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Improving stochastic local search for SAT with a new probability distribution

Introduction

SAT

- Formula F as a set of clauses. Each clause as a set of literals.
- Is there an assignment α, so that $F(\alpha) = 1$?

SLS-solvers

- Operate on complete assignments.
- Try to change an initial assignment α_0 in a way, so that it becomes satisfying after several steps.
- Use local information and heuristics to decide how to modify the assignment in each search step.

Pseudocode of a typical SLS-solver

```
INPUT: formula F, cutoff.
OUTPUT: model for F or UNKNOWN.
SLS(F, cutoff)
    for i=1 to maxTries
        α = α_0 = random assignment;
        for j=1 to maxFlips
            if (α is a model for F) return α;
            var = pickVar();
            α[var] = 1−α[var];
    return UNKNOWN;
```

SLS architectures

- Differ in how to select a variable for flipping.

GSAT

- Considers all variables.

WalkSAT

- Only considers variables from a randomly chosen unsatisfied clause.

DLS

- Similar to GSAT. Additionally uses clause-weighting.

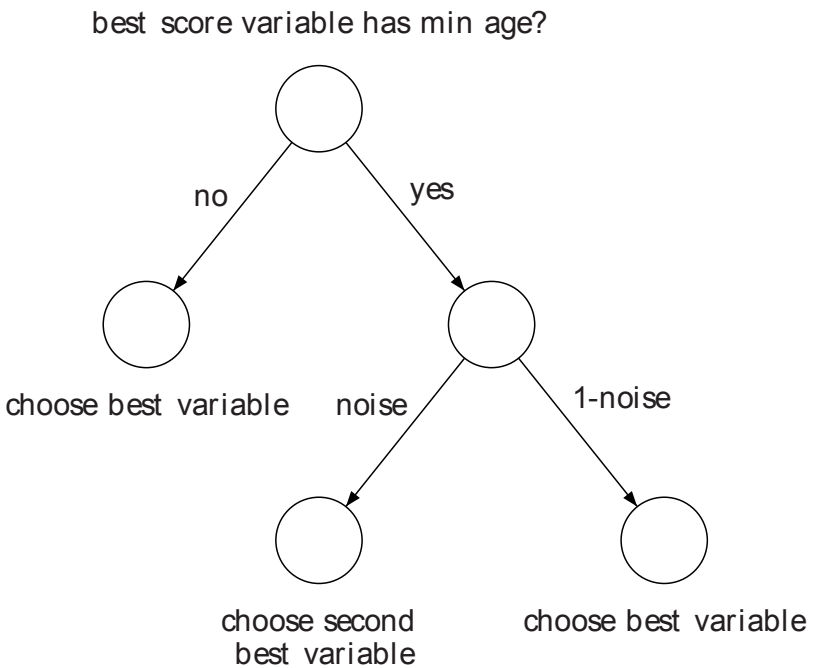
Modern SLS-solvers

- Can not be put into just one category.
- Are hybrids of the different architectures e.g.:
  - adaptG2WSAT = GSAT + WalkSAT + adaptive scheme [Li07]
  - gNovelty+ = adaptG2WSAT + DLS-component [Pham07]
  - TNM = GSAT + WalkSAT-version + two noise schemes to switch between [Wei09]
- they all use a Novelty-version to escape from local minima.

Escaping from local minima

Novelty

- Choose a random unsatisfied clause $c \in F$
- Pick a variable x from c according to the following heuristic:



Escaping from local minima

adaptNovelty+

- Additionally implements a random walk with probability *wp*.
- Uses an adaptive scheme for setting the noise probability [Hoos02].

Novelty as a probability distribution

- This can be seen as a probability distribution p on the variables of c.
- $p(x)$ can take only 4 different values:
  - Always chosen; $p(x) = 1$
  - Chosen, when no noise step occurs; $p(x) = (1 - noise)$
  - Chosen, when a noise step occurs; $p(x) = noise$
  - Never chosen; $p(x) = 0$
- Information used to decide: *score*, *age*
- Does not take into account the exact values of *score* and *age*.
- Does not put different variables into relation.

Example

- $c = \{x_1, x_2, x_3\}$

Case 1

- $score(x_1) = 0, score(x_2) = -3, score(x_3) = -3$
- $age(x_1) = 10^4 - 2, age(x_2) = 10^4, age(x_3) = 10^4 - 1$

Case 2

- $score(x_1) = -1, score(x_2) = -2, score(x_3) = -3$
- $age(x_1) = 10^3, age(x_2) = 10^4, age(x_3) = 10^7$

- Both cases are identical for Novelty+.
- The only chance for x_3 to be selected is by a random walk.

New probability distribution

- Define a probability distribution as a concatenation of continuous functions on the variables of c.
- Choose a random variable according to this distribution.
- The form of this distribution $p(x) := \frac{p_s(x) \cdot p_a(x)}{\sum_{\tilde{x} \in c} p_s(\tilde{x}) \cdot p_a(\tilde{x})}$.
- $p_s(x)$ is a function considering the *score* of x.
- $p_a(x)$ is a function considering the *age* of x.
- $p_a(x), p_s(x) > 0$

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- $p_s(x) := {c_1}^{score(x)}$
- $p_a(x) := \left(\frac{age(x)}{c_3} \right)^{c_2} + 1$
- c_1 controls the influence of *score* on the decision.
- c_2 and c_3 specify, how quick and how strong *age* starts to affect the decision.

Properties of p

Like Novelty

- Prefers variables with high *score*.
- Prefers variables with high *age*.

Additionally

- Takes into account the exact values of *score* and *age*.
- Puts different variables into relation.
- Does not need an explicit random walk.
  - Implicit random walk because of $p_s(x), p_a(x) > 0$.
- Does not need an explicit noise scheme.
  - Growing age automatically leads to new decisions when getting stuck in a certain part of the search space.

Implementation details

Sparrow

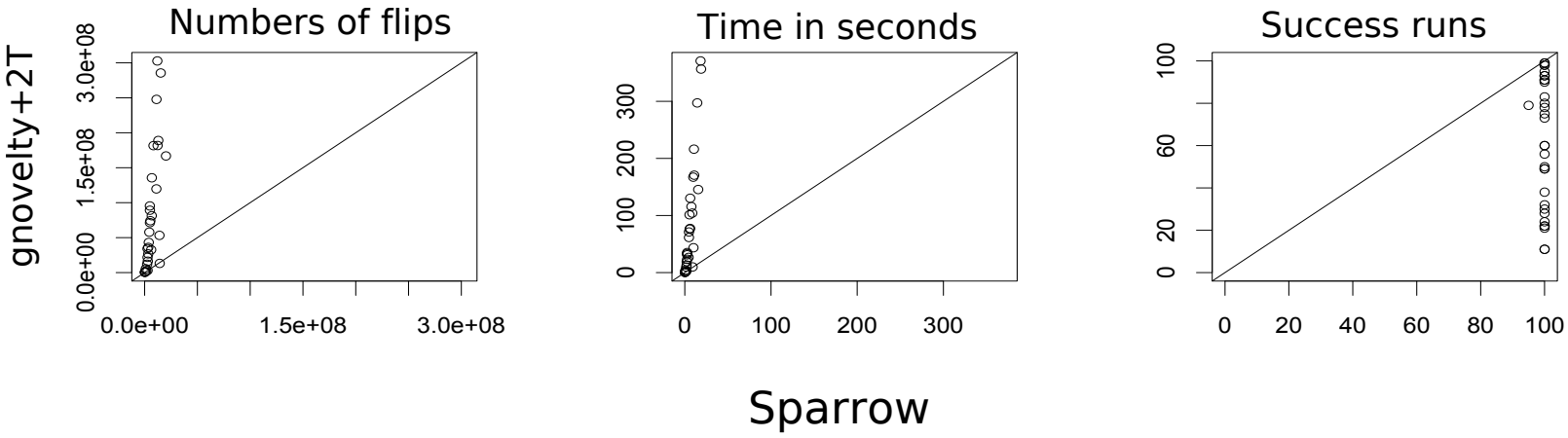
- gNovelty+ as the basic module. gNovelty+ consists of:
  - Gradient-walk as in G2WSAT.
  - adaptNovelty+ to escape local minima.
  - Additive weighting scheme.
- Replaced the adaptNovelty+ component by a WalkSAT-algorithm using the presented probability distribution.
- Specifying the parameters $c_1 = 4, c_2 = 4, c_3 = 10^5$.
  - Constants fixed for all instances.
  - Empirical values.
  - Not optimized yet.

Evaluation

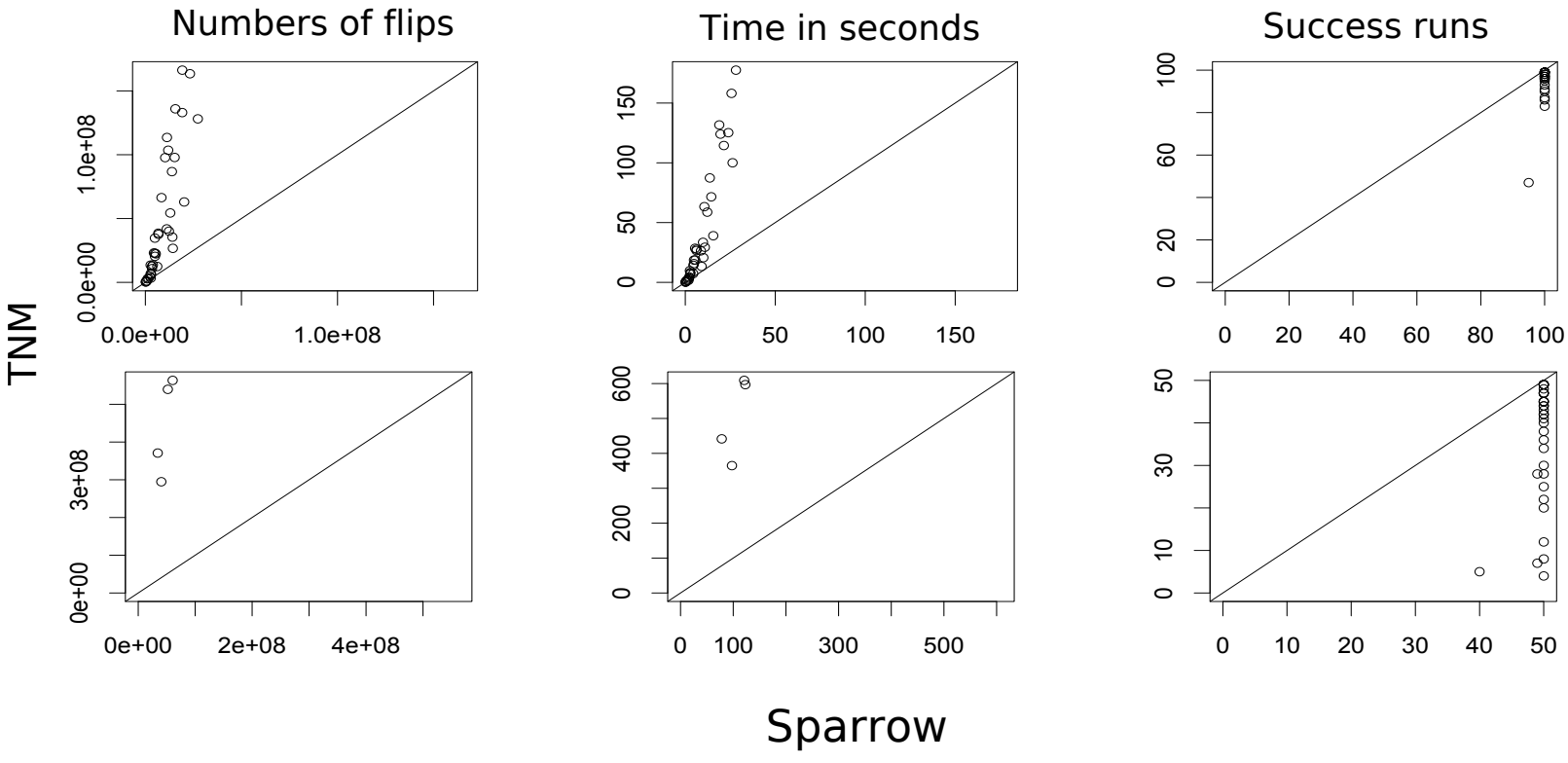
Empirical Tests

- 2 sets of large random 3-SAT instances with $ratio = 4.2$
- Not optimized for k-SAT, $k \neq 3$.
- Benchmark 1:
  - 64 different instances from SAT 2009 Competition (2k - 18k variables)
  - Each instance is solved 100 times (cutoff 1200 sec.)
- Benchmark 2:
  - 40 additional instances from SAT 2009 Competition (20k - 26k variables)
  - Each instance is solved 50 times (cutoff 2400 sec.)

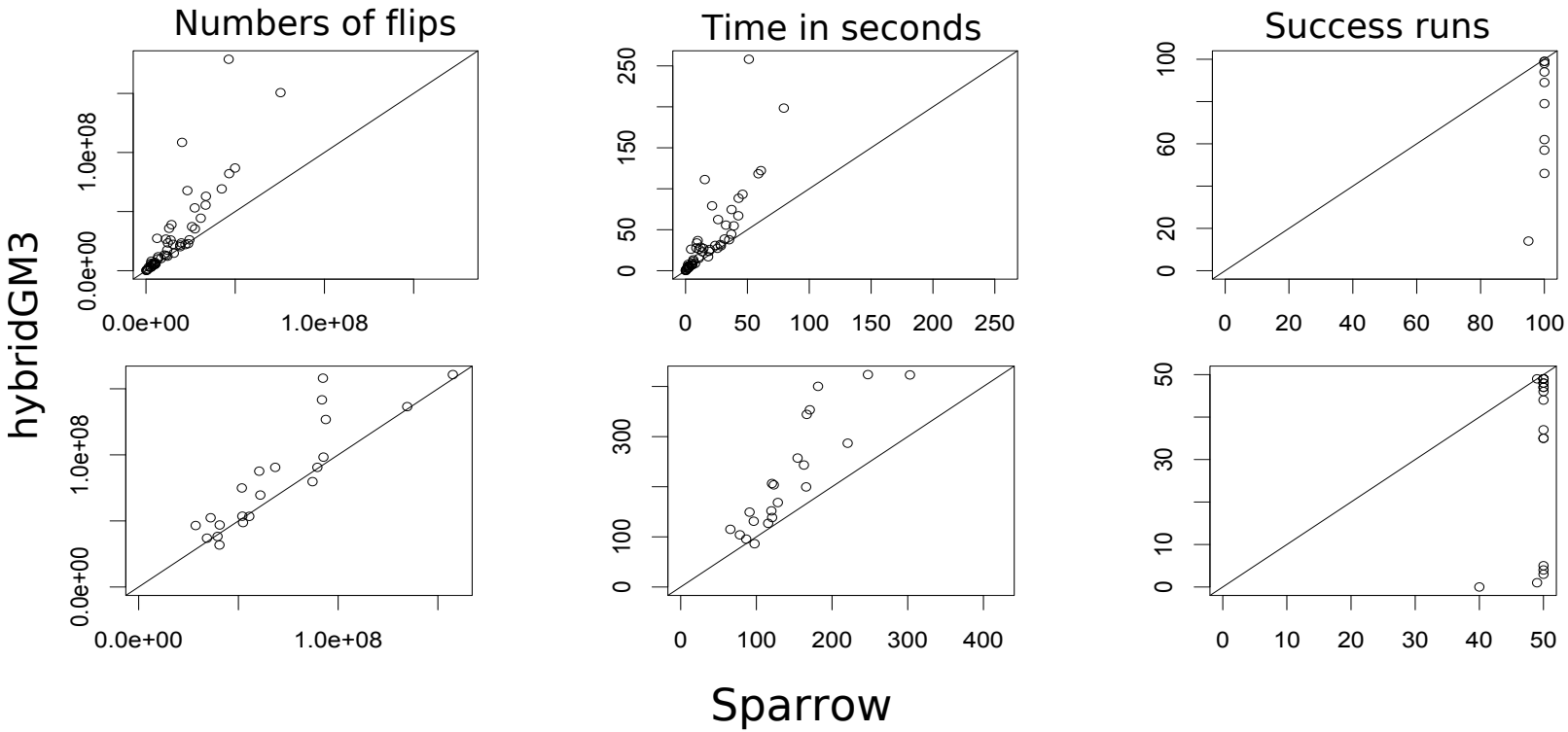
Results: Sparrow vs gNovelty+



Results: Sparrow vs TNM



Results: Sparrow vs hybridGM3



Conclusion

Summary

- Escaping local minima is a key feature for SLS-solvers
- Variable selection in local minima should not be done in a deterministic way
- Detailed differentiation useful for variable selection.

Outlook

- Implement more attributes.
- Arbitrary complex function.
- Variable expressions [Tompkins10].

Thank you for your attention!