Task 9.1 (Tomlinson-Harashima Precoding)
Consider the channel $Y = X + S + Z$ depicted below, where both $S$ and $Z$ are noise terms and $S$ is known to the transmitter.

Now, the channel input $X$ can be encoded to optimize the transmission of a binary message $W \in \{-1, +1\}$ to the receiver. For this purpose, we will use Tomlinson-Harashima precoding and assume $Z = 0$ at all times. The encoding and decoding for the Tomlinson-Harashima scheme is done as follows:

Thus, the channel input is chosen such that
\[ x_i = (w_i - s_i) \mod q, \]
while the output of the decoder is chosen such that
\[ \tilde{w}_i = y_i \mod q. \]

a) How is $q$ selected in the Tomlinson-Harashima precoding scheme?

b) Given the input sequence $w = [-1, -1, +1, -1, +1, +1, +1, -1]$ and the known interference sequence $s = [3.2, -2.4, 0.3, -1.8, -4.2, -1, 1.3, 1.5]$, calculate the transmit sequence $x$, the receive sequence $y$ and the decoded sequence $\tilde{w}$.

c) What is the average transmit power of the sequence $x$? Compare this to the transmit power without the modulo operation, i.e., $x_i = w_i - s_i$. 
Task 9.2 (Degraded Broadcast Channel, BSC)

We consider the degraded broadcast channel consisting of a cascade of two BSCs with error probabilities $p$ and $\delta$, respectively.

In the broadcast case, a superposition code shall be used

$c = c_1 + c_2$

For long codewords, the addition of $c_1$ can be interpreted as an additional BSC with error rate $\gamma = P(1)$.

a) Calculate the capacity $C_1$ for user 1 and $C_2$ for user 2 for the case, that only user 1 or user 2 use the channel exclusively.

b) Show that the rate region that is obtained by applying the superposition code is given by

\[
R_1 \leq h(\gamma(1-p) + (1-\gamma)p) - h(p) \\
R_2 \leq 1 - h(\gamma(1-\epsilon) + (1-\gamma)\epsilon) \quad \text{with} \quad \epsilon = (1-p)\delta + p(1-\delta)
\]

*Hint: Assume that user 1 can decode $c_2$ correctly, see script section 6.3*