# Dynamic Markov Logic Networks as a knowledge base for cognitive technical systems

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17. Oktober 2011

# Overview

Motivation

Markov Logic Networks

Inference in Markov Logic Networks

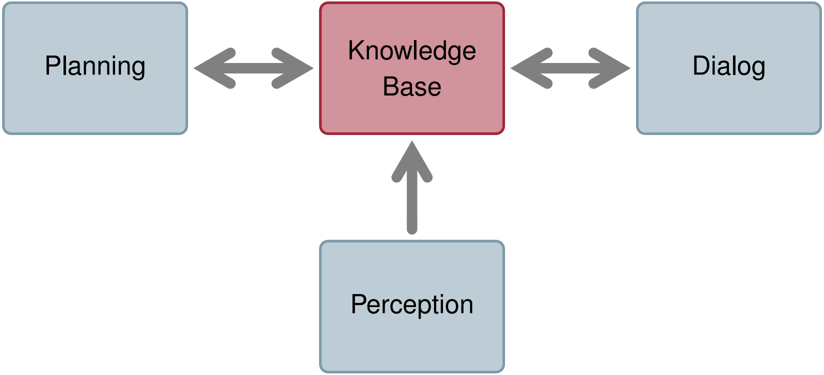
Dynamic Markov Logic Networks

# What is the purpose of a knowledge base?

Representing the knowledge of the system!

- the single purpose of knowledge (for a technical system) is to form a basis for selecting an action
- action is used in the most general form, representing anything that escapes the technical system by the means of an actuator
- knowledge that does not influence the behavior is redundant — it can be removed without any effect to the outside

# Motivation



# How must the knowledge of a cognitive technical system look like?

1. It is uncertain.

- the system is interacting with a natural environment
- parts of the environment are not observable
  (e.g. the emotional state of user)
- input can be imperfect
  (failing sensors, poor classifiers, wrong user input)

# How must the knowledge of a cognitive technical system look like?

1. It is uncertain.
2. It is symbolic.

- knowledge must be accessible / symbolic
- thus, its meaning can be expressed in natural language
- e.g., you cannot simply connect a neural network to the GUI — you must do this through a label giving a meaningful interpretation to the output

# How must the knowledge of a cognitive technical system look like?

1. It is uncertain.
2. It is symbolic.
3. It is dynamic.

- the state of the environment changes
- thus the knowledge has to be “updated”
- a future state strongly depends on a past state

## How must the knowledge of a cognitive technical system look like?

1. It is uncertain.
2. It is symbolic.
3. It is dynamic.
4. It is flexible.

- the model must be flexible and easy to extend
- it should not be fixed to a certain task

# How can we fulfill these criteria?

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3. It is dynamic.
   time is represented explicitly
4. It is flexible.
   state variables are predicates over objects

# Probabilistic models

- the possible states of the world form a probability space $\Omega$
- a probability (mass) function $P : \Omega \to [0, 1]$ represents the belief of the system about what the true state might be
- possible world states assigned a higher probability are more likely the true state

# Learning new facts means making observations

- observations are used to condition the distribution

$$P'(x) = P(x \mid e) = \frac{P(x, e)}{P(e)}$$

- observations change the belief of the system
  (they hopefully improve it, making the true state more probable)

# How to handle state?

- instead of having a very large set of states $\Omega = \{s_1, s_2, \dots\}$
- we can express the state by multiple random variables.
  Informally $\Omega = X_1 \times X_2 \times X_3 \times \cdots \times X_n$
  (obtaining a multivariate distribution)
- and assign meaning (labels) to those variables:
  $X_1 = \mathit{true}$ means "the user is at her desk",
  $X_2 = \mathit{true}$ means "the user is tired"

# How to handle time?

- conditioning a probabilistic model on a fact
  with probability zero is not possible
- contradictory observations can be made when time progresses
  - the user is at her desk
  - the user is away
- unless time is represented explicitly
  - the user is at her desk at 4pm
  - the user is away at 5pm

# Relational models

- a relational model contains objects
  `14:32`, `14:45`, `u#1`, `iPhone`, `iPod`
- and relations between them
  `touches(Time,User,Device)`
- inserting objects into relations forms atoms (or propositions)
  `touches(14:32,u#1,iPhone)`, `touches(14:45,u#1,iPod)`
- dependencies can abstract over objects using variables
  `touches(t,user,device) => near(t,user,device)`
- thus relational models can scale with the number of objects
- time has not to be built-in but can be expressed freely using time-dependent predicates

# Markov Logic Networks

- Markov Logic Networks [1] (MLNs) are a probabilistic, relational model
- dependencies are expressed through first-order logical formulas
- as such they fulfill all presented requirements

[1] Matthew Richardson and Pedro Domingos. "Markov logic networks". In: *Machine Learning* 62.1-2 (2006), pp. 107–136. ISSN: 0885-6125. DOI: 10.1007/s10994-006-5833-1

# An example — users and devices

An informal description of a model about interface devices and users.

- if a user touches a device, she is near the device
- if a user is near a device, she can probably see it
- if a user is near a device, she will probably still be near the device later

# An example — Markov Logic Networks

The same model formalized as a Markov Logic Network.

```
touches(t,user,device) => near(t,user,device).
1.0 near(t,user,device) => sees(t,user,device)
4.0 near(t,user,device) => near(t+1,user,device)
```

---

# Definition: Syntax of Markov Logic Networks

- a Markov Logic Network $L$ is a set of weighted first-order logical formulas

$$L = \{(f_i, w_i) \mid i \leqslant n, f_i \in \text{FOL}, w_i \in \mathbb{R}\}$$

- the formulas can use predicates from sorted logical language and constants from a given constant domain

# Sytnax of MLNs — logical language example

- the logical language defines which propositions can be expressed

```
Predicate near(Time,Person,Device)
Predicate touches(Time,Person,Device)
Predicate sees(Time,Person,Device)
```

---

- objects can be categorized into sorts or types
  (Time, Person, Device)

# MLNs require a finite set of constants

- in addition to the weighted formulas one need to give a finite set of constants for each sort
- using as constants
  `Time = {0,1}, Person = {u#1}, Device = {iPod,iPad}`
  one obtains the atoms for `near(Time,Person,Device)`:
  `near(0,u#1,iPod), near(0,u#1,iPad),`
  `near(1,u#1,iPod), near(1,u#1,iPad)`
- putting the constants in all valid combinations into the predicates, one obtains the set of all ground atoms (the Herbrand base)

# Interpretations are possible worlds

- in logics, an *interpretation* maps atoms to truth values
- an interpretation can be seen as a possible world state that defines the truth of every proposition
- MLNs express the uncertainty over which possible world state is the true world state as a probability distribution over interpretations

# A MLN Defines a Distribution Over Interpretations

- given a formula with free variables, a grounding assigns a constant to each free variable
- given an interpretation $x$, let $n_i(x)$ be the number of true groundings of formula $f_i$ under $x$
- then a MLN defines the following probability distribution over interpretations

$$P(x) = \frac{1}{Z} \prod_i \exp(w_i n_i(x))$$

# A closer look at the example

```
touches(t,user,device) => near(t,user,device).
1.0 near(t,user,device) => sees(t,user,device)
4.0 near(t,user,device) => near(t+1,user,device)
```

---

- line 1: a full-stop marks a deterministic formula
- line 2: the odds of seeing a device when being near it are $\frac{1}{\exp(-1.0)+1} \approx 0.73$
- line 3: the odds of remaining in the vicinity of a device in one time step are $\frac{1}{\exp(-4.0)+1} \approx 0.98$

The equivalence of formula weights and log-odds of the formula being true is only given when formulas do not share atoms.

# Some properties of MLNs

- degeneration to first-order logic when using infinite weights
- no uncertainty over the numbers of objects
- formula weights can be either specified or trained from data-sets

# Inference over MLNs

There are two common inference tasks for MLNs

1. find the most probable interpretation given some evidence $e$ (MAP or MPE)

$$\arg\max_x (P(X = x \mid e))$$

2. compute the marginal probabilities for some $z$ event (assigning to variables $Z \subseteq X$) given some evidence $e$ (MAR)

$$P(Z = z \mid e) = \sum_{x \in \sim\{Z\}} P(X = x \mid e)$$

During the rest of the talk we focus on computing marginal probabilities.

# Incorporating uncertain evidence

- a multivariate probability distribution can only be conditioned on a certain/sure assignment to a variable
- this means an observation can be “`near(15,u#1,iPad)` is true”
- observations obtained from classifiers can be uncertain: “`near(15,u#1,iPad)` is true with a certainty of $p$”
- we can express this by adding this formula with $w = \ln(\frac{p}{1-p})$ to the MLN:
  `w near(15,u#1,iPad)`

# Two ways to infer: MCMC and belief propagation

- inference on MLNs is usually done on an undirected graphical model obtained through grounding all formulas and predicates[2]
- then every algorithm for inference on graphical models can be used
- these algorithms usually fall into two categories
  - message passing / belief propagation
  - Markov Chain Monte Carlo methods
- won't give further details on these algorithms

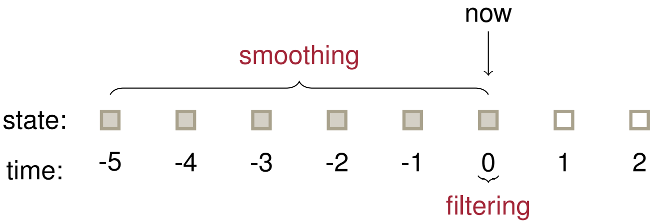
[2] with recent trends towards lifted inference

# Expressing dynamics with MLNs

- predicates can be made dynamic by adding a time index
- DMLNs are general enough to be used to express Dynamic Bayesian Networks with discrete variables
- in this way dynamic Markov Logic Networks (DMLNs) have already been applied in classification tasks
  - Adam Sadilek and Henry Kautz. "Recognizing Multi-Agent Activities from GPS Data". In: *Proceedings of the 24th AAAI Conference on Artificial Intelligence*. 2010, pp. 1134–1139
  - S. Tran and L. Davis. "Event modeling and recognition using markov logic networks". In: *Computer Vision–ECCV 2008* (2008), pp. 610–623

# Which inference task for dynamic, probabilistic models is needed?

- filtering means predicting the *current* state of the world based on past observations
- smoothing means predicting past states in addition (new observations can make past predictions more accurate)




- for an application in a cognitive technical system
  we focus on filtering

# The offline approach to filtering/smoothing in DMLNs

- ground the DMLN from the start up to the current time step and infer on this graph
- this is the way DMLNs have been applied in the literature for dynamic classification tasks
- this includes the smoothing task and can only applied offline

# Online filtering in DMLNs

- we need a method to reuse information gained from inference for the previous time step
- one existing work describes an approximate online inference algorithm for message passing[3]
- how to make online inference for MCMC methods is less clear
- we propose temporal slice inference as an approximative online inference method for DMLNs, working with any inference algorithm for MLNs
- Thomas Geier and Susanne Biundo. "Approximate Online Inference for Dynamic Markov Logic Networks". In: *to appear at ICTAI*. 2011

[3] A. Nath and Domingos. "Efficient Belief Propagation for Utility Maximization and Repeated Inference". In: *Proceedings of the 24th AAAI Conference on Artificial Intelligence*. 2010, pp. 1187–1192

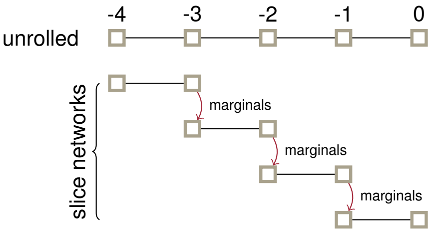
# Temporal Slice inference for DMLNs

- idea is similar to the factored frontier algorithm for DBNs[4]
- it approximates the distribution over the last time step by the marginal probabilities of the variables
- performs only a forward pass over the network, since no smoothing is required

[4] K. Murphy and Y. Weiss. “The factored frontier algorithm for approximate inference in DBNs”. In: *Proceedings of the 17th Conference on Uncertainty in AI*. 2001, pp. 378–385

# Temporal Slice inference for DMLNs (2)

- for each successive pair of time steps, have a temporal slice MLN spanning only these (like a two-step DBN)
- carry over information from the last slice networks by marginal probabilities over single random variables



# Temporal Slice inference for DLMNs (3)

- example slice for $t = 43$

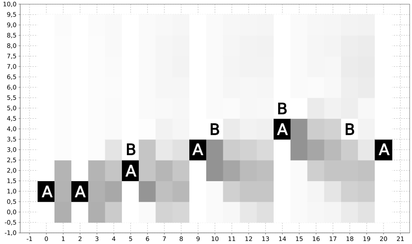
```
touches(43,user,device) => near(43,user,device).
1.0 near(43,user,device) => sees(43,user,device)
4.0 near(42,user,device) => near(43,user,device)

w1   touches(42,u#1,iPod)
w2   touches(42,u#1,iPad)
...
```

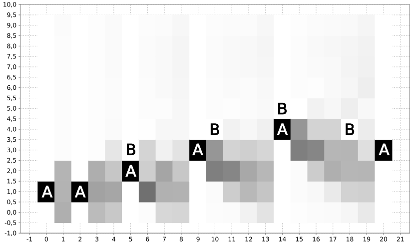
---

- `w1` $= \ln \frac{p_1}{1-p_1}$ with $p_1$ the marginal probability of `touches(42,u#1,iPod)` during the slice for $t = 42$
- the marginals of the slice MLN can then be inferred using any algorithm calculating marginals for MLNs

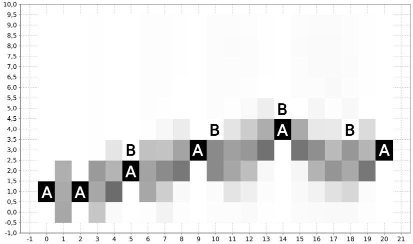
# Approximation error of temporal slice networks




Exact filtering



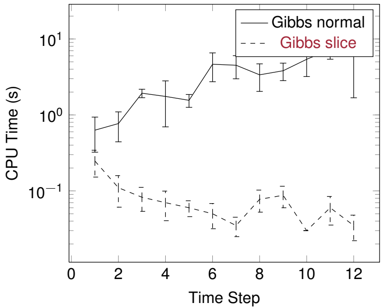

Temporal slice network filtering

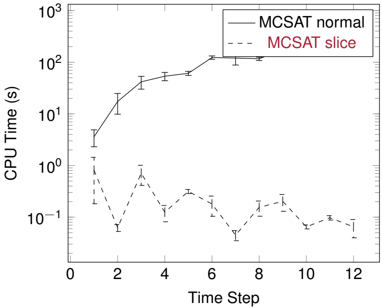



Smoothing

# CPU time for temporal slice inference algorithm

Plots show the CPU time for approximate filtering using temporal slice sampling compared to exact filtering, unrolling the complete network; running on an artificial MLN with random evidence; against number of time steps.



# Online DMLN knowledge base for a companion system

- a module containing the online inference engine has been fitted with a Semaine connection
- it can handle exact observations (with uncertain observations planned)
- it answers queries for marginal probabilities of single atoms

# Future work

- add continuous variables
- further improve inference methods for DMLNs (lifted inference, error-bounded approximation)
- extending and evaluating the model to support output channel selection (together with B3)
- incorporating uncertain observations from classifiers (together with C5) to recognize complex actions
  - a MLN constructed via “expert knowledge” can give hints as to when certain actions are applicable and thus improve classification
  - it allows to incorporate knowledge gained from other sources independently of the trained classifier

# Das Ende

Thank you for your attention.

# The example as a factorgraph

- the distribution defined by a MLN can be represented by a factor graph
- atoms are variable nodes
- ground formulas are factor nodes; connected to the atoms they contain

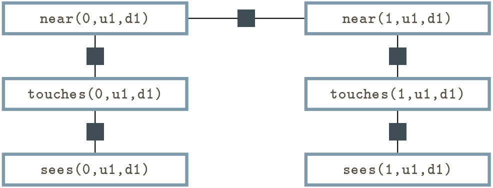



Figure: The factor graph for the example MLN.

# Why Gibbs sampling cannot handle deterministic dependencies

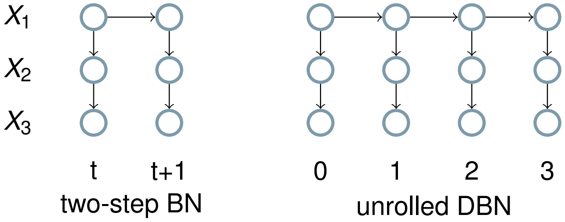
Assume a distribution over two binary random variables with the following probabilities.

| | $a$ | $\neg a$ |
|---|---|---|
| $b$ | 0.5 | 0 |
| $\neg b$ | 0 | 0.5 |

Starting in state $a$, $b$ the Markov chain produced by Gibbs sampling will never escape because it would have to temporarily enter a state with probability 0.

# Excursion: Dynamic Bayesian Networks

DBNs are probabilistic, multivariate, dynamic models.



# How to handle state? (2)

- DBNs “scale in time”
- but like BNs, DBNs are a propositional model, i.e., one has to specify each state variable
- a DBN about a person and her iPhone *is difficult to extend* to a DBN about a person and her iPhone and her iPod
- this is even though iPhones and iPods are very similar
- one needs to create new variables and new dependencies between them

# The Markov Chain Monte Carlo method

Given a MLN $L$ and its corresponding distribution $P_L$.

- we want to compute the marginal probability for some atom $a$ being true
- if we can obtain enough independent samples from $P_L$ then we can estimate $P_L(a = \mathit{true})$ by the frequency over the samples
- MCMC aims to obtain samples from $P_L$ by constructing a Markov chain whose stationary distribution equals $P_L$

# Gibbs sampling as a simple MCMC algorithm

Gibbs sampling works the following.

1. choose a starting interpretation randomly
2. choose an atom randomly
3. resample its truth value given the rest of the interpretation
4. repeat with 2

Gibbs sampling cannot handle deterministic dependencies arising from hard formulas.

# Handling deterministic dependencies with MCSAT

MCSAT improves over Gibbs sampling in handling deterministic dependencies.

- MCSAT is a different MCMC algorithm
- MCSAT re-samples every atom in every step by using slice sampling
- as a subroutine it uses the SampleSAT algorithm to sample uniformly from the satisfying interpretations of a CNF formula
- in contrast to Gibbs sampling MCSAT is able to jump between the modes of the distribution easily

# Obtain all groundings of a formula

- the formula `touches(t,user,device) => near(t,user,device)` contains the free variables `t,user,device`
- a grounding of a formula assigns a constant to each free variable
- a possible grounding of the presented formula is `touches(0,u1,d2) => near(0,u1,d2)`

# How to make a choice — The principle of maximizing expected utility

How to make decisions based on uncertain knowledge?

1. have a set of possible actions $A$
2. assign a utility to each state/action pair

$$u : A \times S \to \mathbb{R}$$

3. the utility $u_a$ for action $a$ is then a random variable
4. choose the action $a$ with the largest expected utility

$$a = \arg\max_{a' \in A} E[u_{a'}]$$

# Evaluation Domains

- Dynamic Smokers

```
1.0 smokes(t,x) => cancer(t,x)
1.0 friends(t,x,y) =>
    (smokes(t,x) => smokes(t,y))
3.0 friends(t,x,y) => friends(t+1,x,y)
3.0 smokes(t,x) => smokes(t+1,x)
```

- Simple Social Force

```
2 at(t+1,a,x) =>
  at(t,a,x-1) v at(t,a,x) v at(t,a,x+1)
1.5 !(at(t,a1,x) AND at(t,a2,x))
```

# Grounded Slice Network