Determined whether a given propositional formula in conjunctive normal form has a satisfying assignment (SAT) is one of the most important problems in computer science. Since all problems in NP can be reduced to SAT, algorithms for this problem (so-called SAT solvers) have become an essential tool for solving many real-world problems in fields like software verification, operations research, or scheduling.

However, it is known that all existing SAT solvers have exponential running times in the worst case. This worst-case behavior is studied in proof complexity, where the search process of a solver is modeled with a proof system. Upper and lower bounds for these proof systems translate to information about the power and limits of SAT solvers. In the first part of this talk, we explore this approach for studying the SAT problem by way of example, focusing on formulas encoding the graph isomorphism problem (GI). We demonstrate a connection between finite model theory and proof complexity, allowing us to obtain exponential lower bounds for refuting GI formulas in resolution (and thus with so-called conflict-driven clause learning solvers).

This bad news in the form of exponential lower bounds seems not to have come to the attention of applied researchers and engineers working on designing and implementing SAT solvers: Remarkably, modern SAT solvers can routinely solve many real-world instances with tens of millions of variables and clauses. In the second part of this talk, we take this more optimistic view on the SAT problem. We show how one can construct a lightweight hybrid solver, improving the performance of stochastic local search solvers by learning additional information which logically entails from the original problem. Furthermore, we find that the resulting runtimes are long-tailed, implying that incorporating additional restarts can further refine all algorithms employing this modification technique.