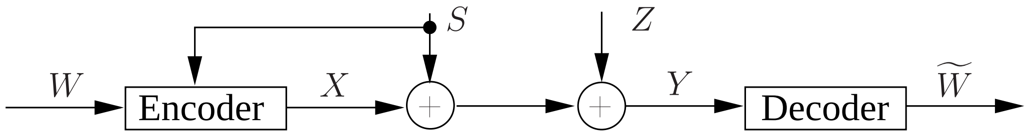
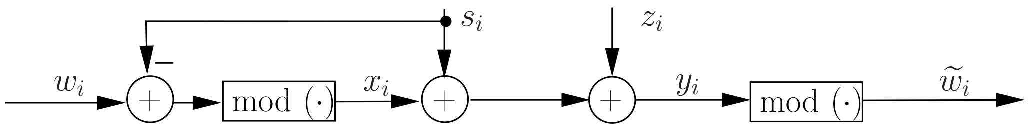
Task 8.1 (Tomlinson-Harashima Precoding)

Consider the channel $Y = X + S + Z$ depicted below, where both S and Z are noise terms and S is known to the transmitter.



Now, the channel input X can be encoded to optimize the transmission of a binary message $W \in \{-1, +1\}$ to the receiver. For this purpose, we will use Tomlinson-Harashima precoding and assume $Z = 0$ at all times. The encoding and decoding for the Tomlinson-Harashima scheme is done as follows:



Thus, the channel input is chosen such that

$$x_i = (w_i - s_i) \bmod q,$$

while the output of the decoder is chosen such that

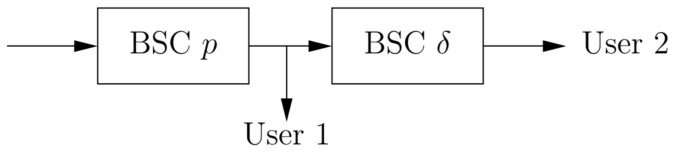
$$\widetilde{w}_i = y_i \bmod q.$$

a) How is q selected in the Tomlinson-Harashima precoding scheme?

b) Given the input sequence $\mathbf{w} = [-1,\ -1,\ +1,\ -1,\ +1,\ +1,\ +1,\ -1]$ and the known interference sequence $\mathbf{s} = [3.2,\ -2.4,\ 0.3,\ -1.8,\ -4.2,\ -1,\ 1.3,\ 1.5]$, calculate the transmit sequence $\mathbf{x}$, the receive sequence $\mathbf{y}$ and the decoded sequence $\widetilde{\mathbf{w}}$

c) What is the average transmit power of the sequence $\mathbf{x}$? Compare this to the transmit power without the modulo operation, i.e., $x_i = w_i - s_i$.

Task 8.2 (Degraded Broadcast Channel, BSC)



We consider the degraded broadcast channel consisting of a cascade of two BSCs with error probabilities p and δ, respectively.
In the broadcast case, a superposition code shall be used

$$\underline{c} = \underline{c_1} + \underline{c_2}$$

For long codewords, the addition of $\underline{c}_1$ can be interpreted as an additional BSC with error rate $\gamma = P(1)$.

a) Calculate the capacity C_1 for user 1 and C_2 for user 2 for the case, that only user 1 or user 2 use the channel exclusively.

b) Show that the rate region that is obtained by applying the superposition code is given by

$$\begin{aligned} R_1 &\leq h(\gamma(1-p) + (1-\gamma)p) - h(p) \\ R_2 &\leq 1 - h(\gamma(1-\epsilon) + (1-\gamma)\epsilon) \text{ with } \epsilon = (1-p)\delta + p(1-\delta) \end{aligned}$$

Hint: Assume that user 1 can decode $\underline{c}_2$ correctly, see script section 6.3