On the Complexity of HTN Plan Verification and its Implications for Plan Recognition

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Ulm University, Institute of Artificial Intelligence

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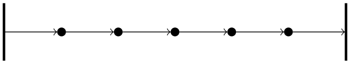
Plan Verification

Are we there yet?



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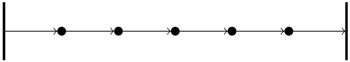

- $\mathcal{O}(n)$ for totally ordered classical plans

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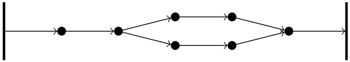

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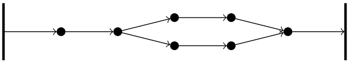

- $\mathcal{O}(n)$ for totally ordered classical plans
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- $\mathbb{NP}$-complete for PO planning (Chapman 1987; Nebel and Bäckström 1994)

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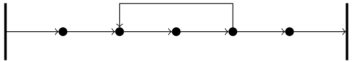

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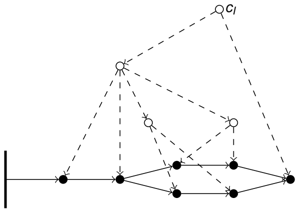

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Why plan verification?

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1. HTN Plan Verification is $\mathbb{NP}$ complete

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1. HTN Plan Verification is $\mathbb{NP}$ complete
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Hierarchical Task Network (HTN) Planning

$$\mathcal{P} = (P, C, c_I, M, L, s_I)$$

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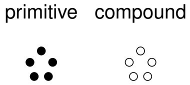

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Hierarchical Task Network (HTN) Planning

$\mathcal{P} = (P, C, c_I, M, L, s_I)$

- P a set of primitive tasks
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A solution $\mathrm{tn} \in Sol(\mathcal{P})$ must

- be a refinement of the initial task

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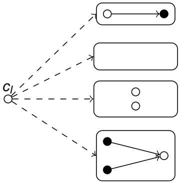

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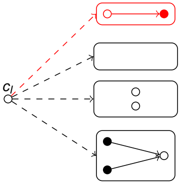

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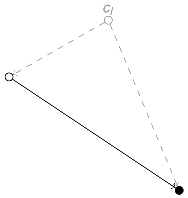

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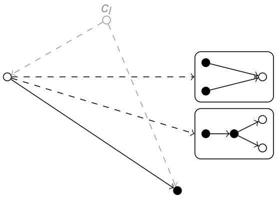

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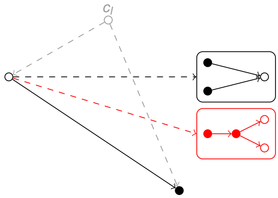

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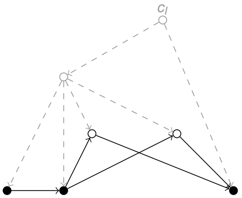

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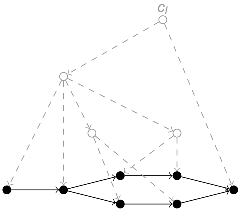

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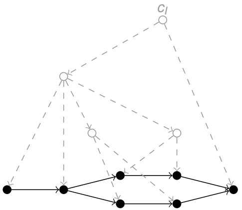

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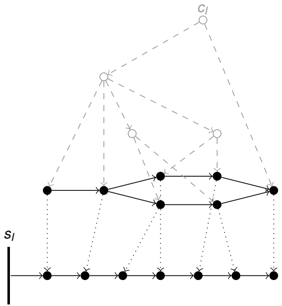

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- $s_I \subseteq L$ the initial state

A solution $\mathrm{tn} \in Sol(\mathcal{P})$ must

- be a refinement of the initial task
- only contain primitive tasks
- have a linearization, executable from the initial state

Plan Verification

Definition (VERIFYTN)

Let $\mathcal{P}$ be a planning problem and tn be a task network.
Decide whether $\mathrm{tn} \in \mathit{Sol}(\mathcal{P})$.

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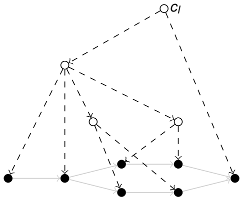

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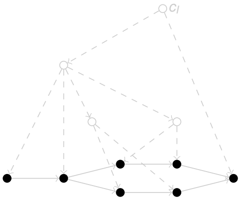

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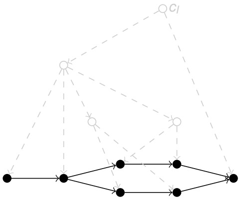

VERIFYTN: $\mathbb{NP}$-hardness

Theorem

VERIFYTN *is* $\mathbb{NP}$*-hard*

[1] (Erol, Hendler, and Nau 1994; Nebel and Bäckström 1994)

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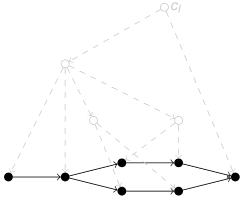
Proof.



Checking whether a partially ordered set of actions has an executable linearization is $\mathbb{NP}$-hard[1].

□

[1] (Erol, Hendler, and Nau 1994; Nebel and Bäckström 1994)

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Adapt the proof by Höller et al. (2014), showing that *Sol*($\mathcal{P}$) form a context sensitive language.

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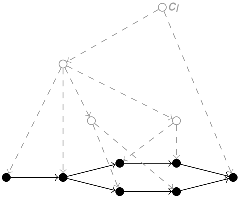

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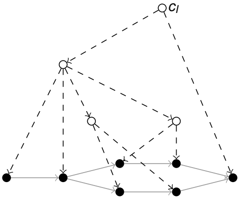
1. guess a linearization and check executability
2. guess decompositions and check them



VERIFYTN: $\mathbb{NP}$-membership

Proof. (continued)

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starting with c_I, guess decompositions and apply them

repeat until tn has been found

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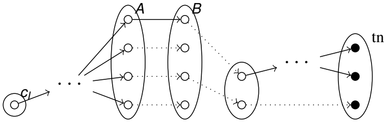

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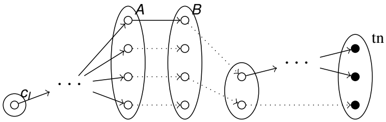

- handle decompositions with $|\mathrm{tn}_m| = 0$

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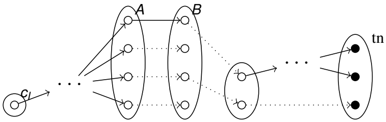

- handle decompositions with $|\mathrm{tn}_m| = 0$
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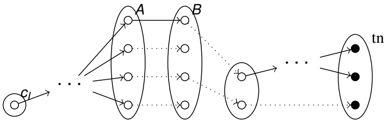

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- result: $|\mathrm{tn}|(|C| + 1)$

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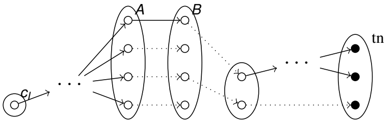

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- abort if more decompositions have been applied

□

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Main reason for $\mathbb{NP}$-hardness:

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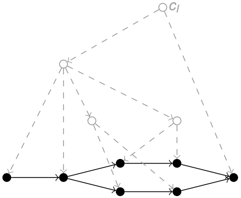
Main reason for $\mathbb{NP}$-hardness:

Find an executable linearization.



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Main reason for $\mathbb{NP}$-hardness:

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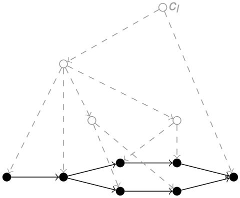
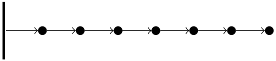
Suppose we already have one.

- by an observation
- by using *hybrid planning*, fusing HTN and POCL





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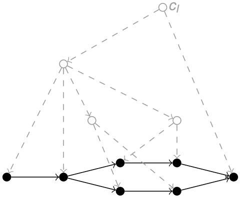

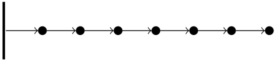

Definition (VERIFYSEQ)

Let $\mathcal{P}$ be a planning problem and ω a sequence of actions.
Decide whether $\exists \mathrm{tn} \in \mathit{Sol}(\mathcal{P})$ and ω is an executable linearization of tn.

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Proof.

Straightforward adaptation of previous proof. □

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Proof (idea).

Reduction from VERTEXCOVER.

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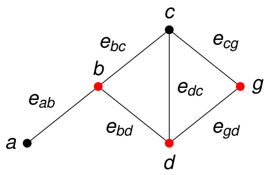
Reduction from VERTEXCOVER.

Reminder: Decide, given a graph $G = (V, E)$ and a number k, whether $\exists V_C \subseteq V$ s.t. $|V_C| \leq k$ and each edge e is adjacent to a node in the cover V_C.



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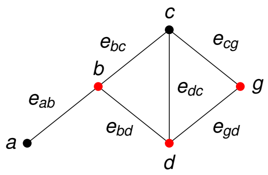

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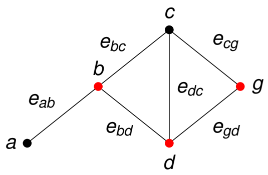

- V_C is chosen by decomposition of vertex-tasks
- ω ensures that $\leq k$ nodes are selected

VERIFYSEQ: $\mathbb{NP}$-hardness

$\mathbb{NP}$-completeness holds even for severely restricted HTN Planning Problems

The constructed domain needs

- neither preconditions nor effects
- no ordering constraints
- no cycles in the decomposition hierarchy
- only a depth of 2 (1 with an initial task network)

Plan Recognition

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- Given observed actions, decide which goal the user pursues

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- Given observed actions, decide whether they can lead to a solution

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- HTNs are commonly used as *Plan Libraries*

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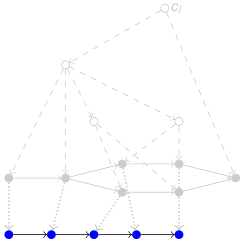

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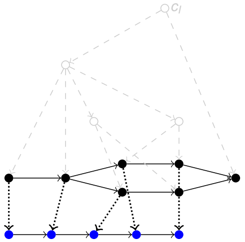

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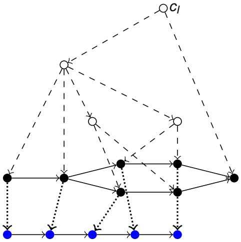

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This is VERIFYSEQ, i.e., still $\mathbb{NP}$-complete.

Conclusion

- HTN Plan Verification is $\mathbb{NP}$-*complete*
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- still $\mathbb{NP}$-*complete* for severely restricted HTN Planning Problems
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A *task network* $\mathrm{tn} = (T, \prec, \alpha)$ is a partially ordered set of tasks

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- Insert primitive tasks from O

HTN Solutions

- A task network tn is an HTN solution iff:
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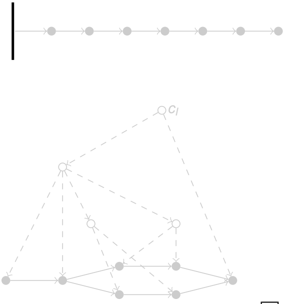

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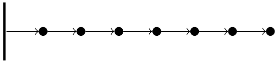

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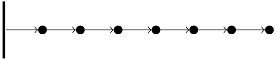

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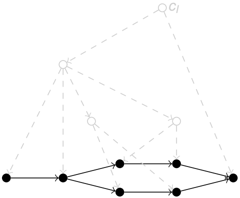

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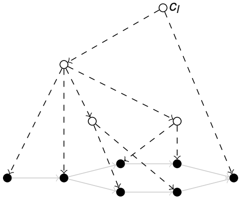
- check executability of ω
- guess a tn with linearization ω
- check whether tn can be decomposed form c_I



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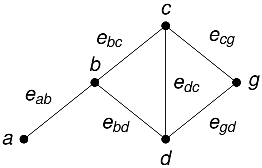
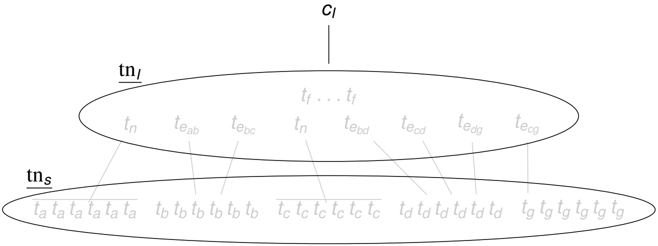
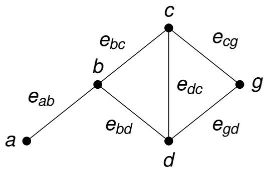
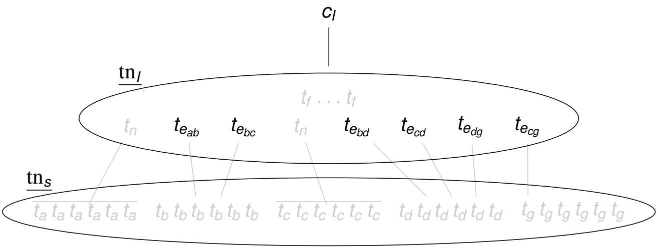




VERIFYSEQ: $\mathbb{NP}$-hardness

represent each edge by an abstract task





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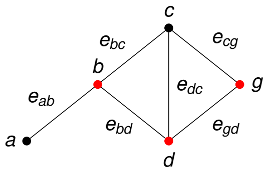

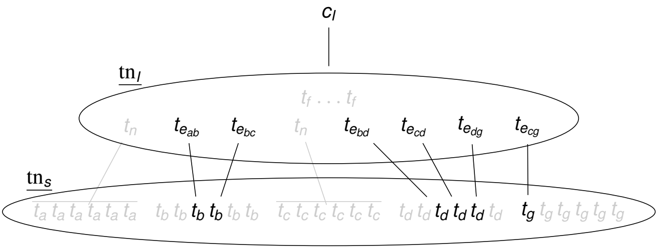

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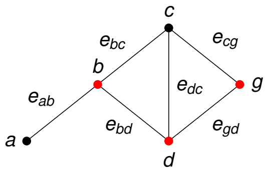

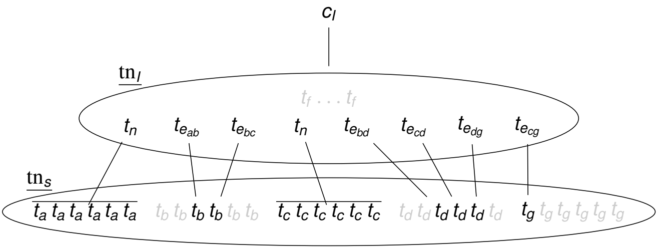

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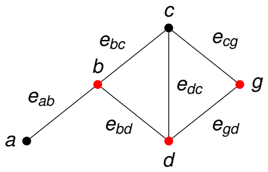

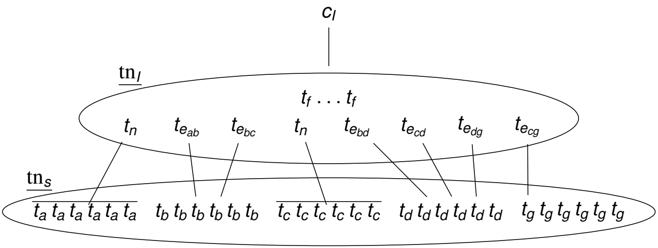

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