A Rule-Based Tool for Analysis and Generation of Graphs Applied to Mason’s Marks

Thom Frühwirth
Ulm University, Germany

http://www.informatik.uni-ulm.de/pm/fileadmin/pm/home/fruehwirth/

Abstract

We are developing a rule-based implementation of a tool to analyse and generate graphs. It is used in the domain of mason’s marks. For thousands of years, stonemasons have been inscribing these symbolic signs on dressed stone. Geometrically, mason’s marks are line drawings. They consist of a pattern of straight lines, sometimes circles and arcs. We represent mason’s marks by connected planar graphs.

Our prototype tool for analysis and generation of graphs is implemented in the rule-based declarative language Constraint Handling Rules (CHR). It features

- a vertex-centric logical graph representation as constraints,
- derivation of properties and statistics from graphs,
- recognition of (sub)graphs and patterns in a graph,
- automatic generation of graphs from given constrained subgraphs,
- drawing graphs by visualization using svg graphics

We started to use the tool to classify and to invent mason’s marks.

1 Tool Description

Mason’s marks are symbols often found on dressed stone in historic buildings. In Europe, they were common from the 12th century on [Fri32, Dav54]. There, one can mainly find mason’s marks from the medieval ages, mostly in churches, cathedrals and monasteries. In one such building, there may be a thousand mason’s marks of hundred different designs. Mason’s marks tend to be simple geometric symbols, usually constructed using rulers and compasses and precisely cut with a chisel. In this way a distinctive sign consisting of straight lines and curves could be produced with little effort.

Our prototype graph analysis and generation tool is currently implemented using CHR in SWI Prolog [WDKTF14]. We assume some basic familiarity with Prolog and Constraint Handling Rules (CHR) [Frü09, Frü15, Frü18a, FR18].
1.1 Representation of Mason Marks as Graphs

We represent mason’s marks (currently without arcs) by connected \textit{planar straight-line graphs}, a drawing of planar graphs in the plane such that its edges are straight line segments [Tam13]. A line (segment) has two nodes, two endpoints, given by Cartesian coordinates. Each point is defined by a pair of numbers written X-Y. For convenience of manipulation, we redundantly represent lines at the same time by polar coordinates, which consist of a reference point (pole), which is the first endpoint of the line, a line length (radius) and an angle (azimuth) in degrees. This leads to the following line constraint:

\[
\text{line}(\text{EndPoint1}, \text{EndPoint2}, \text{LineLength}, \text{Angle})
\]

With polar coordinations, translation, rotation and scaling of lines is easy. With Cartesian coordinates, visualization by translation into \textit{svg} graphics is easy.

1.2 Analysis of Graphs

From a given graph, i.e. its line constraints, we can generate information using propagation rules. For example, one can compute counts for the occurrences of each value in the components of a line (points, lengths, angles) to collect statistical information about the graph.

The constraint \(a(\text{Type}, \text{Count}, \text{Value})\) can be considered as an array entry that contains for each \text{Value} of a certain \text{Type} its \text{Count} of occurrences. Below, the first rule adds such entries for the same \text{Type,Value} pair. The second rule computes relevant information from a single line. (\text{line} is abbreviated to \text{l} in the remainder of the paper.)

\[
% \text{add counts for two entries of the same Type, Value pair}\n\text{a(T,N1,V), a(T,N2,V) } \Rightarrow \text{ N is N1+N2, a(T,N,V).}
\]

\[
% \text{compute statistical information about lines of a graph}\n% \text{Types: l(ine)c(ount), n(ode), l(ine )l(ength), a(ngle)}\n\text{l(P1,P2,L,A) } \Rightarrow \text{ a(cl,1,l),a(n,1,P1),a(n,1,P2),a(ll,1,L),a(a,1,A).}
\]

The first rule is a \textit{simplification rule}. It replaces two matching \text{a} constraints by a new one containing the sum. The second rule is a \textit{propagation rule} that adds a constraints when a line constraint is matched without removing it.

1.3 Pattern Matching of Graphs

We want to find patterns and recognize subgraphs in a graph. To account for scaling and rotation, we introduce two Prolog predicates that we will use in the guard of rules. Here are two examples: how to recognize parallel lines and the subgraph depicted in Figure 1. We use a constraint \text{recognized(What,NodeList)} to record what has been recognized for which nodes.
% two parallel lines have the same angle
l(A,B,L1,A), l(C,D,L2,A) ==> recognized(parallel,[A,B,C,D]).

% recognize subgraph comprised of four lines given in Figure 1
l(A,B,L1,A1), l(B,C,L2,A2), l(E,C,L3,A3), l(C,D,L4,A4) ==>
rotated(([A1,A2,A3,A4],[90,0,90,90]),
scaled([L1,L2,L3,L4],[1,1,1,1]) |
recognized(y_sign,[A,B,C,D,E]).

1.4 Node-Centric Representation of Graphs by Half-Lines

Through exhaustive initial experiments we found that for the encoding and
generation of mason’s marks a node-centric (vertex-centric) representation of
their underlying connected planar straight-line graph is helpful. Each node is
at the center of several lines leaving it. We record the length of these lines and
the angle between neighboring lines in a list for each node of degree larger than
one. For example in node([2,90,1,90,2]), the line lengths are 2, 1, 2 and
the angles between the lines are 90, 90. This constraint represents a turnstile
symbol ⊢, see Fig. 2.

In order to describe larger connected graphs, the node data structure rep-
resentation is extended to allow identifiers for lines. These identifiers are op-
tionally attached to the line-lengths. If such an identifier is shared between two
lines in different nodes it means that these lines are the same, with the two
nodes as endpoints. Such annotated lines we call half-lines, because one needs
a matching pair of them to form a valid line.

When the nodes are translated into line constraints, the subgraphs of the two
nodes connected by this common line are scaled and rotated such that the half-
lines become identical. For example node([2,90,1-I,90,2]), node([3,45,3-I,45,3])
depicts the graph given in Fig. 3. In effect, the node([3,45,3-I,45,3]) is rotated and scaled to meet the half-line identified by 1 in node([1,45,1-I,45,1]). Contrast this with the situation in Figure 4, where the half line
of the first node has changed.
1.5 Exhaustive Generation of Arbitrary Graphs

We can exhaustively generate graphs from a given node-centric representation consisting of node constraints containing half-lines with unique occurrences of free unbound logical variables as identifiers. Different resulting graphs are possible, depending on which half-lines are identified by binding (aliasing) their variable identifiers. Not all such matchings lead to a valid graph that is geometrically possible.

The given graph in node-centric representation is translated into a conjunction of lines, where some of them are half-lines containing identifiers. Two half-lines react with each other by the following rule that contains a disjunction from Prolog in its right hand side. Either the two half-line identifiers can be made identical ($I_1=I_2$) or they must be different (which is enforced by the constraint $\text{diff}$).

```
line(N1,N2,M1-I1,A1), line(N3,N4,M2-I2,A2) =>
(I1=I2, % try identification
 remove(1(N1,N2,M1-I1,A1)), % remove half-lines explicitly
 remove(1(N3,N4,M2-I2,A2)),
 alldiff([N1,N2,N3,N4]), % all nodes must be different
 M is M1/M2, A is A1+180-A2, % compute scaling and rotation
 update(N3,M,A), % scale and rotate N3 graph to fit N1 graph
 N1=N4, N2=N3, % equate nodes of now identical half-lines
 line(N1,N2,M1,A1) % merged line replaces the two half-lines
);

diff(I1,I2) % or otherwise half-lines must be different
```

Scaling and rotation is applied to the complete subgraph in which the second half-line occurs using the constraint $\text{update(Node,Scaling,Rotation)}$. Finally, the nodes of the two half lines are identified and a new proper full line without identifier replaces the two merged half-lines. (At this point, the node points are still unbound variables, their coordinates have no values yet.)

1.6 Random Generation of Graphs for Mason Marks

We have encoded a number of mason’s marks from [Rzi81] in our node-centric representation, in particular for the Ulm Minster (see Figure 5). For random generation of similarly shaped mason’s mark graphs we randomly choose node constraints from these mason’s marks satisfying certain conditions. The resulting graph may not always be valid due to unmatched half-lines. Figure 6 shows some examples of mason’s marks generated in this way. For more, see [Frü18b].
2 Conclusions

We have shortly presented our prototype tool to analyse, generate and draw straight-line graphs based on a novel node-centric representation of graphs using constraints. We have applied the tool to the domain of stonemason’s marks [Frü18b]. For a more complete coverage of mason’s marks, we need to add the representation of arcs and other curves.

This application shows the power of declarative modeling, handling, analysis and generation of pictorial information using a logic-based programming language that results in compact and concise code. In principle, our tool can be applied to any problem domain that admits a modeling as graphs.

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References


