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July 1, 2009

A novel approach to combine a SLS- and a DPLL-solver for the satisfiability problem

Overview

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  - Motivation
  - Preliminary Study
- Search Space Partition
  - Definition
  - Using SSPs to check intensification
  - *hybridGM*
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Motivation for developing hybrid SAT-solver

Major solving paradigms for the SAT-problem

- DPLL
  - \+ Complete: can solve sat and unsat problems
  - \+ Good at solving industrial and crafted problems
  - \- Not good on random formulas
  - \- Use large amounts of memory
- SLS
  - \+ Fast at solving random sat problems
  - \+ Little memory consumption
  - \- Incomplete
  - \- Scale bad on industrial and crafted problems

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Considerable effort has been undertaken to combine these two solving paradigms for gaining: robustness, completeness or speed

Preliminary study towards hybridisation

Our approach

- Look for the weaknesses of a solver and try to overcome it with a solver following the other paradigm
- We have chosen a SLS-solver for our analysis : *gNovelty+* (winner of SAT2007 competition category random sat)

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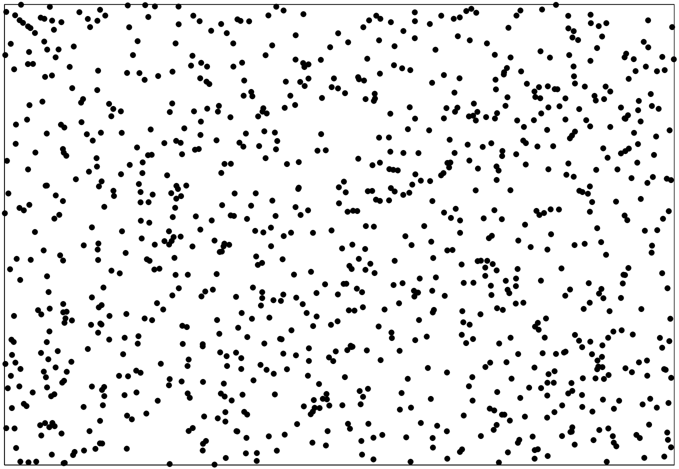
Search Properties of a SLS-solver

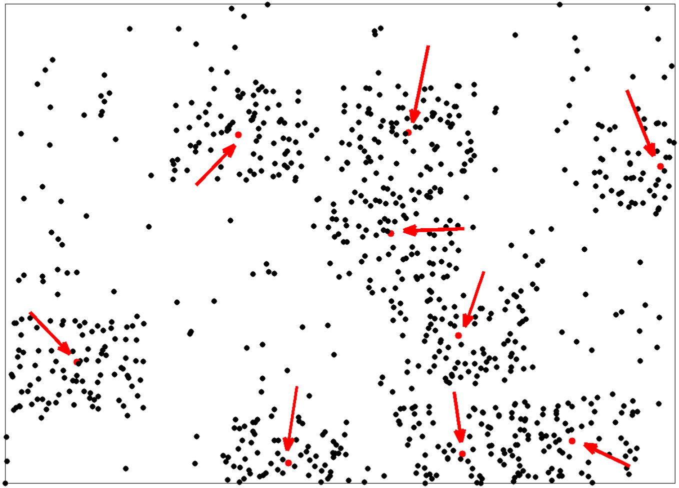
1. Diversification: How good is the search space coverage?
2. Intensification: How good is the search around high quality points?

An Example for Good Diversification



The search space represented two-dimensionally.
Black dots: points visited by the solver during its search

An Example for Good Intensification



Red dots: Good quality local minima

Diversification analysis

Our approach

- Try to cluster the points visited by the SLS $\rightarrow$ impossible due to large data (formula with 4000 variables $\rightarrow$ up to 10^8 flips during the search)
- Confine to search space points where the objective function has low values (i.e. local minima and their neighborhood)
- Save all this points in a bloom filter
- Check how many points are in the close neighborhood of the saved points

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Results

Not more than 2% of the points lay in the close neighborhood of the saved ones → good diversification

Intensification analysis

Why is intensification around good local minima important?

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How to check if the intensification is sufficient

Search the complete Hamming neighborhood (within a certain distance) of local minima for solutions: possible only for very small formulas

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Solution

Create a new neighborhood relation that can be searched up "fast" $\longrightarrow$ Search Space Partition (SSP)

Some Definitions

Complete Assignment of a Formula F

$\alpha \in \mathbb{B}^n$ where $\mathbb{B} = \{0, 1\}$ is called complete assignment of the variables $\{x_1, \dots, x_n\}$ of F

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Flip Trajectory

$(t_1, \ldots, t_w)$ where $t_i \in \{x_1, \ldots, x_n\}$ denote the variables being flipped by the SLS-algorithm during its search (w denotes the total number of flips)

Search Space Partition

Definition

Given a complete assignment α_j of F visited by the solver in the j-th flip and the flip trajectory $(t_1, \ldots, t_w)$ we construct a partial assignment β by starting with $k = 0$ and $\beta = \alpha_j$ and then repeat:

$\beta[t_{j+k}] = ?$ and $\beta[t_{j-k}] = ?$ where $t_{j\pm k}$ are the variables of the flip trajectory until $|\beta|_? \geq c \cdot n$ where c is some fixed constant $c \in (0, 1)$

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Example

$\alpha_7 = (0,0,1,1,0,1,0,1,1,1)$ assignment for F with 10 variables ($c = 0.5$)

flip trajectory $(x_2, x_6, x_1, x_9, x_1, x_6, \underline{\mathbf{x_1}}, x_3, x_9, x_1, x_1, x_8, x_3, \ldots)$

for $k = 0$ we have

$\beta = (?,0,1,1,0,1,0,1,1,1)$

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for $k = 1$ we have

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for $k = 2$ we have

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for $k = 3$ we have

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for $k = 4$ we have

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for $k = 5$ we have

$\beta = (?,0,?,1,0,?,0,?,?,1)$

Using SSPs to check intensification

Algorithm for checking intensification

1. Run SLS-solver and save a portion of the actual flip trajectory
2. Create SSPs around good quality local minima on the fly
3. Apply the partial assignment of the SSP on the formula to get a simplified sub formula
4. Solve the sub formula with a complete DPLL-solver to gain certainty that there is no solution

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Details

1. The construction of SSPs is done on the fly in linear time
2. The size can be controlled by the constant c
3. If the DPLL-solver finds a solution within a SSP $\rightarrow$ a new faster hybrid solver


#Unsat
Clauses
1
Flip

Implementation details

hybridGM

- SLS-solver: *gNovelty+* (SAT 2007 Comp. version slightly modified)
- DPLL-solver : *march_ks* (bug fixed version with some slight modification by Marijn Heule)
- Build SSPs around local minima with only 1 unsatisfied clause
- The constant c starts with $1/2$ and increases in steps of $1/20$ if the sub formula is too simple for *march_ks*

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Empirical Tests

- Different instances from SAT 2007 Competition
- Each instance is solved 100 times (cutoff 2000 sec.)
- In case of 5-SAT and 7-SAT there was sometimes a memory leak

Some Empirical Results

Instance	gNovelty+	adaptG2-WSAT0	hybridGM3 (gNov,March)	Gain
SAT 2007 Competition random instances				
unif2p-p0.7-v3500-c9345-S1568322528-08	10%	2.91 | 1.63	9.63 | 7.82 (9,91)	**>1**
unif2p-p0.7-v6500-c17355-S1097641288-15	97.64 | 69.35	3.75 | 1.61	4.31 | 2.57 (20,80)	**22.65**
unif2p-p0.7-v6500-c17355-S152598520-02	226.64 | 168.00	10.23 | 1.90	2.17 | 1.66 (27,73)	**104.44**
unif2p-p0.8-v1295-c4027-S1762612346-15	136.06 | 94.03	1.13 | 0.79	2.65 | 2.20 (9,91)	**51.34**
unif2p-p0.8-v1665-c5178-S1363528912-04	20.84 | 16.16	5.11 | 3.33	5.21 | 4.65 (73,27)	**4.00**
unif2p-p0.8-v2405-c7479-S1163137157-19	8.18 | 6.14	4.00 | 2.27	9.67 | 8.01 (28,72)	0.85
unif-k3-r4.261-v650-c2769-S1159448555-06	0.46 | 0.29	0.24 | 0.19	0.67 | 0.53 (76,24)	0.69
unif-k3-r4.2-v10000-c42000-S1173369833-06	73.87 | 50.01	11%	7.28 | 5.61 (23,77)	**10.15**
unif-k3-r4.2-v13000-c54600-S1416986890-04	331.30 | 250.70	0%	22.02 | 17.58 (23,77)	**15.05**
unif-k3-r4.2-v16000-c67200-S1600965758-04	18%	0%	73% (18,55)	**>1**
unif-k3-r4.2-v16000-c67200-S1826381479-08	550.04 | 457.84	0%	38.00 | 33.06 (23,77)	**14.47**
unif-k3-r4.2-v19000-c79800-S1106616038-10	74%	0%	92.02 | 70.75 (39,61)	**>1**
unif-k3-r4.2-v19000-c79800-S1875179522-13	470.69 | 381.00	0%	25.59 | 20.93 (25,75)	**18.39**
unif-k3-r4.2-v4000-c16800-S1178874381-13	8.11 | 6.02	184.52 | 110.63	4.99 | 4.00 (46,54)	**1.63**
unif-k3-r4.2-v4000-c16800-S1580061366-10	2.51 | 1.98	19.21 | 13.11	1.18 | 1.01 (42,58)	**2.13**
unif-k3-r4.2-v7000-c29400-S102550125-14	42.92 | 31.46	82%	6.85 | 5.70 (28,72)	**6.27**
unif-k3-r4.2-v7000-c29400-S1312035429-13	288.97 | 184.15	1%	246.31 | 160.57 (40,60)	**1.17**

Conclusion

Summary

- We have introduced the term of a Search Space Partition for SLS-solvers based on their search trajectory
- We propose a simple generic approach to combine a SLS- and a DPLL-solvers for a speedup with the help of SSPs
- Empirical results show that this approach is promising (SAT Competition 2009 results ...?)

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Outlook

- Dynamical adaptation scheme for when the SSPs are built
- Improve *hybridGM*'s performance at solving 5-SAT and 7-SAT

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A novel approach to combine a SLS- and a DPLL-solver for the satisfiability problem

Pseudocode of *hybridGM*

```
INPUT: formula F, cutoff.
OUTPUT: model for F or UNKNOWN.
hybridGM(F, cutoff) {
    α = α_S = random assignment;
    numFlips = 0;
    c = 0.5;
    barrier = 1;
    collectSSP = FALSE;
    while (numFlips < cutoff) {
        var = pickVar();
        α[var] = 1 − α[var];
        numFlips++;
        if (α is model for F) return α;
        if (numUnsatClauses == barrier) {
            β = α;
            collectSSP = TRUE;
            j = numFlips;
            k = 0;
        }
        if (collectSSP == TRUE) {
            β[ variableIndex( T_S(F, α_S)[j + k] ) ] = ?;
            β[ variableIndex( T_S(F, α_S)[j − k] ) ] = ?;
            k++;
        }
        if (|β|_? ≥ cn) {
            μ = March_ks(F, β);
            if (μ is model for F) return μ;
            else if (unaryConflictOccurred() == TRUE)
                c = c + 0.05 · n;
            collectSSP = FALSE;
        }
        updateParameters(); //noise, scores
    }
    return UNKNOWN;
}
```