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Solutions to exercise sheet 8

Solution to Task 8.1 (Tomlinson-Harashima Precoding)

a) In Tomlinson-Harashima precoding the input alphabet is extended, such that the points are equidistant. Here, we have $W \in \{-1, +1\}$, i.e., the extension is such that -1 is represented by $\dots, -5, -1, 3, 7, \dots$ while $+1$ is represented by $\dots, -7, -3, 1, 5, \dots$. Thus, the interval we are repeating ranges from -2 to 2. As q has to be chosen as the length of this interval we have $q = 4$.

b) The output range of the modulo functions in THP is chosen as $[-\frac{q}{2}, \frac{q}{2})$, so that we obtain the following sequences

$$\begin{array}{rcccccccccl} \mathbf{x} = [& -0.2 & 1.4 & 0.7 & 0.8 & 1.2 & -2.0 & -0.3 & 1.5 &] \\ \mathbf{y} = [& 3 & -1 & 1 & -1 & -3 & -3 & 1 & 3 &] \\ \widetilde{\mathbf{w}} = [& -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 &] \end{array}$$

Thus, the sequence $\widetilde{\mathbf{w}}$ is equivalent to the input sequence $\mathbf{w}$.

c) The average transmit power of the transmit vector $\mathbf{x}$ is given by

$$\frac{1}{8}\sum_{i=1}^{8} |x_i|^2 = 1.3638$$

If the modulo operation is omitted, the transmit vector would be

$$\mathbf{w} - \mathbf{s} = [-4.2,\ 1.4,\ 0.7,\ 0.8,\ 5.2,\ 2.0,\ -0.3,\ -2.5],$$

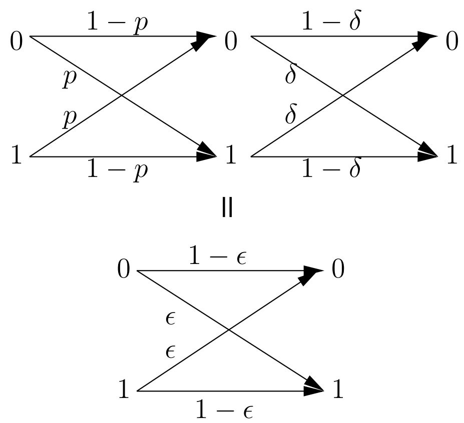
which has an average power of

$$\frac{1}{8}\sum_{i=1}^{8} |w_i - s_i|^2 = 7.2638.$$

Thus, the modulo operation reduces the transmit power enormously.

Solution to Task 8.2 (Degraded Broadcast Channel, BSC)

a) A cascade of two BSCs can be transformed into a single BSC:



with $\epsilon = (1-p)\delta + p(1-\delta)$. Thus $C_1 = 1 - h(p)$ and $C_2 = 1 - h(\epsilon)$.

b) User 2:
We treat $\underline{c}_1$ as an additional error on $\underline{c}_2$ (see script, section 6.3), i.e. an additional BSC with error probability $P(1) = \gamma$. This means we again have a cascade of two BSCs with error probabilities γ and ϵ, which gives us a BSC with error probability $(1-\gamma)\epsilon + \gamma(1-\epsilon)$.
$\Rightarrow R_{2,BC} \leq 1 - h((1-\gamma)\epsilon + \gamma(1-\epsilon))$

User 1:
For superpositioning, the channel input can be written as

$$X = X_1 + X_2,$$

where X_i is the bit that shall be transmitted to user i and $f_{X_i}(1) = \gamma$. As user 1 has a better reception than user 2, it can decode $\underline{c}_2$ and thus calculate

$$\widetilde{Y}_1 = Y_1 - X_2 = X_1 + Z,$$

where Y_1 is the received signal of user 1 and Z is the possible bit error from the first BSC with $f_Z(1) = p$.
The entropy of $\widetilde{Y}_1$ is calculated as

$$\begin{aligned}
&f_{\widetilde{Y}_1}(1) = f_{X_1}(1) f_Z(0) + f_{X_1}(0) f_Z(1) = (1-p)\gamma + (1-\gamma)p =: q, \\
&f_{\widetilde{Y}_1}(0) = 1 - q, \\
&H(\widetilde{Y}_1) = h(q).
\end{aligned}$$

And together with

$$H(\widetilde{Y}_1|X_1) = H(Z) = h(p)$$

we get

$$\begin{aligned} R_{1,BC} &\leq I(X_1, \widetilde{Y}_1) = H(\widetilde{Y}_1) - H(\widetilde{Y}_1|X_1) \\ &= h(q) - h(p) = h(\gamma(1-p) + (1-\gamma)p) - h(p). \end{aligned}$$