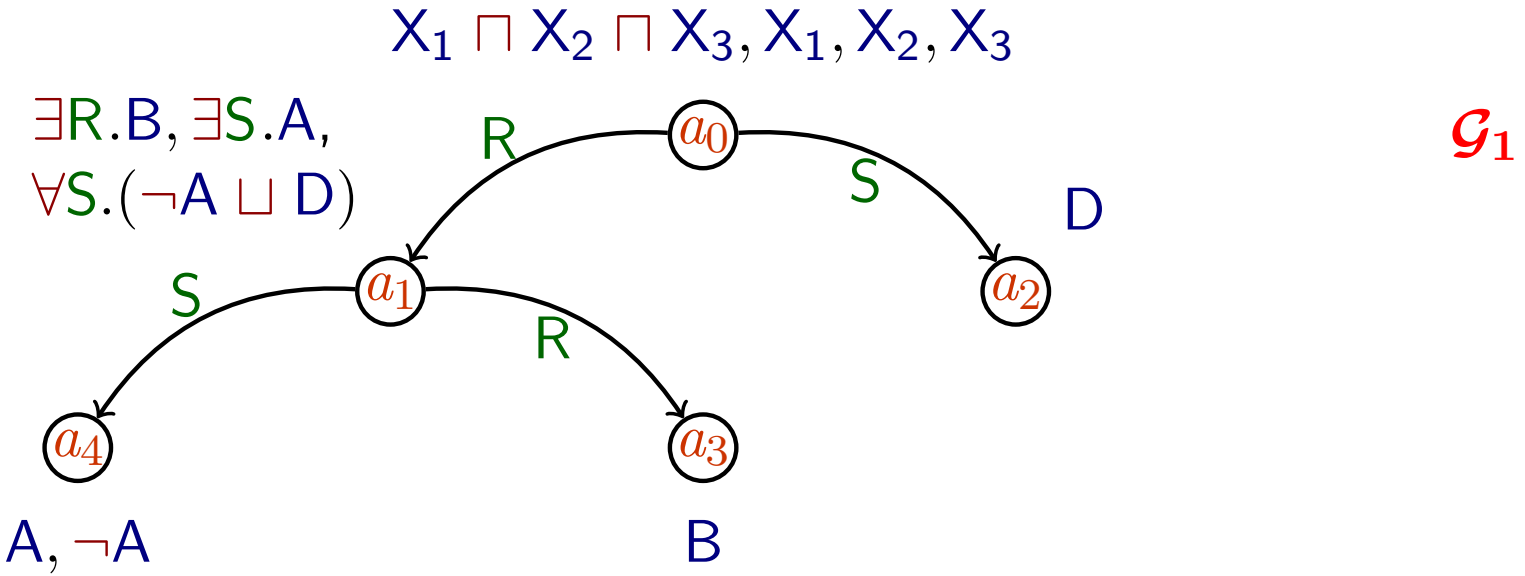

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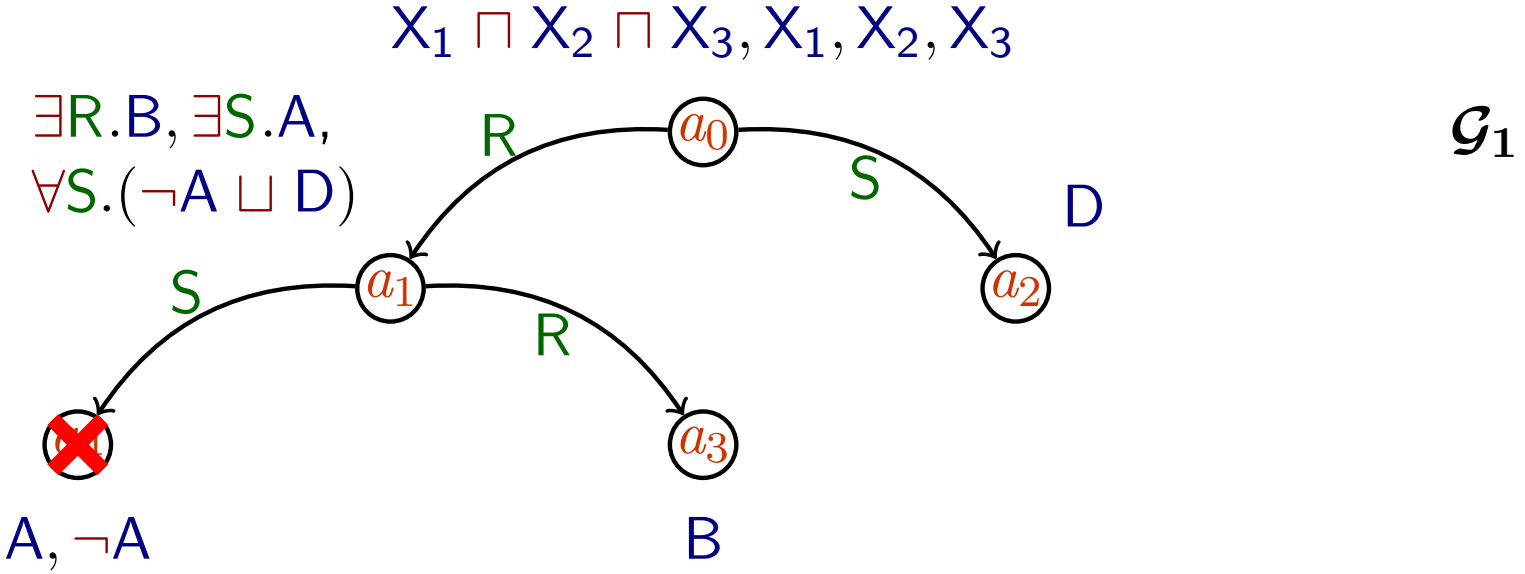

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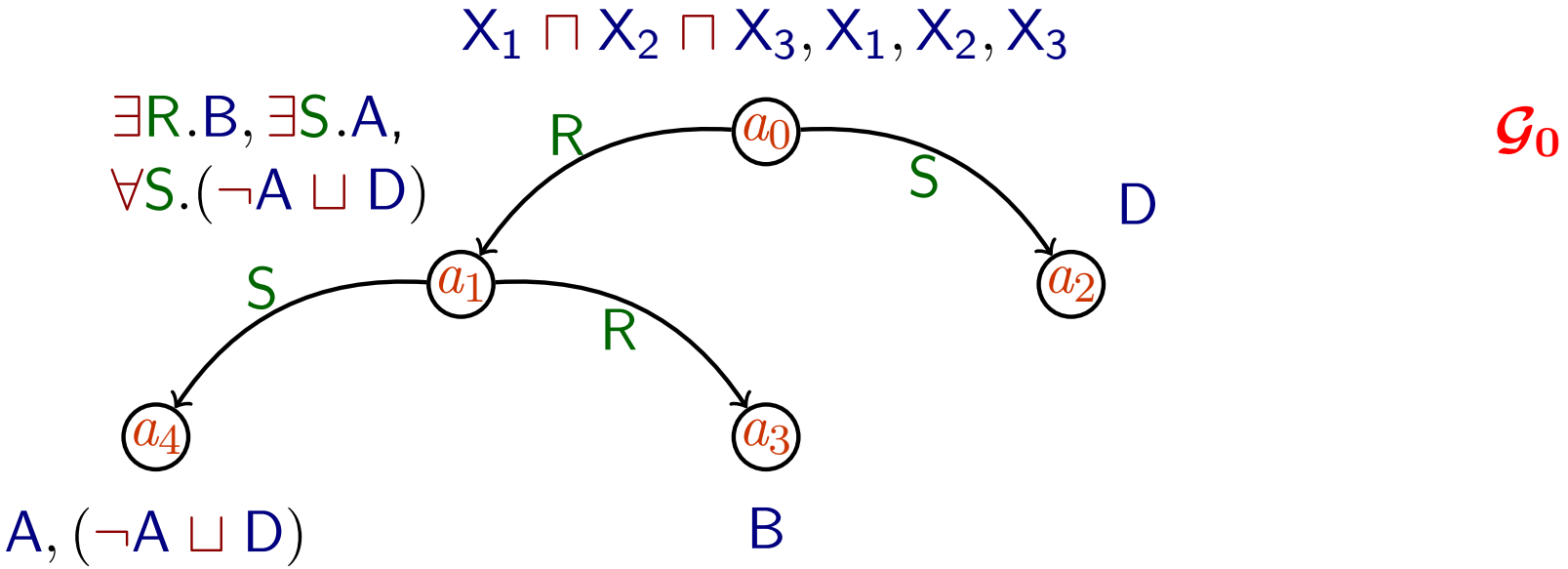

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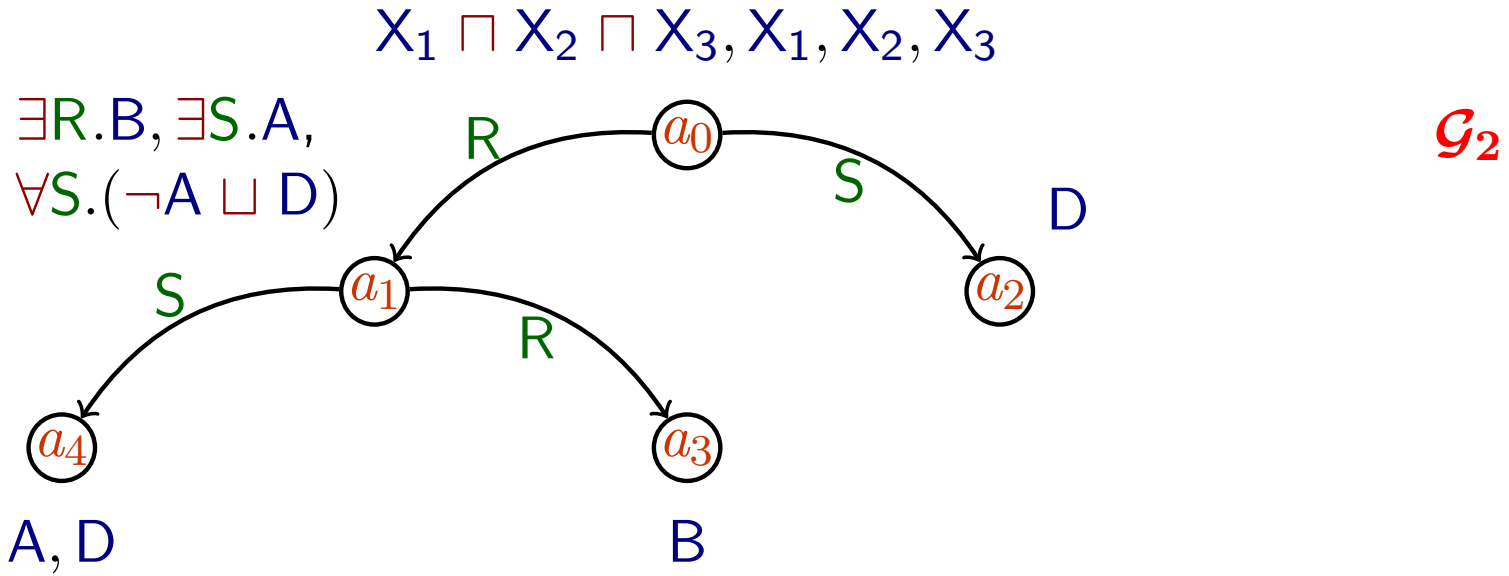

$\mathcal{G}_2$

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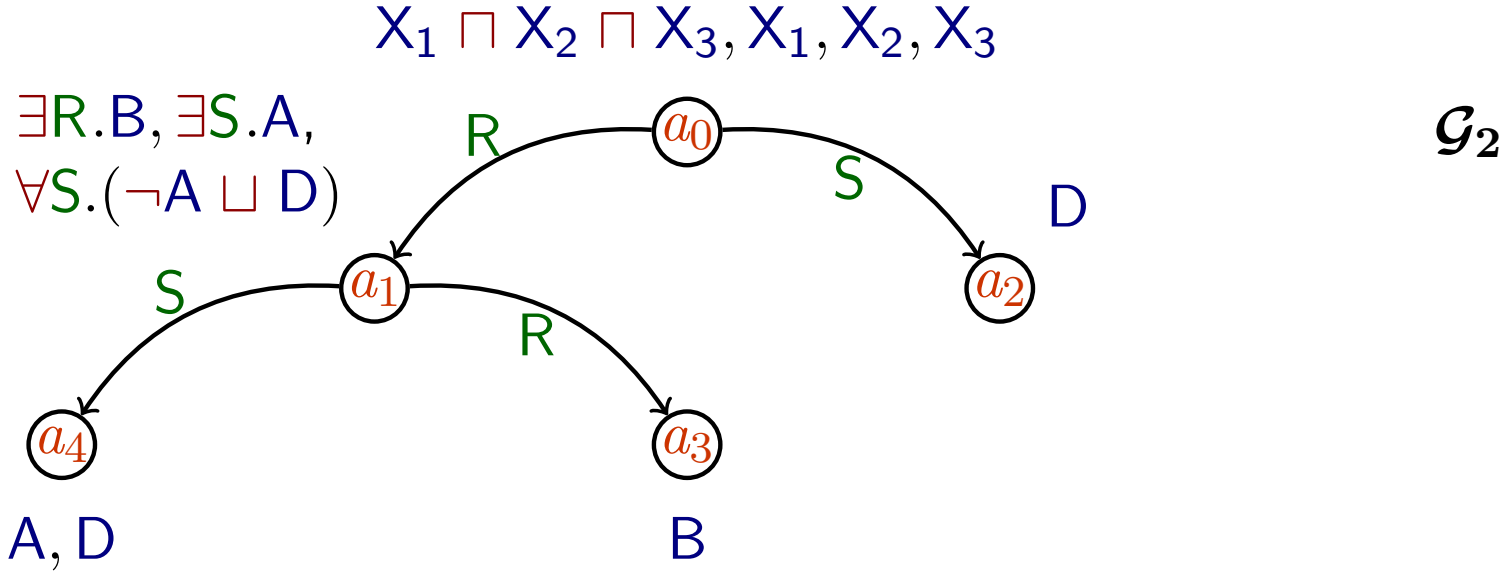

$\mathcal{G}_2$ is a complete clash-free completion graph $\Rightarrow$ X is satisfiable

Tableau Algorithm for $\mathcal{ALC}$: Rule Ordering

Multiple expansion rules could be applicable

- Irrelevant for correctness
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Question: which are cheap and which are costly?

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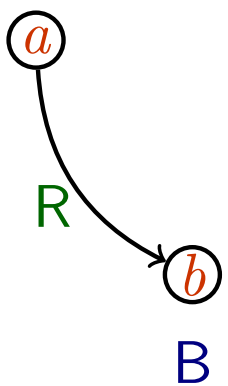

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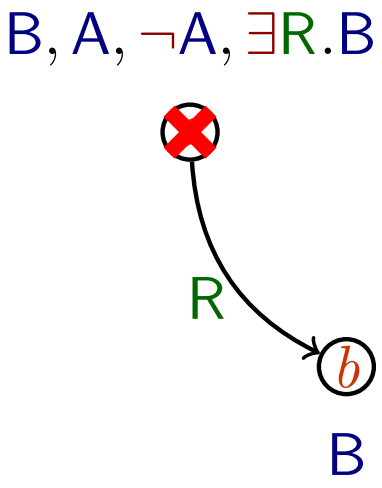

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Disjunctive Non-Determinism

The $\sqcup$-rule is the only one to create two graphs out of one

Observations:

- copying isn't necessary
  sufficient to consider only a single graph at a time
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Example: $\exists R.B \sqcap (C \sqcup \neg D)$

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$@$

Good order: $\sqcup$ after $\sqcap$, $\exists$

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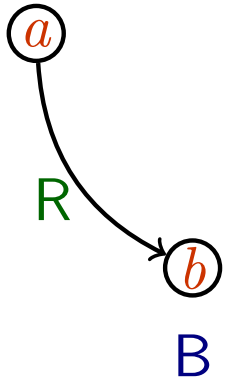

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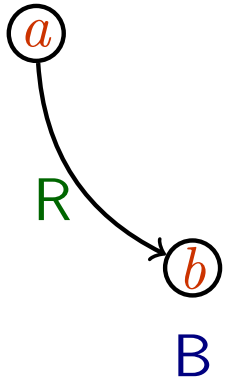

Good order: $\sqcup$ after $\sqcap$, $\exists$

$\sqcap$, $\exists$, $\sqcup$

3 rule applications

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a

Bad order: $\sqcup$ before $\exists$

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$\exists R.B, C \sqcup \neg D$

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$\exists R.B, C$

$@$ $\mathcal{G}_1$

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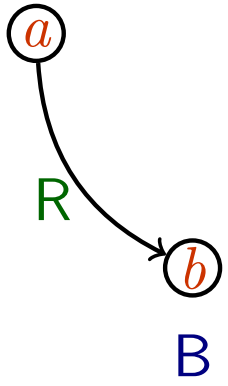
$\exists R.B, C$



$\mathcal{G}_1$

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Example: $\exists R.B \sqcap (C \sqcup \neg D)$

$\exists R.B, \neg D$

$@a$ $\mathcal{G}_2$

Bad order: $\sqcup$ before $\exists$

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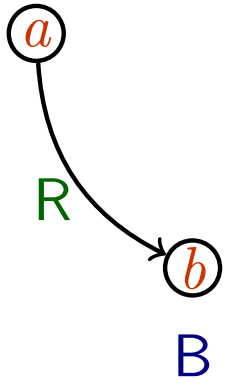

$\mathcal{G}_2$

Bad order: $\sqcup$ before $\exists$

$\sqcap$, $\sqcup$, $\exists$ $(\mathcal{G}_1)$, $\exists$ $(\mathcal{G}_2)$

4 rule applications

No reason to do the same work twice

Tableau Algorithm for ABox Consistency

ABox: $\mathcal{A}$ is a finite set of assertions $a : C$ or $(a, b) : \mathsf{R}$

Consistency: is there interpretation $\mathcal{I}$ which satisfies all assertions?

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A slight generalization of the concept satisfiability algorithm:

- Start with $\mathcal{G}_{\mathcal{A}}$ such that:
  - each individual in $\mathcal{A}$ corresponds to its own node in $\mathcal{G}_{\mathcal{A}}$
  - $a\colon C \in \mathcal{A}$, then $C \in L(a)$
  - $(a, b)\colon \mathsf{R} \in \mathcal{A}$, then an R-edge from a to b is in $\mathcal{G}_{\mathcal{A}}$
- Apply the tableau expansion rule exhaustively
  - clashes in all graphs $\Rightarrow$ inconsistent
  - otherwise $\Rightarrow$ consistent

ABox Consistency: Example

$$\mathcal{A} = \{a\colon (\mathsf{B} \sqcap \exists \mathsf{R}.\neg \mathsf{F}),\quad b\colon (\mathsf{A} \sqcap \forall \mathsf{S}.\forall \mathsf{R}.\mathsf{F}),\quad (b, a)\colon \mathsf{S}\}$$

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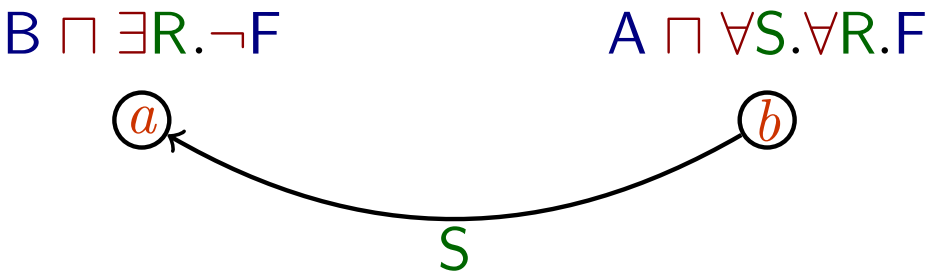

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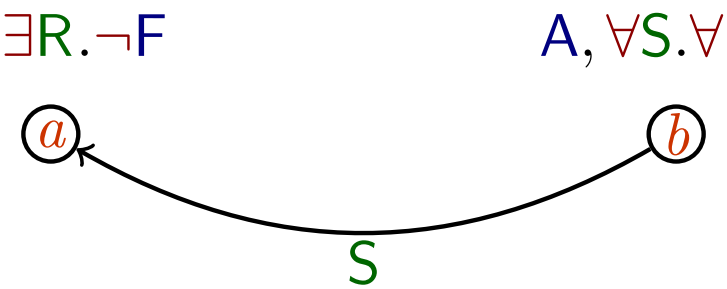

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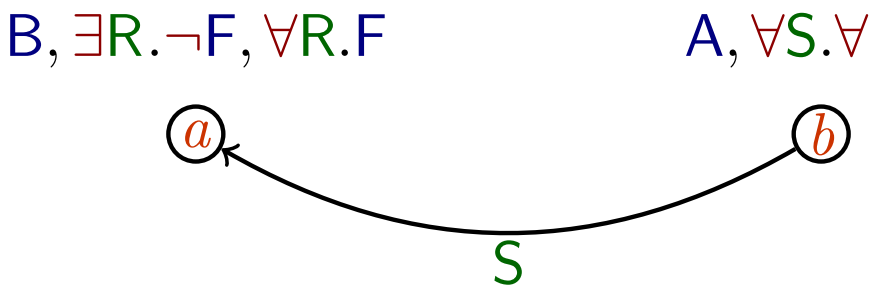

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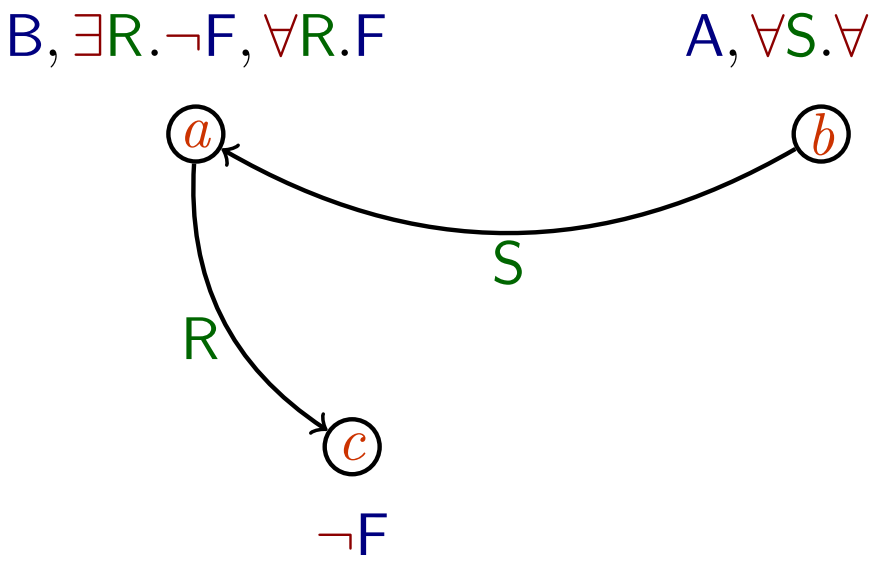

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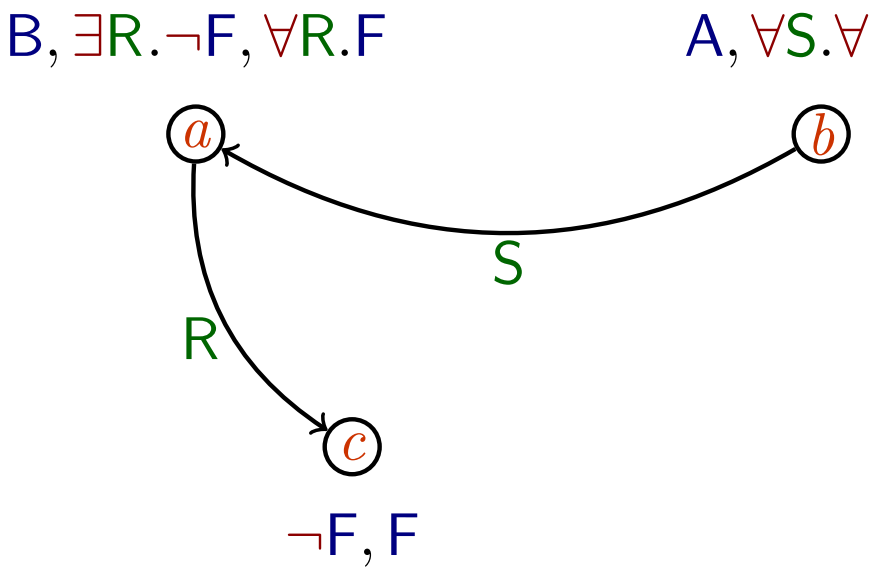

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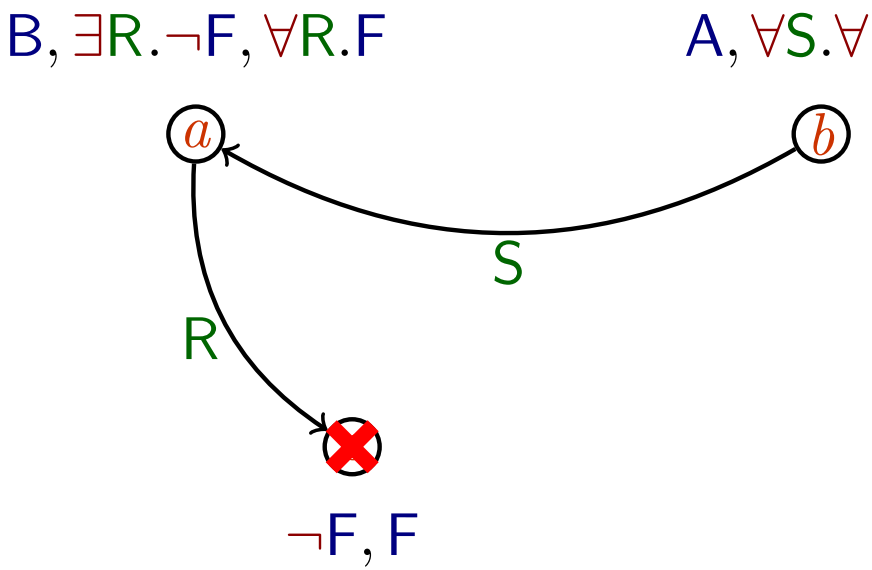

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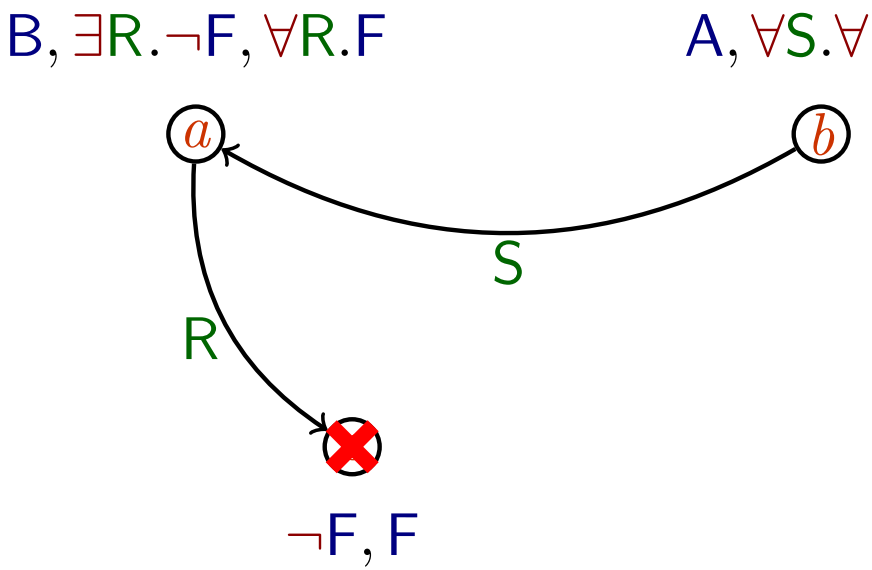

The same optimizations work

The same theoretical properties hold (soundness, completeness, termination, complexity)

Reminder: Reasoning Problems in DLs

$\mathcal{ALC}$ Tableau with TBoxes

Two Essential Optimizations

TBoxes Enter the Picture

TBox: $\mathcal{T}$ is a finite set of concept subsumption axioms $C \sqsubseteq D$

Problem: decide consistency of ontology $\mathcal{O} = (\mathcal{T}, \mathcal{A})$
a.k.a. $\mathcal{A}$ consistency w.r.t. $\mathcal{T}$
"is data consistent w.r.t. schema?"

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We consider cases:

- Acyclic simple TBoxes (easy)
- General TBoxes (not-so-easy)

Simple TBoxes and Reachibility

Axioms are restricted to: $\mathsf{A} \sqsubseteq C$ and $\mathsf{A} \equiv C$, where

A is a concept name and C is a concept expression

Each A (defined concept) occurs only once on the left

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Question: how to check acyclicity?

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Ontology consistency can be reduced to ABox consistency
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- $\mathsf{A} \equiv C$: replace A by C
- $\mathsf{A} \sqsubseteq C$: replace A by $C \sqcap \mathsf{A}^*$
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Unfolding is a completely syntactic procedure

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Now $(\mathcal{T}, \mathcal{A})$ is consistent if and only if $\mathcal{A}$ is consistent
(same technique works for concept satisfiability, subsumption, etc.)

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How much bigger can the unfolded concepts be?

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$A_1 \equiv \exists R_1.A_0 \sqcap \exists R_2.A_0$,
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Unfolding can be done lazily
B unfolded only when B or $\neg$B is in $L(v)$ for some v

General TBoxes

TBox is a set of general concept inclusion axioms (GCIs) $C \sqsubseteq D$

$C \equiv D$ is a syntactic sugar for $\{C \sqsubseteq D,\ D \sqsubseteq C\}$

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Question: would the following rule be sufficient?

GCI_0-rule: if $C \in L(a)$ for some a and $C \sqsubseteq D \in \mathcal{T}$
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i.e. can we view GCIs as IF-THEN rules?

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$\mathcal{T} \equiv \{\mathsf{A} \sqcap \mathsf{B} \sqsubseteq \mathsf{C}\}$
$\mathcal{A} \equiv \{a\colon \mathsf{A}, \quad a\colon \mathsf{B}\}$

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- Fixes this sort of problems
- Messy, many useless conjunctions
- Particularly messy for disjunctions (what would the rule be?)

Local GCI Rule: Bigger Problem

$\mathcal{T} \equiv \{C \sqsubseteq \mathsf{D}, \quad \mathsf{D} \sqsubseteq \mathsf{F}, \quad \neg C \sqcup \mathsf{F} \sqsubseteq \mathsf{X}\}$
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Reasoning by cases: in every $\mathcal{I}$, $a^{\mathcal{I}} \in C^{\mathcal{I}}$ or $a^{\mathcal{I}} \in (\neg C)^{\mathcal{I}}$

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Local GCI Rule: Bigger Problem

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Lesson: GCIs can be inherently global and apply to all individuals

Global GCI Rule

Instead we use the following global GCI rule:

GCI-rule if $C \sqsubseteq D \in \mathcal{T}$ and $\dot{\neg} C \sqcup D \notin L(a)$ for some a, then add $\dot{\neg} C \sqcup D$ to $L(a)$

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We retain the local rule :

GCI_0-rule if $\mathsf{A} \sqsubseteq C \in \mathcal{T}$ and $\mathsf{A} \in L(a)$ and $C \notin L(a)$ for some a,
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Ontology Consistency: Example

$\mathcal{T} \equiv \{\mathsf{A} \sqsubseteq \exists \mathsf{S}.\mathsf{C}, \quad \mathsf{B} \sqsubseteq \forall \mathsf{R}.\forall \mathsf{S}.\mathsf{D}, \quad \mathsf{C} \sqcap \mathsf{D} \sqsubseteq \bot\}$

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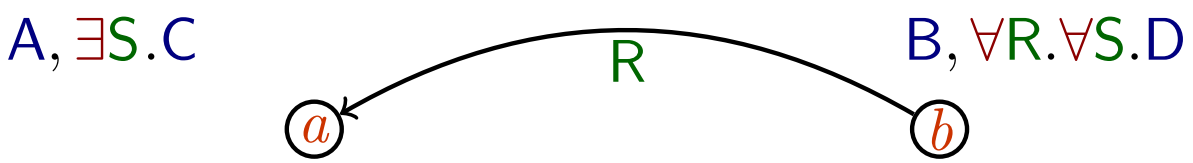

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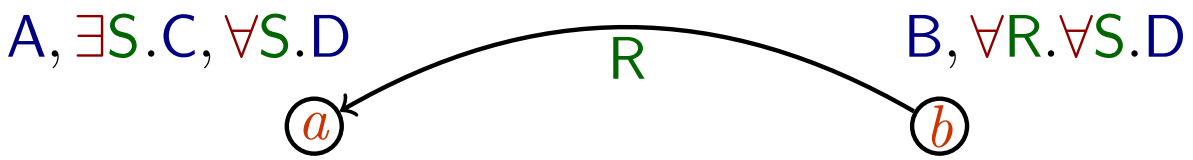

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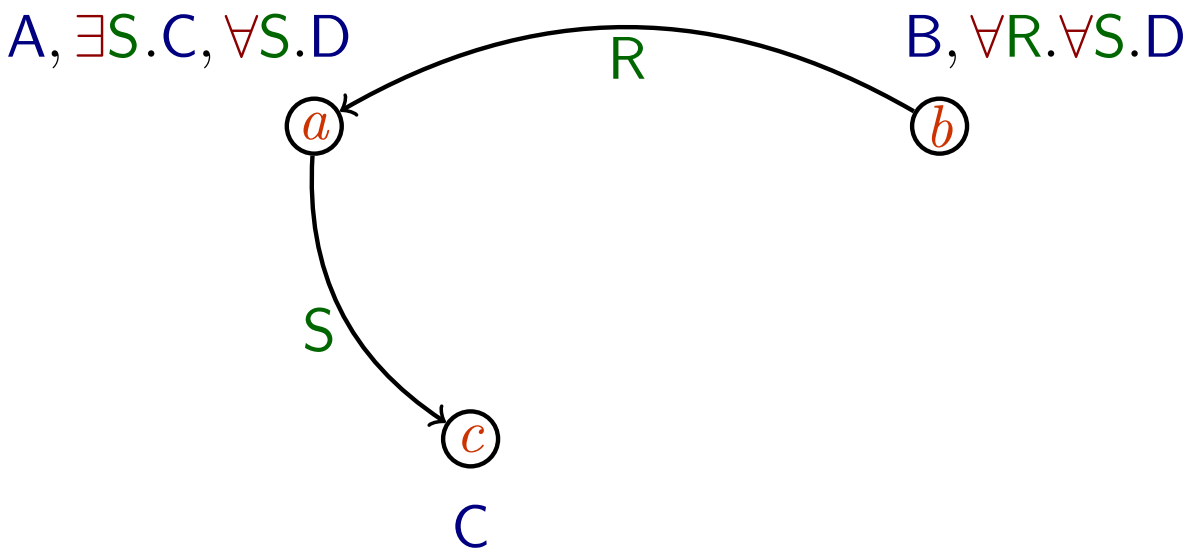

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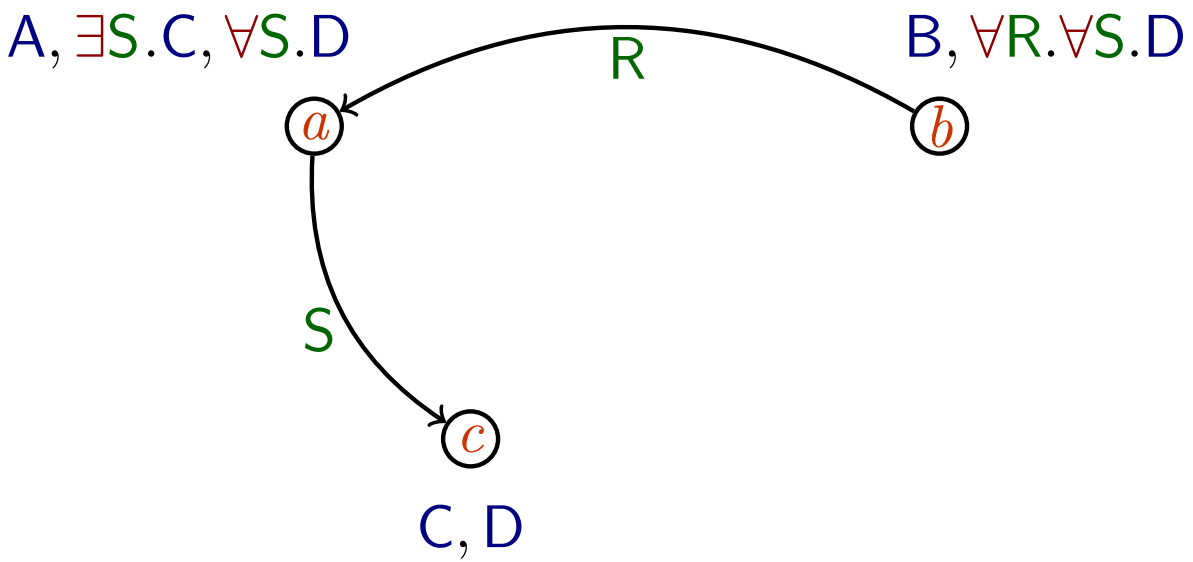

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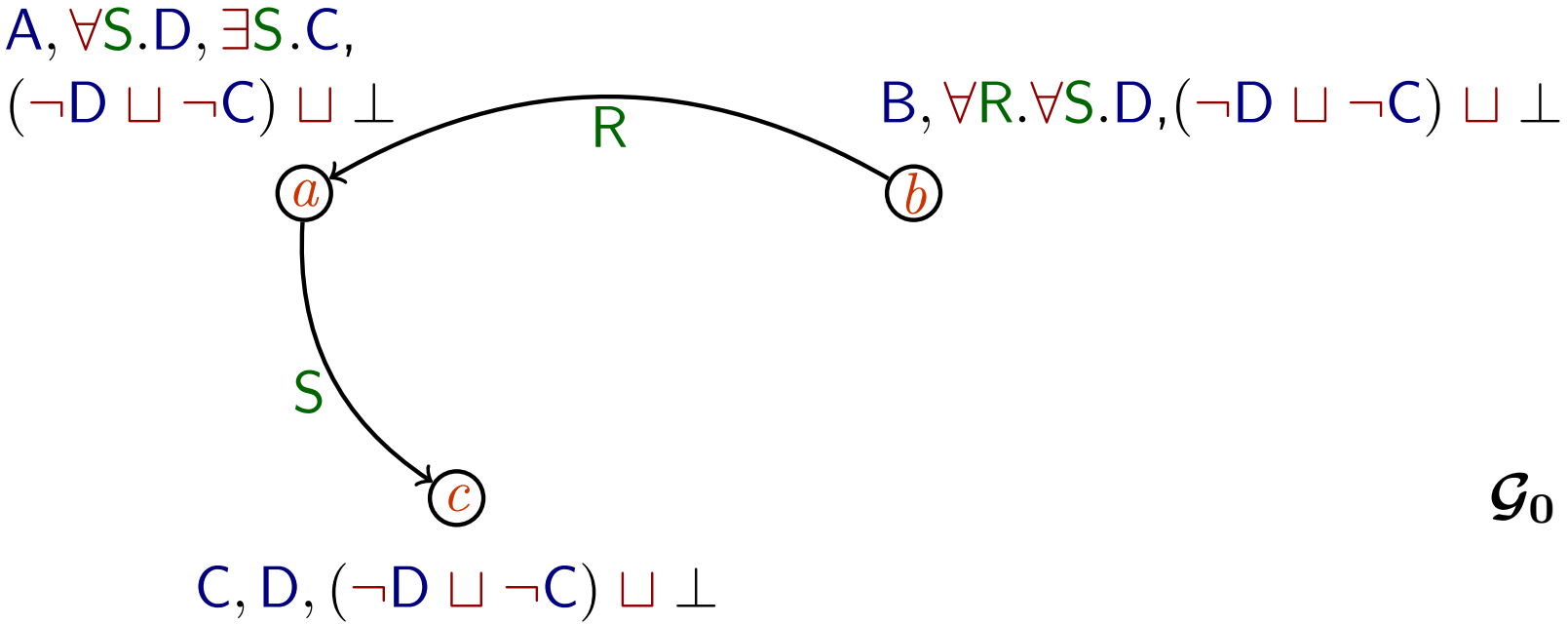

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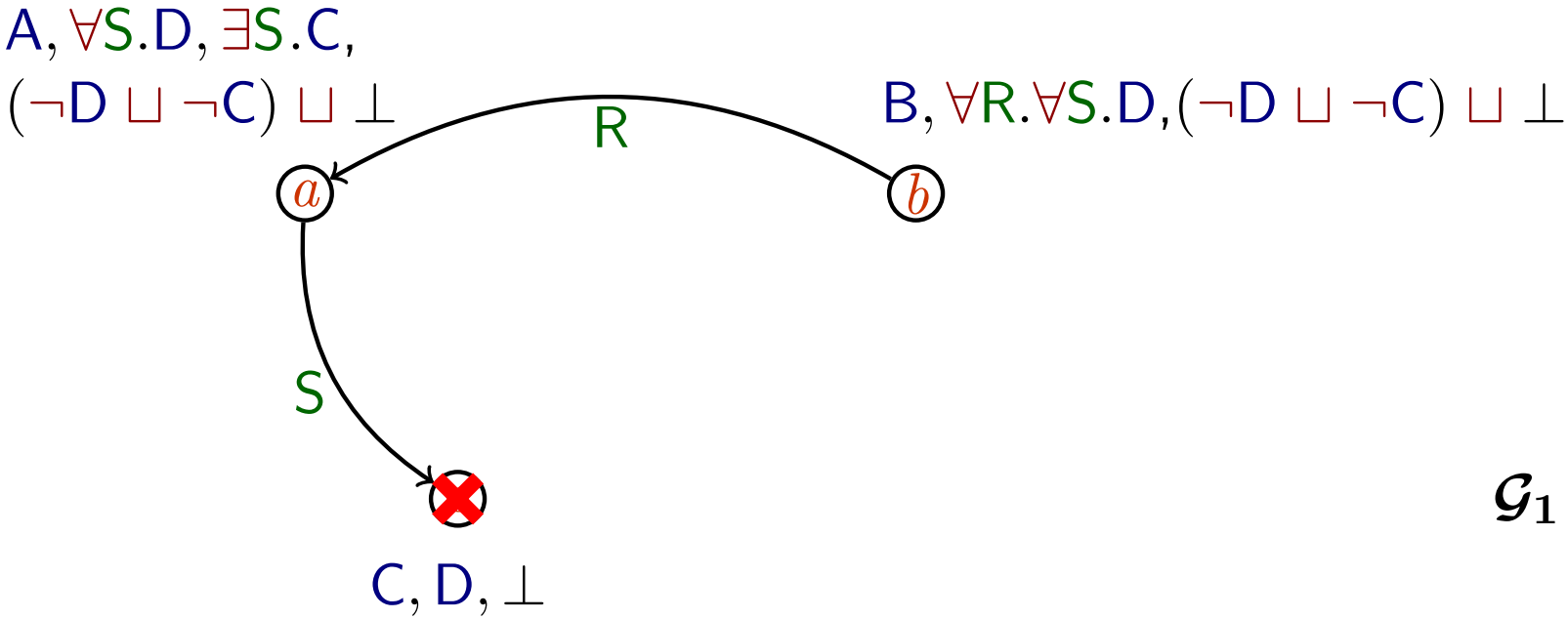

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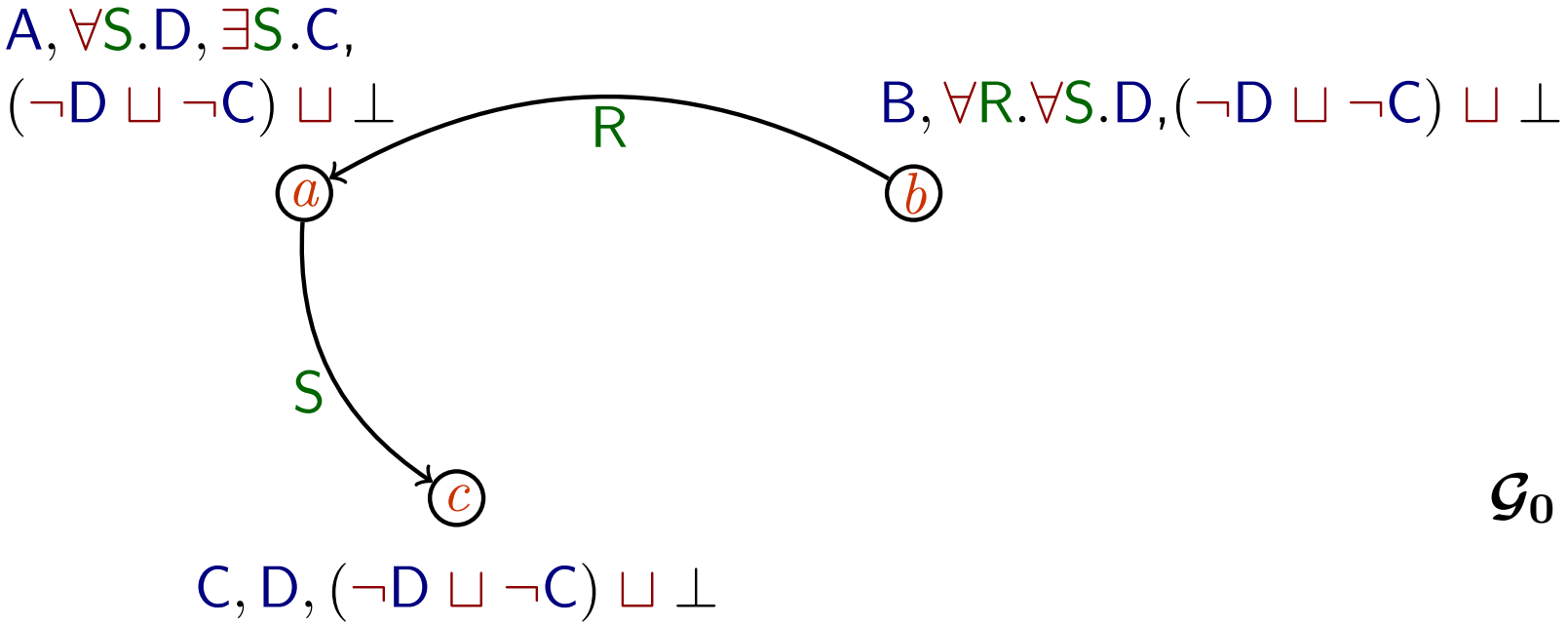

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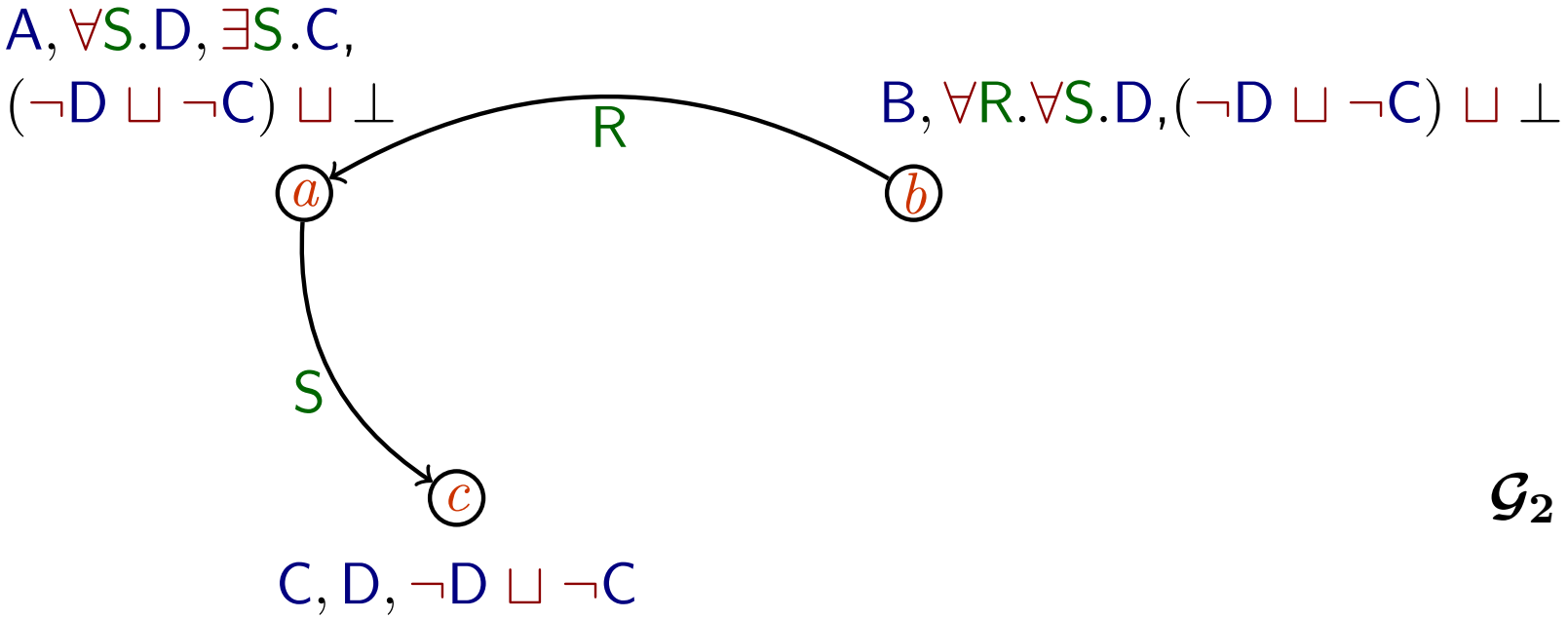

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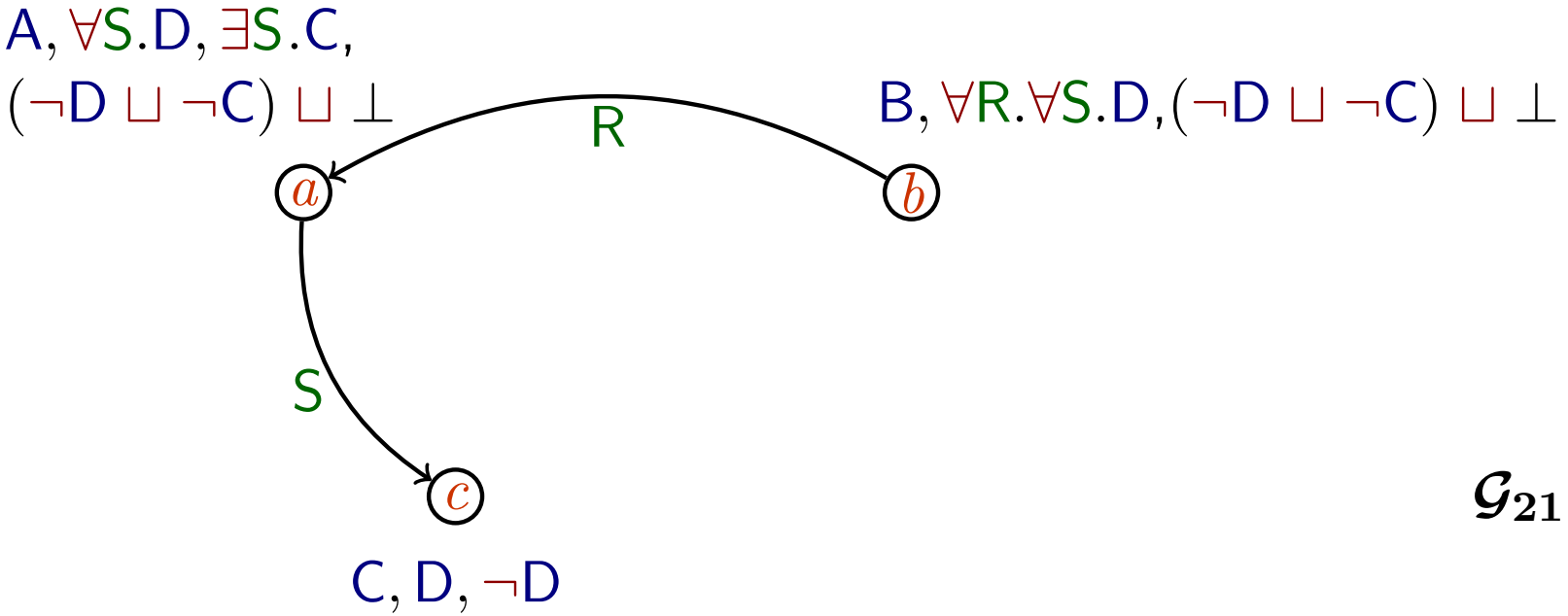

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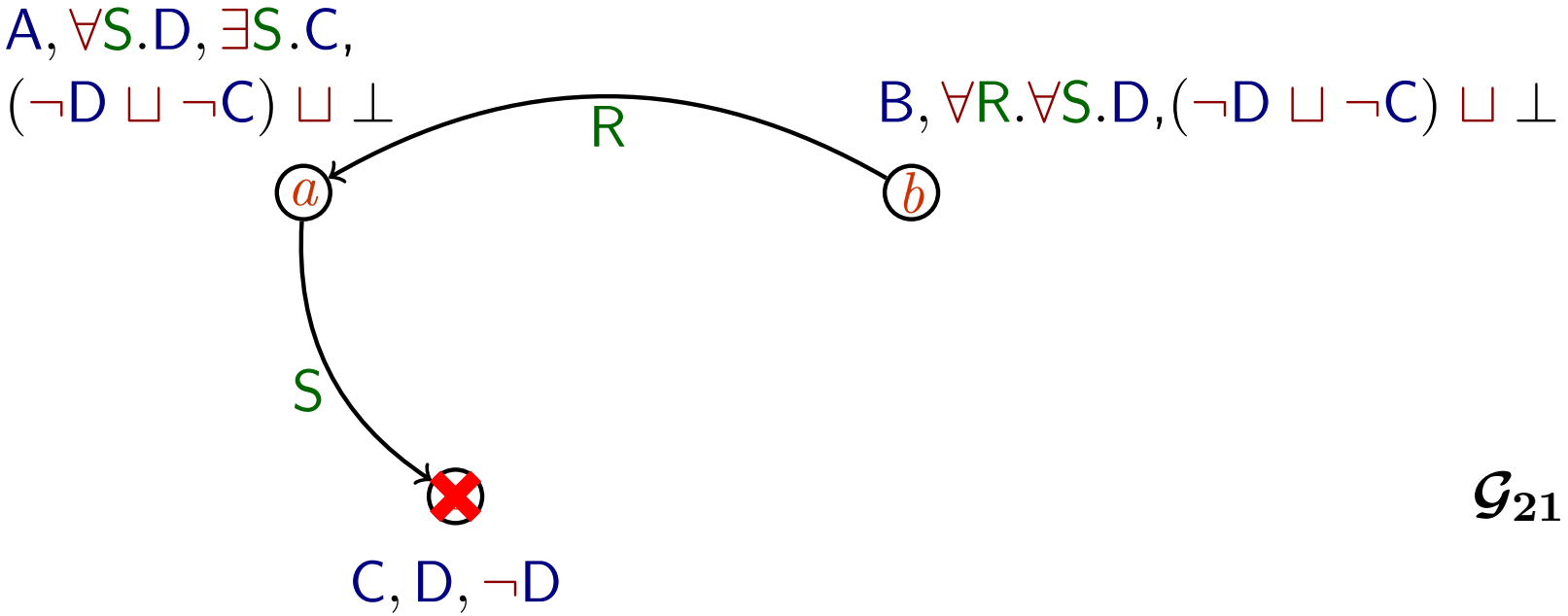

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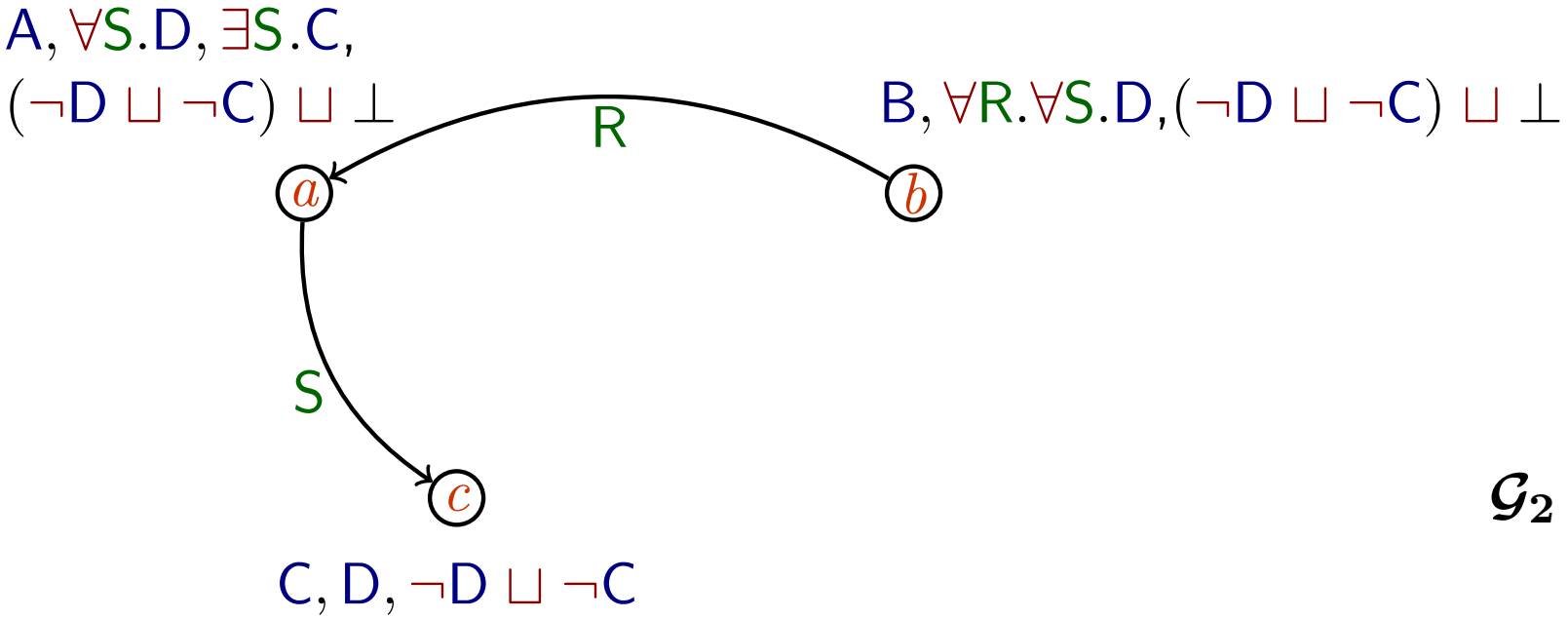

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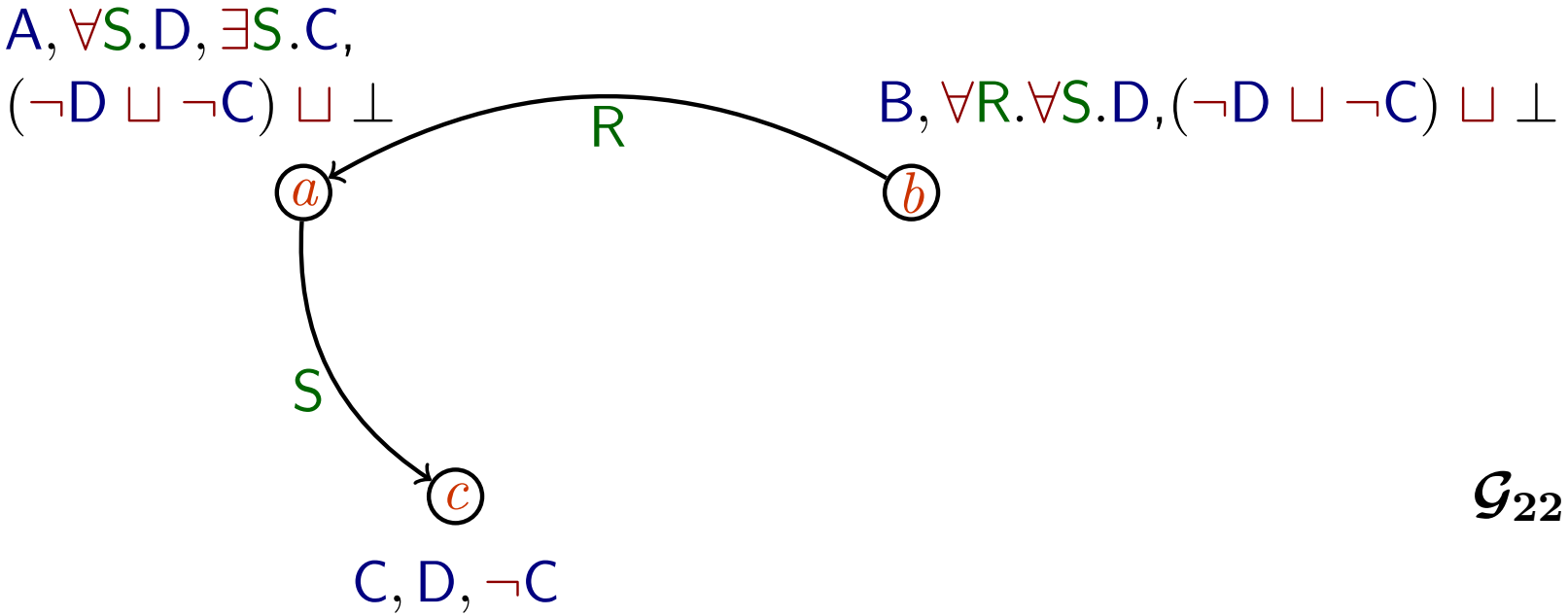

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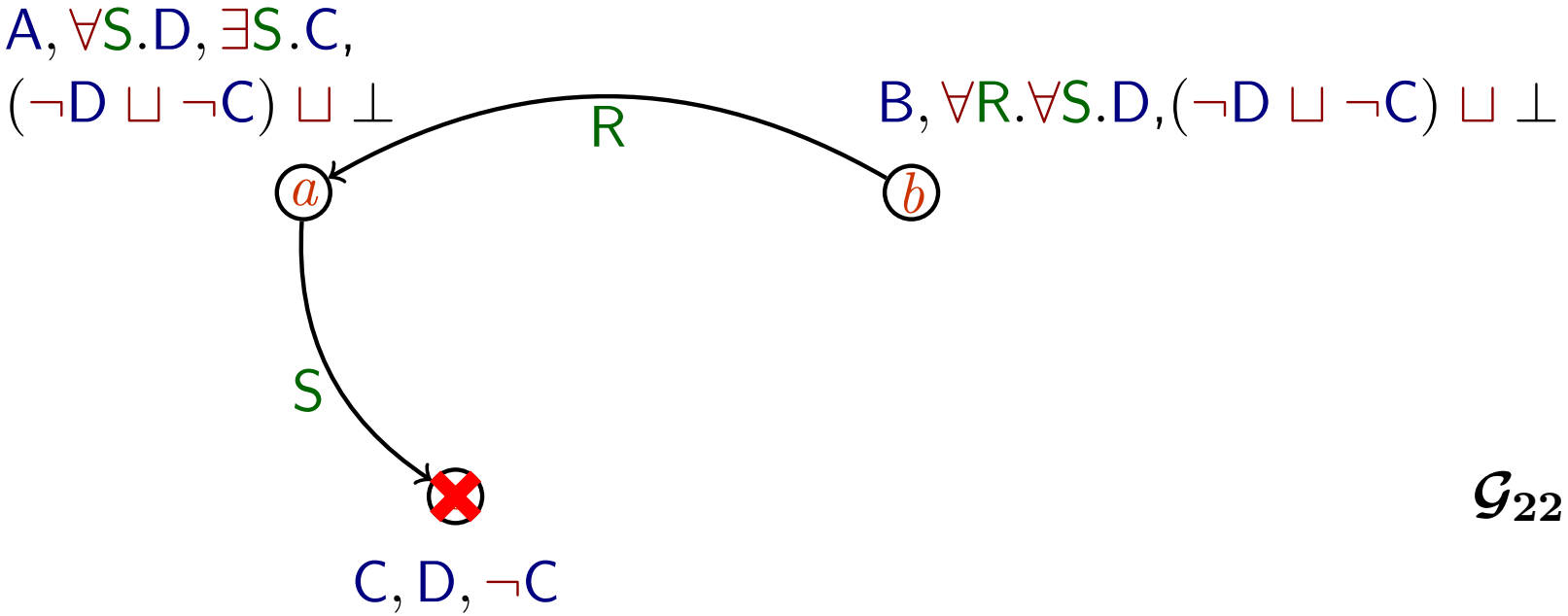

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It's All About Choices...

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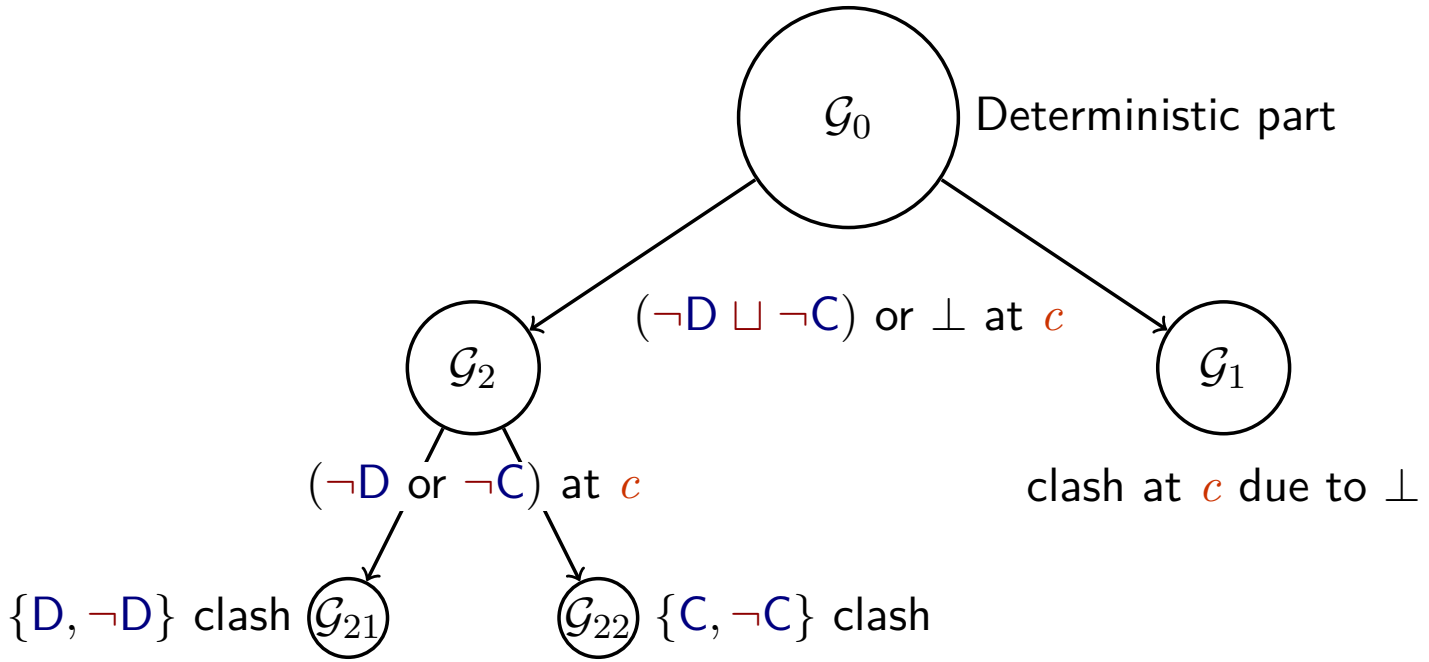

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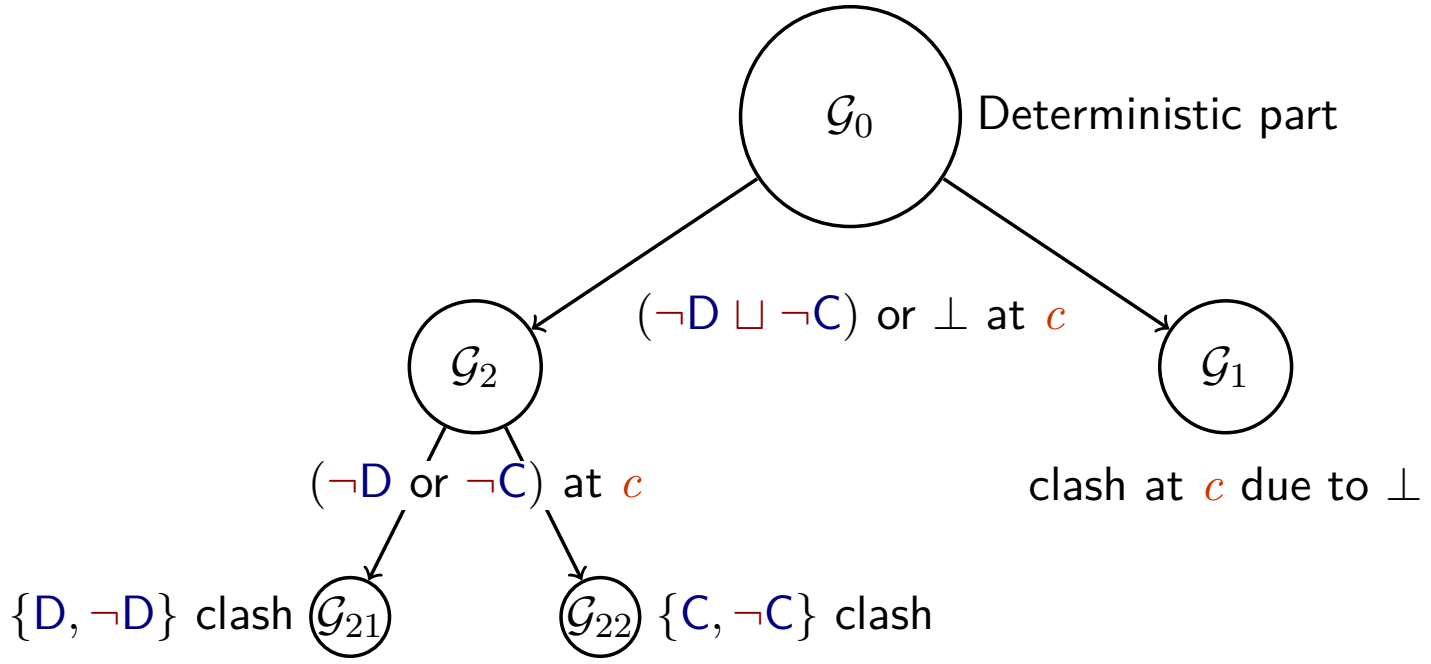

The space of choices explodes huge and very-very fast (lots of backtracking)

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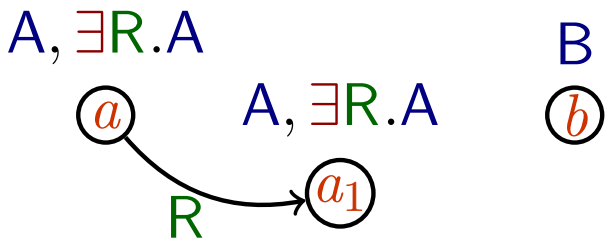

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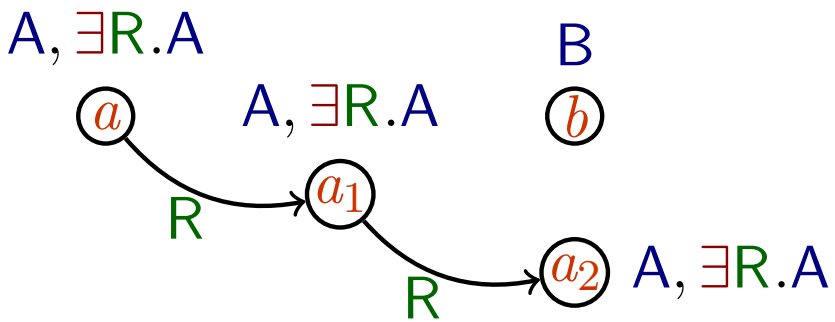

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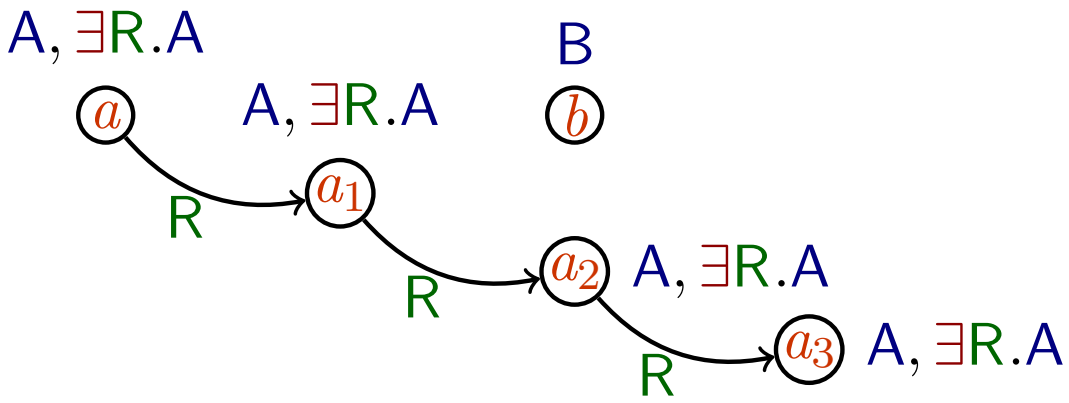

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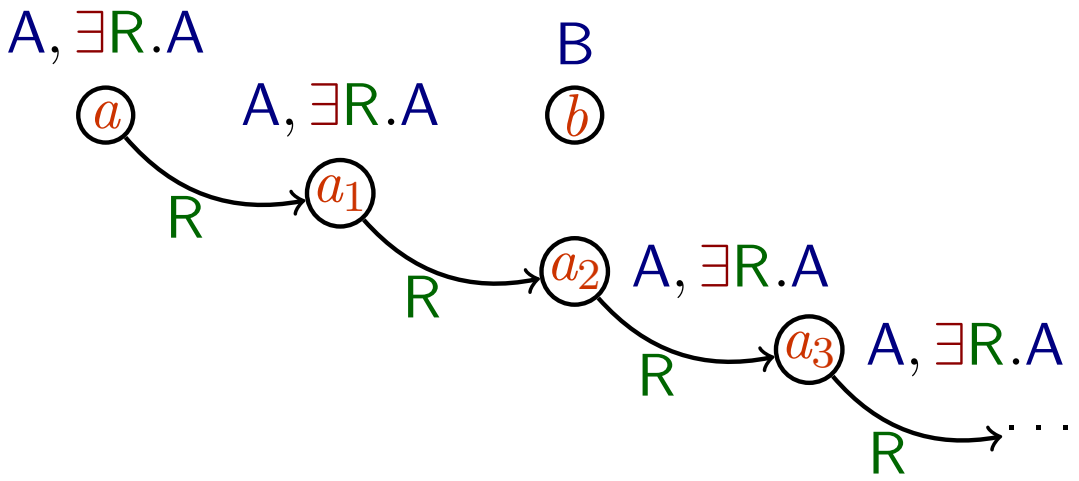

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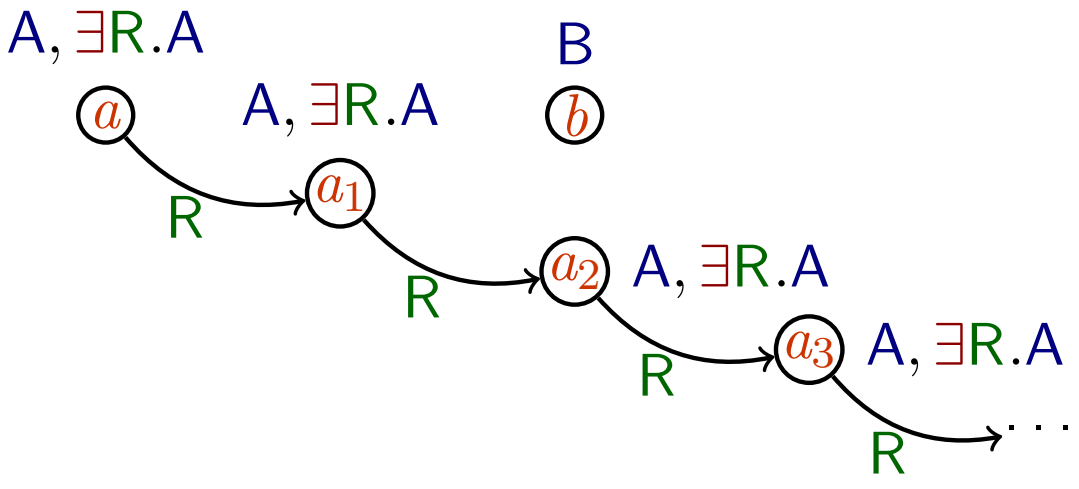

The GCI propagates $\exists \mathsf{R}.\mathsf{A}$ along the infinite chain

This expansion does not terminate on cyclic TBoxes

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$\sqcap$-rule: if $\exists R.C \in L(a)$ for some non-blocked a and there is no b s.t. $C \in L(b)$ and an R-edge from a to b is in $\mathcal{G}$

then create a new node b, add an R-edge from a to b and add C to $L(b)$

Theoretical Properties

The tableau algorithm with the global GCI rule and blocking is:

- Sound
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for standard reasoning problems in $\mathcal{ALC}$ (w.r.t. general TBoxes)

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Implemented in FaCT++, RacerPro, Pellet, Konclude,...

Reminder: Reasoning Problems in DLs

$\mathcal{ALC}$ Tableau with TBoxes

Two Essential Optimizations

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Two essential optimizations:

- Backjumping: being smarter about non-determinism
- Absorption: reducing non-determinism

Absorption of General Concept Inclusions

Remember: for $\mathcal{T} = \{C_i \sqsubseteq D_i \mid 1 \leq i \leq n\}$,
where no C_i is a concept name,

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and $\mathsf{A} \sqsubseteq D \in \mathcal{T}$ and $D \notin L(a)$

then add D to $L(a)$

GCI-rule if $C \sqsubseteq D \in \mathcal{T}$
and $\dot{\neg} C \sqcup D \notin L(a)$ for some non-blocked a,

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Observation: many GCIs are of the form $\mathsf{A} \sqcap X_1 \sqcap X_2 \sqcap \ldots \sqsubseteq C$
for concept name A and arbitrary concepts X_i
e.g., $\mathsf{Human} \sqcap \ldots \sqsubseteq C$ or $\mathsf{Device} \sqcap \ldots \sqsubseteq C$

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Localise GCIs to concept names by rewriting $\mathsf{A} \sqcap X \sqsubseteq C$ into equivalent $\mathsf{A} \sqsubseteq \neg X \sqcup C$

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In the best case nearly all GCIs are absorbed

In the worst case nothing changes

Absorption: Example

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No real ontology with complex concepts on the left can be classified without absorption

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Recall: when a clash is encountered, the non-deterministic algorithm backtracks i.e., returns to the last non-deterministic choice and tries the other possibility

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Backjumping attempts to recognize such interactions

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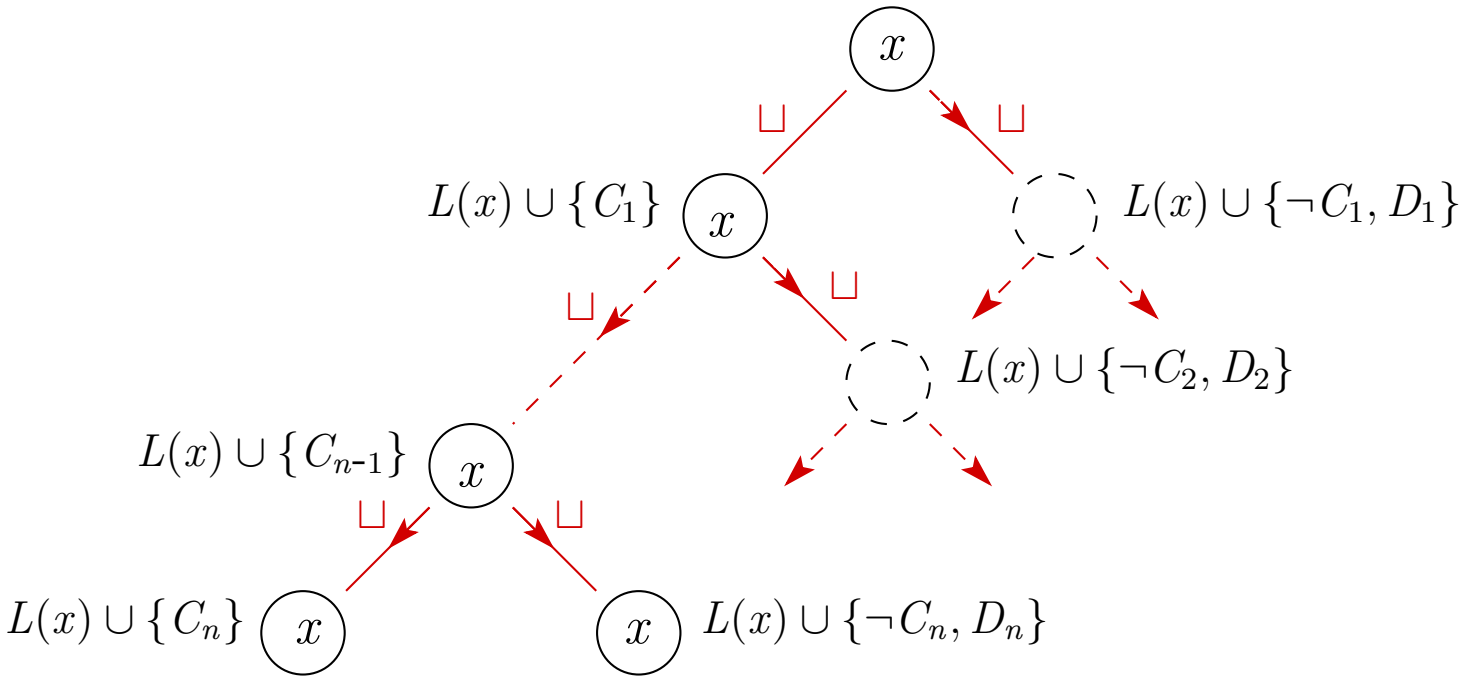

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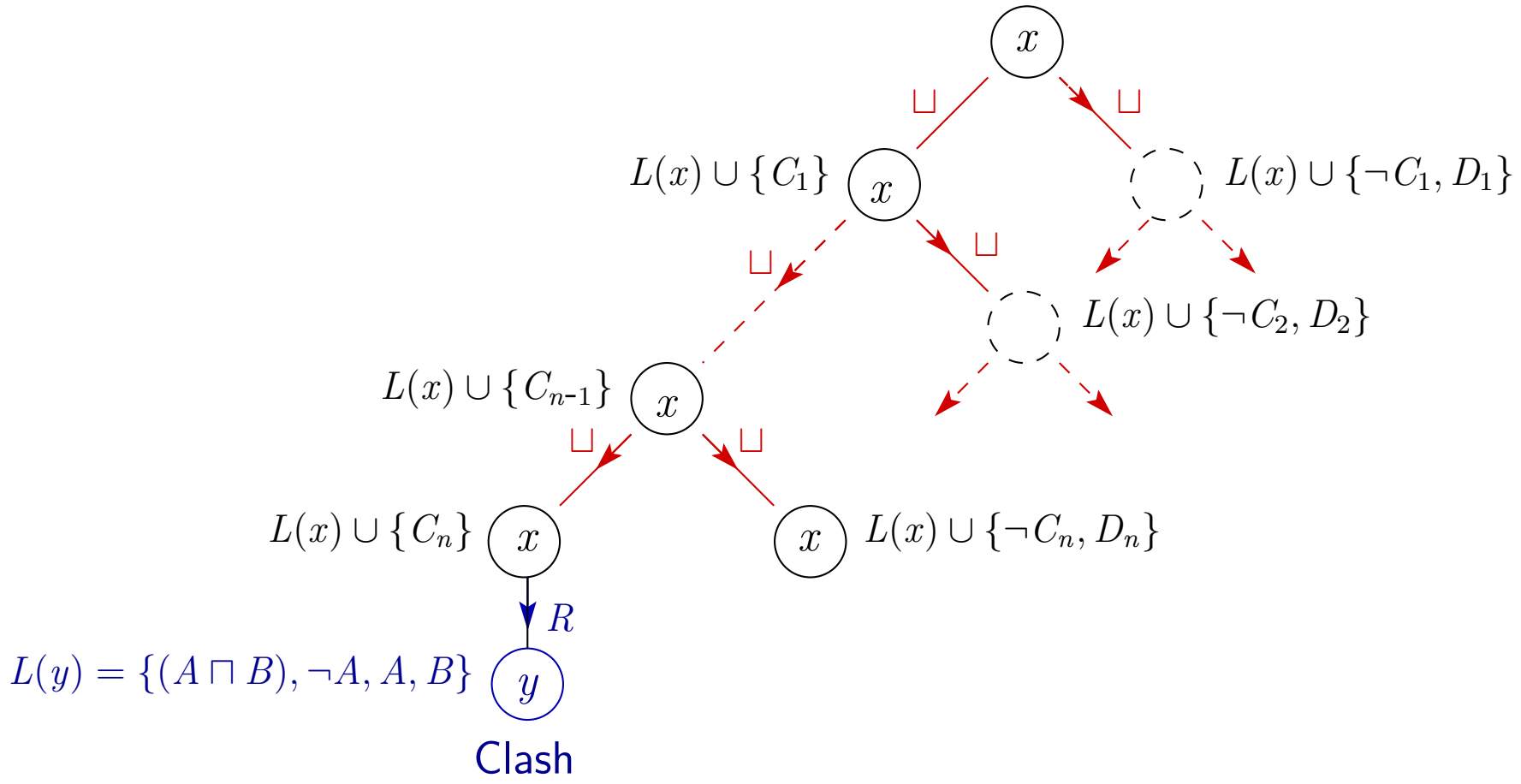

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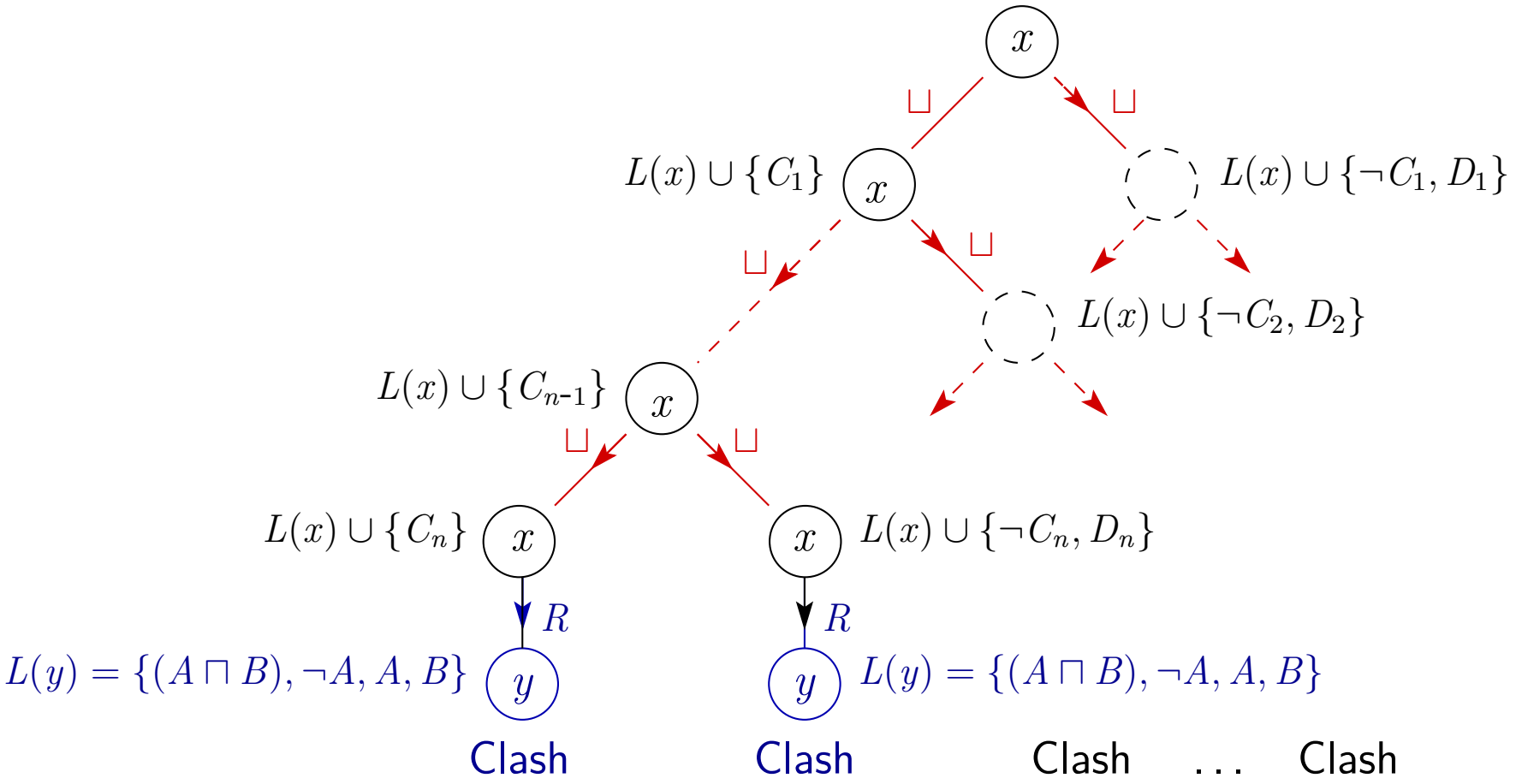

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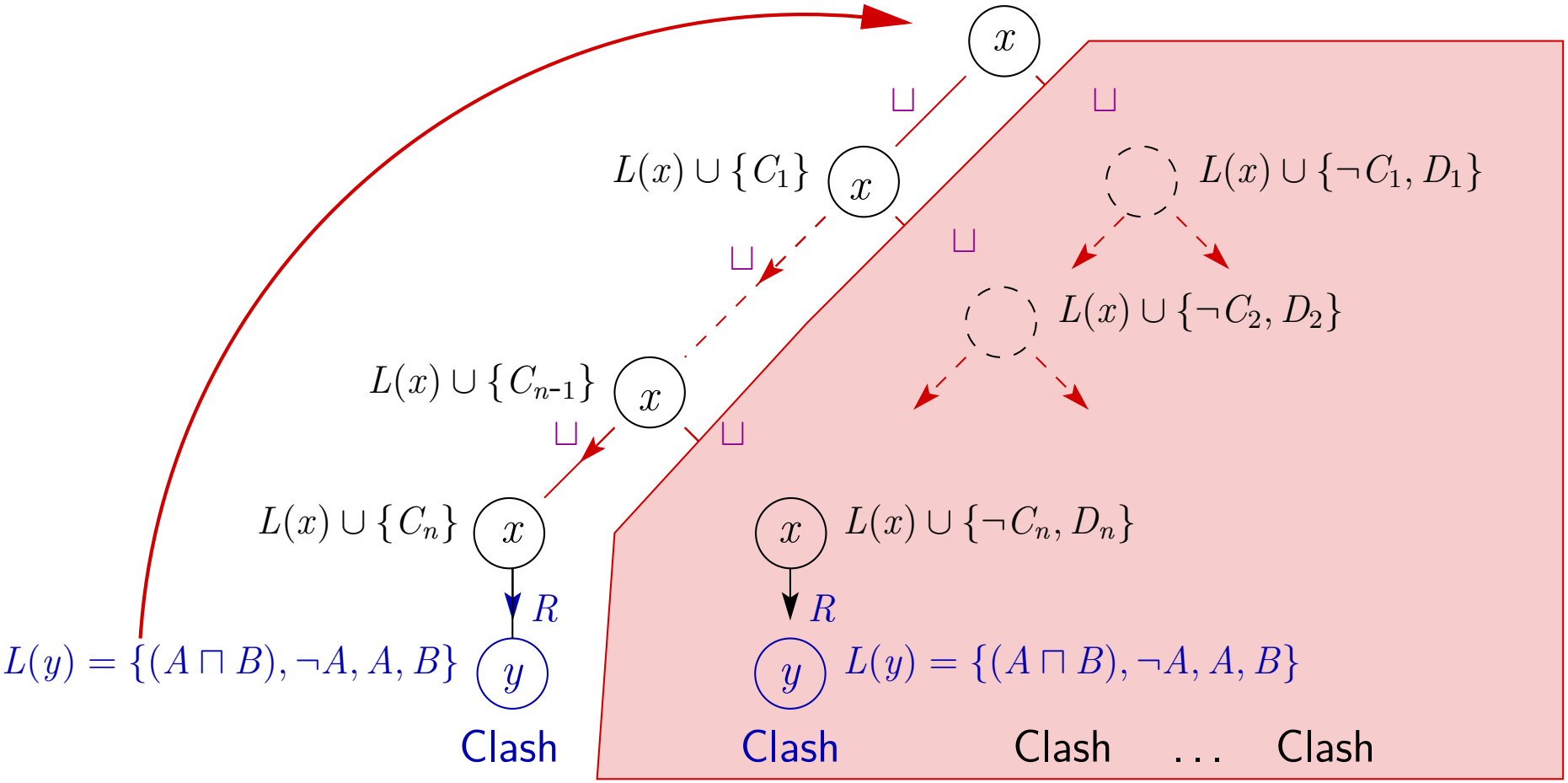

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- Cheap rules first
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$\sqcap$-rule: $\exists R.(A \sqcap B), (C_1 \sqcup D_1), \ldots, (C_n \sqcup D_n), \forall R.\neg A$

$\exists$-rule: $A \sqcap B, \neg A \subseteq L(v)$, which is a clash

Another Backjumping Example

$$(\exists R.(A \sqcap B) \sqcup C_0) \sqcap ((C_1 \sqcup D_1) \sqcap \ldots \sqcap (C_n \sqcup D_n)) \sqcap \forall R.\neg A$$

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- Naive tableau will systematically try each C_i or D_i

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Clause/choice ordering and other SAT heuristics help further

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- standard reasoning problems for $\mathcal{ALC}$ ontologies
- tableau algorithm for $\mathcal{ALC}$ ontologies that
  - requires blocking for termination
  - is a decision procedure for the standard reasoning problems
  - works on a set of labeled graphs in a non-deterministic way with backtracking
  - is implemented in state-of-the-art reasoners
- some essential optimizations

Tomorrow: Classification algorithm and performance experiments

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Ask your questions!