Exercise 1. Compare the following CHR programs, which consist of one of the given rules by posing the given queries. Check your answers with the system’s answers. Make sure, you understand why seemingly innocuous rules produce different answers.

\begin{verbatim}
c1 @ c(X), c(X) <=> q(X,X).
c2 @ c(X), c(Y) <=> r(X,Y).
c3 @ c(X), c(X) ==> q(X,X).
c4 @ c(X), c(Y) ==> r(X,Y).
\end{verbatim}

Queries:
\begin{itemize}
  \item a) c(X), c(X)
  \item b) c(X), c(Y)
  \item c) c(X), c(Y), X=Y
\end{itemize}

More variants:
\begin{verbatim}
q1 @ p(X,Z), q(Z,Y) <=> q(X,Y).
q2 @ q(Z,Y), p(X,Z) <=> q(X,Y).
q3 @ p(X,Z), q(Z,Y) ==> q(X,Y).
q4 @ q(Z,Y), p(X,Z) ==> q(X,Y).
q5 @ p(X,Z) \ q(Z,Y) <=> q(X,Y).
q6 @ q(Z,Y) \ p(X,Z) <=> q(X,Y).
\end{verbatim}

Queries:
\begin{itemize}
  \item d) p(A,B), q(B,C)
  \item e) p(A,B), q(B,C), p(D,A)
\end{itemize}

Comment on the system’s answers for queries a) to e).
Comment on the system’s answers for the rule q5 and the following two queries.
\begin{itemize}
  \item p(X,C), p(Y,C), q(C,A) und
  \item p(Y,C), p(X,C), q(C,A).
\end{itemize}

Exercise 2. Implement the constraints \texttt{less/2} (encoding $<$) und \texttt{leq/2} (encoding $\leq$) and their mutual relations/interactions in CHR. You may find the lecture’s CHR program for the $\leq$ constraint helpful.

For an example query, take your last name as a sequence of variables with $\leq$ constraints between succeeding characters.
The name \texttt{Fruehwirth} translates to the query
\begin{verbatim}
F \text{leq} R, R \text{leq} U, U \text{leq} E, E \text{leq} H, H \text{leq} W, W \text{leq} I, I \text{leq} H
\end{verbatim}
with answer \begin{verbatim}
F \text{leq} E, H=E, I=E, R=E, T=E, U=E, W=E.
\end{verbatim}
Tests should include (at least) three more queries consisting of combined \texttt{less} and \texttt{leq} constraints.

Exercise 3. Use the CHR Constraint \texttt{leq/2} from the previous exercise for a “constrain-and-generate” version of the sorting example from assignment #1-2. To this end, replace the clpq-Constraint $<=$ by the CHR-constraint \texttt{leq}.

Your tests should (at least) include the following queries
\begin{verbatim}
?- \text{permsortCHR([1,A,3],[1,3,7])}.
?- \text{permsortCHR([2,A],X)}.
?- \text{permsortCHR([A,B,A],X)}.
?- \text{permsortCHR(List,[1,X,3])}.
?- \text{permsortCHR([1,X,Y],[X,1,Y]), \text{permsortCHR([4,5,10], [Z,Y,W])}}.
\end{verbatim}