Task 6.1 (DMC properties)

Consider a discrete memoryless channel.

a) Give a definition for a

(i) uniformly dispersive channel
(ii) uniformly focusing channel
(iii) strongly symmetric channel

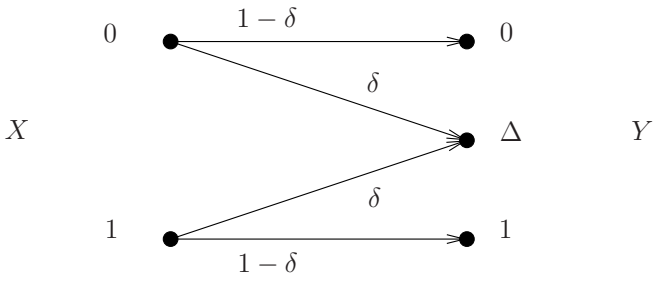
b) Give an example for a channel which is

(i) uniformly dispersive but not uniformly focusing
(ii) neither uniformly dispersive nor uniformly focusing
(iii) strongly symmetric

Task 6.2 (Channel capacity of symmetric channels)

Definition: A DMC is called *symmetric*, if its outputs can be partitioned into n sets, each with $|Y_i|$, $i = 1, \ldots, n$ outputs, so that the channel can be divided into n strongly symmetric channels each with $|X|$ inputs and $|Y_i|$ outputs and each of these channels is chosen with probability q_i.

a) Consider the binary erasure channel (BEC):



This channels erases the transmitted symbol with probability δ. Classify the BEC with respect to its symmetries and give a partition into strongly symmetric channels of the BEC.

The channel capacity of a symmetric channel is given by

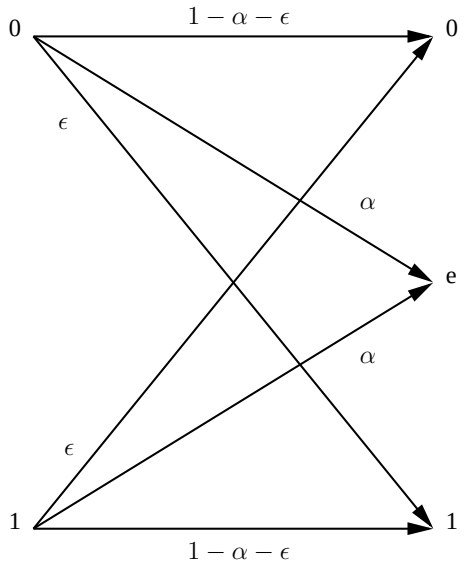
$$C_{sym} = \sum_{i=1}^{n} q_i C_i,$$

where q_i is the probability for the i-th strongly symmetric component channel and C_i is its capacity.

b) Give the channel capacity of the BEC.

Task 6.3 (Binary Symmetric Erasure Channel)

Consider the Binary Symmetric Erasure Channel (BSEC) that has both erasures and errors. Let the probability of error be ϵ and the probability of erasure be α, so the channel is given as follows:



a) Find the capacity of this channel

b) Specialize to the case of the binary symmetric channel ($\alpha = 0$)

c) Specialize to the case of the binary erasure channel ($\epsilon = 0$)