Evidence for Long-Tails in SLS Algorithms

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Motivation

Recent development in SLS solvers [LW20]:

- `GapSAT` solves not the original instance but a modified yet logically equivalent one
- Empirically shown: on average, this improves the performance of state-of-the-art SLS solvers

Overview of Our Results

1. A **lognormal** distribution perfectly characterizes the hardness distribution of such modified instances
   $\Longrightarrow$ The hardness is **long-tailed**
2. Restarts are useful for long-tailed algorithms

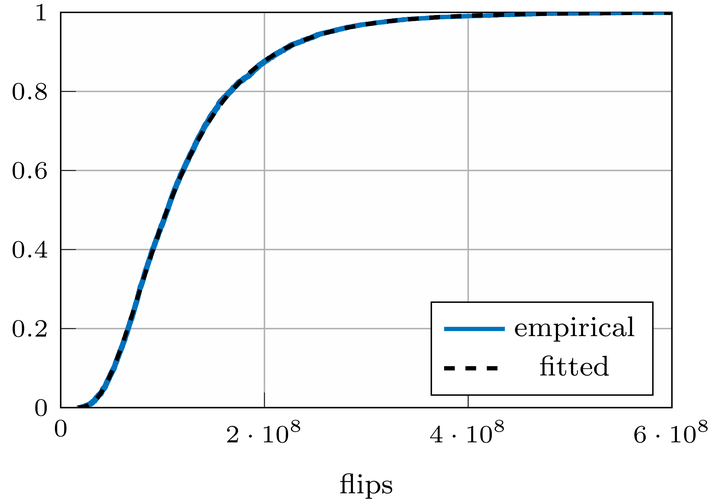
Method: Modification of Instances by `Alfa`

- Model the addition of a set of logically equivalent clauses L to a formula F and the subsequent solving of the amended formula $F^{(1)} := F \cup L$ by an SLS solver
- Use width-4-bounded resolution to generate L

Empirical Results

From formula F, generate 5000 modified instances $F^{(1)}, \ldots, F^{(5000)}$ and solve each 100 times with different seeds. Record the means $\mathsf{mean}(F^{(i)})$. Plot, among others, the empirical cumulative distribution function

$$\widehat{F}_{5000}(t) := \frac{1}{5000} \sum_{i=1}^{5000} \mathbb{1}_{\{x_i \leq t\}}, \quad t \in \mathbb{R}.$$



Test goodness-of-fit with χ^2- and bootstrap-test (see [LW21]).

Conjectures

Strong Conjecture. The runtime of `Alfa` with `SLS` $\in \{$`SRWA`, `probSAT`, `YalSAT`$\}$ follows a lognormal distribution.

Definition ([FKZ11])**.** A positive, real-valued random variable X is *long-tailed*, if and only if

1. $\forall x \in \mathbb{R}^+ : \Pr[X > x] > 0$, and
2. $\forall y \in \mathbb{R}^+ : \lim_{x \to \infty} \frac{\Pr[X > x+y]}{\Pr[X > x]} = 1$.

Weak Conjecture. The runtime of `Alfa` with `SLS` $\in \{$`SRWA`, `probSAT`, `YalSAT`$\}$ follows a long-tailed distribution.

Theoretical Result

Theorem. Let X be a positive, long-tailed random variable with continuous pdf f and hazard rate function r. Assume that

- either $\mathrm{E}[X] = \infty$ holds;
- or the limits $\lim_{t \to \infty} r(t)$ and $\lim_{t \to \infty} t^2 \cdot f(t)$ both exist.

In both cases, restarts are useful for X.

Corollary of the Weak Conjecture. Restarts are useful for `Alfa` with `SLS` $\in \{$`SRWA`, `probSAT`, `YalSAT`$\}$.

Full Paper

References

[FKZ11] Sergey Foss, Dmitry Korshunov, and Stan Zachary. *An Introduction to Heavy-Tailed and Subexponential Distributions*, Volume 6. Springer, 2011.

[LW20] Jan-Hendrik Lorenz and Florian Wörz. On the effect of learned clauses on stochastic local search. In *Proceedings of the 23rd International Conference on Theory and Applications of Satisfiability Testing (SAT '20)*, pages 89–106. Springer, 2020.

[LW21] Florian Wörz and Jan-Hendrik Lorenz. *Evidence for Long-Tails in SLS Algorithms*. Full Paper at arXiv:2107.00378.