On Reversal and Transposition Medians

Martin Bader | June 25, 2009

Genome Rearrangements

- During evolution, the gene order in a chromosome can change
- Gene order of two land snail mitochondrial DNAs

Cepaea nemoralis

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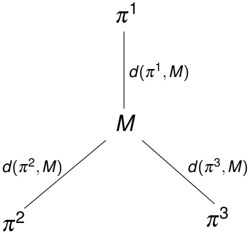
- Reconstruct evolutionary events
- Use as distance measure
- Use for phylogenetic reconstruction

The Median Problem

- Given gene orders π^1, π^2, π^3
- Find M where $\sum_{i=1}^{3} d(\pi^i, M)$ is minimized



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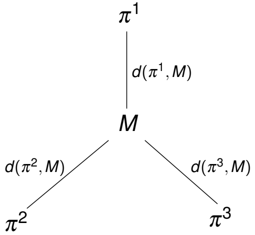

- NP-hard even for the most simple distance measures

Our contribution

- Exact algorithms for the Transposition Median Problem

  Exact algorithm for the weighted Reversal and Transposition Median Problem

  (Extension of Reversal Median solver, Caprara 2003)

- Improved exact algorithm for pairwise distances

  (Improvement of Christie 1998)

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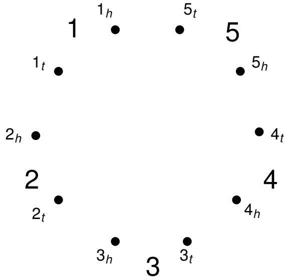

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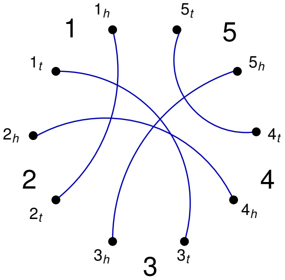

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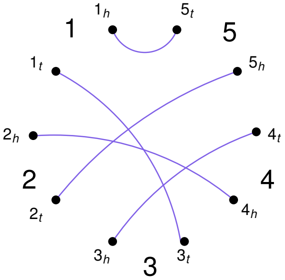

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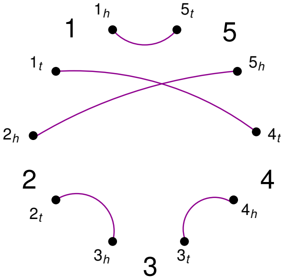

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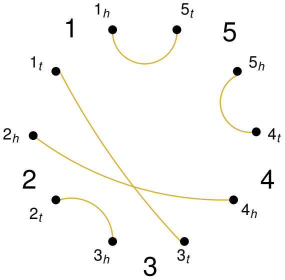

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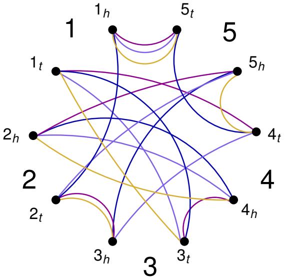

Cycles and distances

- Edges of two colors form cycles

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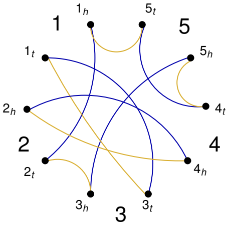

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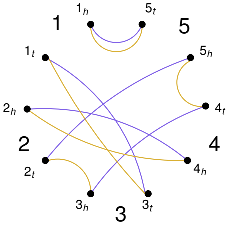

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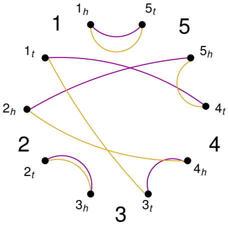

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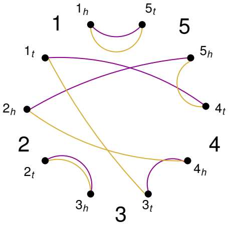

- Distances closely related to number of cycles

$$d_r = n - c + h + f$$

$$d_t \geq \frac{n - c_{odd}}{2}$$

$$d_w \geq \frac{w_t}{2}(n - c_{odd} - (2 - \frac{2w_r}{w_t})c_{even})$$

Sketch of the algorithm

- Solve Cycle Median Problem
- Verify solution

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  - Start with empty M
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  - Estimate lower bound for partial solution
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  - NEW: Consider cycle lengths
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- Verify solution
  Calculate edge weights
  - … either by an exact algorithm for pairwise distances
  - … or by an approximation algorithm (faster)

Experiments

- Create random input
  - Start with *id* of size n ($n = 37$ and $n = 100$)
  - Create 3 sequences of operations of length r ($2 \leq r \leq 15$)
  - Use these sequences to obtain π^1, π^2, and π^3

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  - Start with *id* of size n ($n = 37$ and $n = 100$)
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  - Use these sequences to obtain π^1, π^2, and π^3
- Testing
  - Most inputs could be solved within a few seconds
  - Verifying solutions with approximation algorithm is very accurate
  - Much faster than previous algorithm for the Transposition Median Problem (Yue et al. 2008)

Conclusion

We presented an algorithm that ...

- can solve the TMP and wRTMP exactly
- is fast enough for practical use
- is FREE SOFTWARE (GPL v3.0)

$\Rightarrow$ download it from

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Thanks for your attention!