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Task 9.1 (Broadcast vs. TDMA, AWGN Channel)

Two users A and B want to receive data from a time discrete AWGN channel. User A needs a rate $R_A = 0.5$ and the rate R_B of user B shall be optimized.
User A has a noise power of $N_A = 0.5$, while user B has a better channel with noise power $N_B = 0.1$. The available transmit power shall be $P = 1$ at all times.

a) Calculate the maximum achievable rate $R_{B,TD}$ if TDMA with constant transmit power is applied.

b) Determine the maximum achievable rate $R_{B,BC}$ for the broadcast case and "cloud modulation". Here, two modulation alphabets with average power αP and $(1-\alpha)P$ are superpositioned.

Task 9.2 (TDMA in the Multiple-Access Channel)

We consider a two-user Gaussian Multiple-Access Channel,

$$Y = X_1 + X_2 + Z,$$

where Y is the channel output, X_1 and X_2 are the channel inputs and Z is Gaussian noise with zero-mean and variance N. The inputs are real and restricted by the power constraints $E(X_i^2) \leq P_i$ for $i = 1, 2$.

a) Calculate the capacities C_1 and C_2 if either user 1 or user 2 use the channel exclusively.

b) In contrast to the successive decoding scheme discussed in the lecture, we will consider a time division multiple-access (TDMA) scheme here. In this scheme, user 1 uses the channel exclusively in fraction α of the time, while in fraction $1-\alpha$ of the time user 2 uses the channel exclusively. Let $\widetilde{P}_i$ $(i = 1, 2)$ be the transmit powers of the users while they use the channel. Calculate $E(X_1^2)$ and $E(X_2^2)$ for the TDMA scheme.
Hint: The expectation is now also over time

c) What are the maximum values for $\widetilde{P_1}$ and $\widetilde{P_2}$ that can be used such that the power constraints $E(X_i^2) \leq P_i$ are still satisfied?

d) Calculate the achievable rates R_1 and R_2 of the users for the TDMA schemes if the transmit powers are chosen as calculated in c).

e) Show that $\alpha^* = \frac{P_1}{P_1+P_2}$ maximizes the achievable sum rate $R_\Sigma = R_1 + R_2$. Compare this maximum sum-rate to the achievable sum-rate for the successive decoding scheme.

f) Sketch the achievable rate regions for both the TDMA scheme and the successive decoding scheme for $P_1 = 15$, $P_2 = 7$, $N = 1$.