Exercise 1 (CLP Cryptoarithmetic Puzzle).
Replace distinct letters by distinct digits (numbers have no leading zeros), s.t. the following calculation holds.

<table>
<thead>
<tr>
<th>T</th>
<th>E</th>
<th>S</th>
<th>T</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>F</td>
<td>E</td>
<td>S</td>
<td>T</td>
</tr>
<tr>
<td>+</td>
<td>D</td>
<td>E</td>
<td>I</td>
<td>N</td>
</tr>
<tr>
<td>=</td>
<td>K</td>
<td>R</td>
<td>A</td>
<td>F</td>
</tr>
</tbody>
</table>

a) Stick to the example presented in the course and use the library clpfd.
b) Write a predicate distinct(L) which is true if all elements from list L have a value from \{0, 1, \ldots, 9\} and are (pairwise) distinct. Use the library clpq.

CCLP

We use (a subset of) the Constraint Handling Rules (CHRs) to program in the CCLP paradigm. The following CCLP-clause

\[ H \leftarrow C : D \mid G \]

is written by

\[ N @ H \Leftarrow C , D \mid G \]

where N @ is an optional name for the rule.

1. Read the SICStus manual on how to use CHRs. Before using CHR rules, the CHR library and a handler must be set by \:- use_module(library(chr)). handler h.
2. In order to differentiate (built-in) predicates from constraints, the latter have to be declared, e.g. constraints con/1.
3. The rule \[ N @ H \Leftarrow C \mid G \] is a short form of \[ N @ H \Leftarrow C , true \mid G \].

Exercise 2 (Comparison of CLP and CCLP).

Compare the following CLP- (in the left column) and CCLP-programs (in the right column), which consist of one of the given rules by posing the queries given below. Check your answers with the system’s answers. Make sure, you understand why seemingly innocuous rules produce different answers.

\[ p(a) :- true. \]
\[ p(X) :- X=a. \]
\[ p(X) :- X = a, X = b. \]
\[ p1 @ p(a) \Leftarrow true \mid true. \]
\[ p2 @ p(X) \Leftarrow X=a \mid true. \]
\[ p3 @ p(X) \Leftarrow true \mid X=a. \]
\[ p4 @ p(X) \Leftarrow true , X = a \mid true. \]
\[ p5 @ p(X) \Leftarrow X = a , X = b \mid true. \]

Queries: (a) p(a), (b) p(b), and (c) p(C).

Exercise 3. Implement the following three variants of the CCLP program \texttt{min} (minimum) in CHR:

- Variant 1:
  \[ \text{min1}_1(X,Y,Z) \leftarrow \top \mid X \leq Y, Z = X \]
  \[ \text{min1}_2(X,Y,Z) \leftarrow \top \mid Y \leq X, Z = X \]
• Variant 2:
  \[ \text{min}_2(X, Y, Z) \leftarrow X \leq Y \mid Z = X \]
  \[ \text{min}_2(X, Y, Z) \leftarrow Y \leq X \mid Z = X \]

• Variant 3:
  \[ \text{min}_3(X, Y, Z) \leftarrow X \leq Y : Z = X \mid \top \]
  \[ \text{min}_3(X, Y, Z) \leftarrow Y \leq X : Z = X \mid \top \]

Test and explain the different responses of the variants by posing the following six queries (only one at a time).

\[
\begin{align*}
\text{min}(1, 2, C) & \quad \text{min}(A, 2, 1) & \quad \text{min}(A, 2, 3) \\
\text{min}(A, A, B) & \quad \text{min}(1, 2, 1) & \quad \text{min}(1, 2, 3)
\end{align*}
\]