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Exercise sheet 1

Notation:

$$\mu_X = E[X]$$
$$\mu_{g(X)} = E[g(X)]$$
$$\sigma_X^2 = E[(X-\mu_X)^2]$$

Task 1.1 (Introduction to probability theory)

Given a random experiment 'toss of a manipulated coin', with the events H: 'head' or T: 'tail'. Let the probability for the event H be $P(H) = 1/3$. First, we consider three independent tosses.

a) Specify the elementary events and the corresponding probabilities for three independent coin tosses.

b) Let X be a random variable that counts how often head was tossed in the three independent tosses. Calculate the probability distribution $f_X(x)$.

c) Let Y be a random variable that counts how often head was tossed in the first of the three tosses. Specify the probability distribution $f_{XY}(x,y)$ in form of a table.

Task 1.2 (Conditional Probability, Statistical Independence)

Usually 80% of the students pass the Channel Coding exam. 85% of the students pass the Applied Information Theory exam. 95% which pass the Channel Coding exam also pass the Applied Information Theory exam. (This excercise uses fictive values.)

a) How many students pass both exams?

b) How many of the students pass at least one of the exams?

c) How many of the students which pass the Applied Information Theory exam also pass the Channel Coding exam?

d) Are passing the Channel Coding exam and passing the Applied Information Theory exam independent?

Task 1.3 (Total Probability)

An urn with 4 green, 5 red and 3 blue balls is given. Assume we draw 2 balls without putting the first one back. With which probability is the second ball green?

Task 1.4 (Expected value I)
A random variable X takes the values x_i with corresponding probabilities p_i

x_i	0	1	3
p_i	0.5	0.25	0.25

Calculate the mean values $\mathrm{E}[X]$, $\mathrm{E}[f_X(X)]$, $\mathrm{E}[-\log_2 f_X(X)]$ and the variance σ_X^2.

Task 1.5 (Expected value II)
Show with help of the expected value operator that the following dependencies hold. Use the linearity of the expected value.

a) The mean square is the sum of the variance and the squared mean, i.e.,

$$\mu_{X^2} = \sigma_X^2 + \mu_X^2.$$

The expected value of the product is the sum of the covariance and the product of the mean values, i.e.,

$$\mu_{XY} = \sigma_{XY}^2 + \mu_X \, \mu_Y.$$

b) When adding two random variables $Z = X + Y$, the mean values add up as well, i.e.,

$$\mu_Z = \mu_X + \mu_Y.$$

If the random variables X and Y are uncorrelated, then the variances add up, too, i.e.,

$$\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2.$$

Task 1.6 (Binomial distribution, Geometric distribution)
Assume that the probability that a bit gets transmitted incorrectly is 0.01.

a) What is the probability that exactly 2 errors occur when transmitting a sequence of 50 bits?

b) What is the probability that at least one bit is received erroneously?

c) What is the probability that there are at least 100 bits transmitted correctly before one error occurs?