Task 5.1 (MAP- and ML-Decoding)

Consider a binary source with probabilities $f(0) = 0.75$ and $f(1) = 0.25$ and no source coding. The source symbols are given as blocks of length 3 *bit* which are encoded using a triple bit parity check code $\mathcal{C}(6, 3, 2)$. The generator matrix G of the code $\mathcal{C}$ is given as

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

After transmission over a BSC with error probability p the word $y = (111011)$ is received.

a) Give the maximal number of detectable and correctable errors of the code $\mathcal{C}$.

b) Decode the word y in the code $\mathcal{C}$ by minimizing the Hamming-distance.

c) Decode the word y in the code $\mathcal{C}$ using ML-decoding, with respect to p.

Assume the encoded sequence was part of the subset $M = \Big\{(000), (100), (110), (111)\Big\}$.
Let $\mathcal{C}_M = \{c = i \cdot G \mid i \in M\}$ be the corresponding subset of the code $\mathcal{C}$ and let $p = 1/3$.

d) Decode the word y in $\mathcal{C}_M$ using MAP-Decoding.

Task 5.2 (Reed-Muller Codes and Incremental Redundancy)

Let $\mathcal{C}$ be the Reed-Muller Code $\mathcal{R}(1, 3)$. Let $i = (i_1\ i_2\ i_3\ i_4)$ be the information bits.

a) Give a generator matrix of the code $\mathcal{C}$ such that

$$\begin{aligned} (i_1\ i_2\ i_3) &\to u \\ i_4 &\to v \\ c &= (u|u + v)\,. \end{aligned}$$

b) Transform your matrix into a generator matrix for a punctured code at positions $\{2, 6\}$.

c) Consider a punctured code at positions $\{5, 6, 7, 8\}$. Give the name and parameters of this code.

Consider a transmission scheme using the punctured code $\mathcal{C}_p$ from b). Let the received word be $y_1 = (101011)$ and the incremental redundancy be $y_2 = (01)$.

d) Compare the number of errors that code $\mathcal{C}$ can correct to the number of errors that code $\mathcal{C}_p$ can detect.

e) Decode y_1 in $\mathcal{C}_p$ with the help of y_2 (if needed).