

Universität Ulm
Fakultät für Informatik



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Heiko Neumann

Universität Ulm

Luiz Pessoa

Universidade Federal do Rio de Janeiro

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Visual Filling-in and Surface Property Reconstruction¹

Heiko Neumann[†] and Luiz Pessoa[‡]

[†]Universität Ulm,

Fakultät für Informatik, Abteilung Neuroinformatik,

Oberer Eselsberg, D-89069 Ulm, Germany

[‡]Universidade Federal do Rio de Janeiro,

Programa de Engenharia de Sistemas e Computação – COPPE Sistemas,

Ilha do Fundao, Rio de Janeiro, RJ 21945-970, Brasil

Abstract

The visual system is constantly confronted with the problem of integrating local signals into more global arrangements. This arises given the local nature of early cell responses, be them to local luminance, motion, or retinal position differences, or discontinuities. Consequently, from sparse, local measurements, the visual system must somehow generate the most likely hypothesis consistent with them.

In this paper we study the problem of the reconstruction of achromatic surface properties, namely brightness. We show that the filling-in of brightness through a diffusion process can be linked to the general framework of feature reconstruction through regularization theory that is widely used in computer vision. In particular, it will be shown that brightness filling-in is a means of reconstructing smooth information from local contrast data that minimizes first order derivative information. Our investigation provides a formal link between modeling perceptual data for biological vision and the mathematical frameworks of regularization and linear spatially variant diffusion.

In terms of regularization theory, the reconstruction of surface properties through filling-in from local signals proceeds by minimizing a functional which utilizes model and data terms. Minimizing the functional takes into account a *confidence* value ascribed to the data terms, or local measurements. Typically, the absence of local measurements implies zero confidence. The present analysis of filling-in in terms of regularization theory suggests a new diffusion mechanism that effectively employs confidence information. Simulations on psychophysical stimuli illustrate the mechanism's potential.

1 Introduction

Experimental studies indicate the existence of distinct perceptual sub-systems in human vision, one that is concerned with *contour extraction* and another that assigns *surface properties* to bounded regions. One example concerns illusory contour stimuli where an independence of illusory contour clarity, or sharpness of the contours, and the brightness of the illusory figure is found (see Leshner, 1995). The emerging picture from the experimental investigations is one in which fast processes extract shape outlines which are subsequently conjoined with texture, color or brightness information by the action of slower "painting" mechanisms (Elder and Zucker, 1998; Rogers-Ramachandran and Ramachandran, 1998; cf. Grossberg and Mingolla, 1985). The two sub-systems

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are not independent, however, but interact to determine perception. The importance of object contours (or boundaries) for proper surface perception was vividly demonstrated by experiments with stabilized images (Krauskopf, 1963; Yarbus, 1967). In the classic study by Krauskopf (1963), an inner green ring was surrounded by a red annulus. When the red-green boundary was stabilized on the retina (so that it always maintained a fixed position on the eye), subjects reported that the central disk disappeared and the whole target, disk plus annulus, appeared red. In other words, the annulus color appeared to fill in the rest of the target. Krauskopf's observers perceived red even though "green" light was striking the corresponding region of the retina. These and other results (e.g., Land, 1983) suggest that even under natural viewing conditions the perceived color of a surface depends not only on the light reflected from the surface but on the change in light across the boundary of the surface or surface region. These observations are at odds with the principle of joint boundary-region processing for surface segmentation proposed in a variety of computer vision models (Blake and Zisserman, 1987; Mumford and Shah, 1989; Schnörr and Sprengel, 1994).

Models of brightness perception were among the first to explore the dichotomy of boundary and surface sub-systems. Based on stabilized image studies, Gerrits and Vendrik (1970) proposed that the perception of brightness depends on filling-in processes that occur within separate ON and OFF channels. The two channels would be involved in "brightness" and "darkness" processes, respectively. These ideas were formalized by Cohen and Grossberg (1984) and Grossberg and Mingolla (1985) who proposed, as part of the FACADE theory, the Boundary Contour System/Feature Contour System (BCS/FCS). The BCS is responsible for determining the boundaries of surfaces in general situations (such as in illusory contours). The FCS computations determine the appearance of the incoming stimulus (such as brightness, hue and depth). The two systems are involved in complementary computations and, as indicated by the image stabilization studies, the final boundary signals produced by the BCS are used to regulate a filling-in process that occurs within the FCS system. Grossberg and Todorović (1988) showed how this proposal is capable of qualitatively accounting for several brightness phenomena, including simultaneous contrast, brightness assimilation, the 2-D Craik-O'Brien-Cornsweet effect, the Hermann grid, and Mondrian displays. Recent simulations of the model by Arrington (1994) have also shown that it is capable of accounting for data on the temporal dynamics of brightness perception, as studied by Paradiso and Nakayama (1991). A recent extension of the model motivated by phenomena exhibiting brightness gradients (Pessoa *et al.*, 1995) was used to account for stimuli such as trapezoidal and triangular Mach bands (see Pessoa 1996a,b), low- and high-contrast missing fundamental stimuli, sinusoidal waves, among others.

In this paper we show that the filling-in of brightness through a diffusion process may be understood in terms of the general framework of feature reconstruction through regularization theory that is widely used in computer vision. In particular, it will be shown that the diffusion process is a means of reconstructing brightness information from local contrast data that minimizes first order variations, or first-derivative information. This

guarantees that brightness is reconstructed in a smooth manner, as suggested by perceptual data. The explicit link to regularization theory also illustrates that the diffusion of information that has been used for brightness perception may also prove useful in other domains. In particular, depth and motion information may potentially be integrated within the filling-in framework.

Based on our formalization of diffusive filling-in within the framework of regularization theory, we introduce a new version of filling-in. In the standard regularization approach, reconstruction proceeds by minimizing the difference of model and data terms. Such differences are taken into account in proportion to the *confidence* ascribed to the data terms, or local measurements. Thus typically, the absence of local measurements implies zero confidence. Below we show that standard diffusive filling-in assumes constant (high) confidence everywhere, even at positions lacking the support of measurements. Based on these observations we propose a modification of the standard filling-in equation that more effectively employs confidence information and illustrate its behavior on simulations of psychophysical brightness stimuli.

2 Filling-in and Feature Reconstruction

Many of the objects we perceive have roughly uniform regions of surface color, brightness and depth. At the same time, cells in the visual cortex in general do not respond to uniform regions, but rather to discontinuities (Hubel and Wiesel, 1962, 1968). In other words, many neurons respond more strongly to *boundaries* than to regions or surfaces. Consider Fig. 1. In the vicinity of object borders contrast signals are produced by the visual system. How should the appearance of inner regions be determined given the absence of direct neural support? To introduce concepts, we consider the task of generating a continuous representation of surface layout as one of *painting* (or *coloring*; Mumford, 1994) an empty region that is bounded by the local measurements at region boundaries (compare with Fig. 1(c)). The task thus consists of the reconstruction of surface properties from sparse data, i.e., generating from local measurements continuous representations of surface layout. Individual surfaces occur at different sizes and with various layouts. Therefore, any such mechanism has to be insensitive to such size and shape differences. Filling-in models suggest that bounded local contrast measures be used in the determination of surface appearance through a process of lateral spreading, or diffusion.

A model of complementary boundary and surface systems (BCS/FCS) was proposed by Grossberg and colleagues (Cohen and Grossberg, 1984; Grossberg and Mingolla, 1985; Grossberg and Todorović, 1988). In a nutshell, BCS/FCS processing occurs as follows. The input distribution is initially processed by center-surround mechanisms that code luminance ratios at image edges. Such contrast signals are analogous to retinal ganglion cell responses. Contrast signals are then used in two ways. They are used in the extraction of boundaries (BCS), as in other edge detection algorithms. Contrast signals are also fed to a filling-in stage (FCS) where they undergo a process of lateral spreading or diffusion (Fig. 2, left). Because the activity within the filling-in

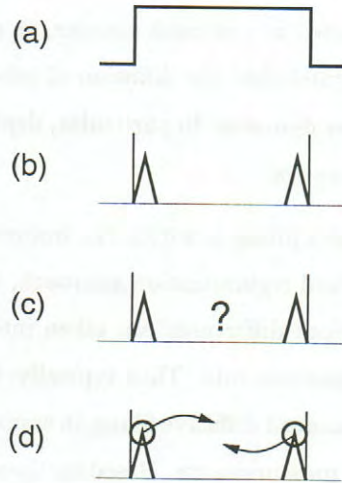


Figure 1: Profile of an example stimulus arrangement with a light region in the center and a darker background (a). Processing of such stimulus utilizing initial center-surround interaction produces local activations in the vicinity of the contrast (b). For simplicity we only display idealized contours (thin lines) and ON-center responses for the central light patch. For the reconstruction of a dense representation of surface quality the empty inner region must be somehow filled with activation (c). The suggested mechanism that we focus on in our analysis takes the initial contrast responses near the contrast and paints the rest of the region by a process of lateral spreading (d).

stage is the model's correlate of perceived brightness, we will call the activity at this stage *brightness signals*. The diffusion process is extremely fast, so for most everyday situations, the final equilibrated configuration of brightness signals corresponds to the percept². Moreover, diffusion is regulated by boundary signals from the BCS that block lateral spreading, effectively constraining brightness signals to remain within bounded regions.

For concreteness, let us consider how a simple BCS/FCS scheme can explain brightness perception in a simplified figure-ground scene (see Fig. 1). For simplicity we employ a 1-D representation of the stimulus and assume only an ON channel is present (Fig. 2, right). The luminance difference between the disk and the surround produces contrast signals associated with the dark-to-light transitions – just as retinal ganglion cells. These signals will occur just inside the luminance plateau – in a system with an OFF channel, OFF responses would occur just outside the center region. Contrast signals constitute an important first step in the determination of brightness. As stated above, they are used in two ways. They generate boundary signals (often, but not exclusively associated with edges) that determine the regions of influence of the initial contrast measurements. Contrast signals are also sent to the filling-in stage. There, filling-in signals undergo lateral diffusion. Initially, filling-in signals (dashed lines in Fig. 2, right) are equivalent to contrast signals. With time, the distribution of activity of filling-in signals changes as they spread laterally. Spreading occurs as long as signals are not stopped by a boundary. Hence, the strong boundaries separating center and surround regions

²At the same time, temporal brightness phenomena correspond to the temporal evolution of brightness signals (while they diffuse) within the filling-in stage (see Arrington, 1994).

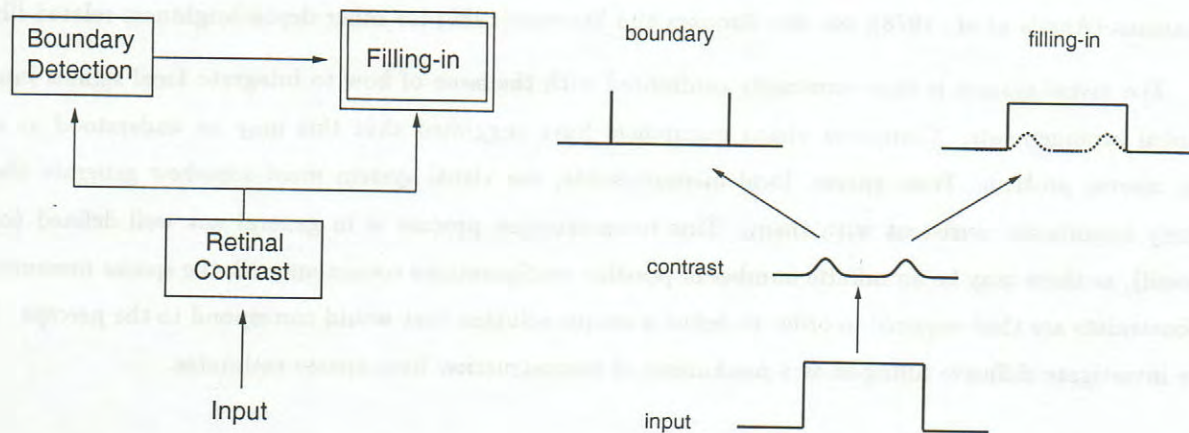


Figure 2: Macroscopic BCS/FCS model and boundary/feature processing. Left: Contrast signals are input to boundary detection (BCS) and filling-in (FCS) stages. Filling-in is regulated by boundary signals that constrain signals to occur within bounded regions. The activity within the filling-in stage is the model's correlate of perceived brightness. Right: One-dimensional schematic representation of the processing of a center-surround display. The initial filtering of the stimulus by small circularly symmetric ON-cells (similar to retinal cells) leads to the generation of contrast signals. These, by their turn, are used to extract boundaries and to start a filling-in process. The initial filling-in signals (dashed line) undergo bounded spreading, eventually producing a plateau distribution of activity (solid line). Activity within the filling-in stage corresponds to perceived brightness.

effectively isolate the center from the surround. Eventually, after the brightness signals spread, a uniform plateau of brightness signals (continuous line) ensues defining a center region as lighter than the background. In all, the BCS/FCS in particular, as well as other filling-in proposals (Davidson and Whiteside, 1971; Hamada, 1984; Arrington, 1996), may be viewed as attempts to bridge the gap from local contrast responses (ratios) to more continuous spatial representations (which in the BCS/FCS comprise the equilibrated filling-in signals).

The task of integrating local measurements into more global percepts is not limited to brightness perception. The domains of depth and motion perception are confronted with similar challenges. Using random dot stereograms, Julesz (1971) showed that the visual system strives to find a smooth surface layout compatible with the disparity information. This holds even in cases where only a minor fraction of points gives rise to localized disparity signals (so-called 5% stereograms; Julesz, 1971, p.122). Hence, instead of perceiving individual dots floating in depth, the subject perceives a *surface* in depth that is consistent with the local estimates. Recent evidence suggests that depth is assigned to the interiors of bounded homogeneous regions in which only the *vertically* oriented boundaries provide a source of local disparity (Nakayama and Shimojo, 1992). The depth assignment revealed by these and other studies may be mediated by a filling-in mechanism similar to the one for the determination of brightness. Starting from sparsely localized disparity estimates, a smooth surface in depth would be generated. Functionally similar underlying processes for brightness and depth are further suggested by similarities between stereo-depth and brightness effects. For example, in the standard Craik-O'Brien-Cornsweet (COC) effect, a "cusp" edge separating two same-luminance regions generates the percept of a brightness step. This illusion also holds when the stimulus is defined in depth. In this case, a step in depth is seen for the

plateaus (Anstis *et al.*, 1978); see also Brookes and Stevens (1989) for other depth-brightness related illusions.

The visual system is thus constantly confronted with the issue of how to integrate local signals into more global arrangements. Computer vision researchers have suggested that this may be understood as solving an *inverse problem*. From sparse, local measurements, the visual system must somehow generate the most likely hypothesis consistent with them. This reconstruction process is in general not well defined (or well-posed), as there may be an infinite number of possible configurations consistent with the sparse measurements. Constraints are thus required in order to define a unique solution that would correspond to the percept. Below, we investigate diffusive filling-in as a mechanism of reconstruction from sparse estimates.

3 Filling-in as a Mechanism to Solve an Inverse Problem

3.1 The Basic Filling-in Equation

The filling-in equation analysed in this section has been successfully used as a building block of models of brightness and surface depth perception (e.g., Grossberg and Todorović, 1988; Gove *et al.*, 1995; Pessoa *et al.*, 1995; Grossberg and McLoughlin, 1997). As a starting point, we consider a mechanism of filling-in that is based on the lateral spreading of the initial measurements. Within the empty space of bounded regions, activity can freely diffuse whereas at boundaries the diffusion stops. The boundary information that specifies surface layout can be used to control the efficiency of the lateral spreading or diffusion. The diffusion mechanism can thus deal with the problem of “painting” regions of different sizes. Important information about surface appearance is captured by the local measurement of luminance *contrast ratio*. The “painting” operation should therefore fill a region with local ratio information measured at the boundaries irrespective of the size of the region. In order to take into account such a constraint, Cohen and Grossberg (see also Grossberg and Todorović, 1988) suggested a non-conservative filling-in mechanism which utilizes a steady (or clamped) source of local input signals, c . The resulting filling-in activity will be denoted by v . In addition, in order to take into account a neurally plausible implementation based on leaky integrators, a passive decay of membrane potential is also incorporated. In all, the (already discretized) equation for the filling-in mechanism reads

$$\dot{v}_i = -Kv_i + c_i + \sum_{j \in \mathcal{N}_i} (v_j - v_i) \rho_{ij} \quad (1)$$

(see Cohen and Grossberg, 1984; Grossberg and Todorović, 1988). In this equation the input source and the passive decay rate are denoted c and $-Kv$, respectively; i and j are spatial position indices. The third component – sum over a nearest-neighbor coupling \mathcal{N}_i – corresponds to the diffusion component controlled by a spatially varying diffusivity which is modulated by the activation w of the topographically organized boundary system. An instance of this modulation function is given by $\rho_{ij} = \rho / (1 + f(w_i, w_j))$ such that the filling-in equation remains linear. In the modulation function the parameter ρ is a constant diffusivity effective in cases of

zero-amplitude input from boundary activation and $f(\cdot)$ is a function to connote the combination of discretized boundary activation at different sites (in the simulations we used $f(\cdot) = a(w_i + w_j)$).

3.2 Filling-in, Diffusion and Regularization Theory

In this section we first derive a continuous version of filling-in whose discretized form corresponds to the mechanism presented in Eqn. 1. This allows us to subsequently relate the result to the frameworks provided by diffusion and regularization theory, both widely used in computer vision. In terms of regularization the basic underlying observation is that the filling-in of surface features (e.g., brightness) from sparse local measurements can be considered as an inverse problem of reconstruction which is ill-posed. In order to find a solution to this problem it has to be regularized by imposing constraints on the space of admissible solutions. In the case of brightness reconstruction the nature of such constraints should also take into account perceptual data from psychophysical experiments (see Section 5).

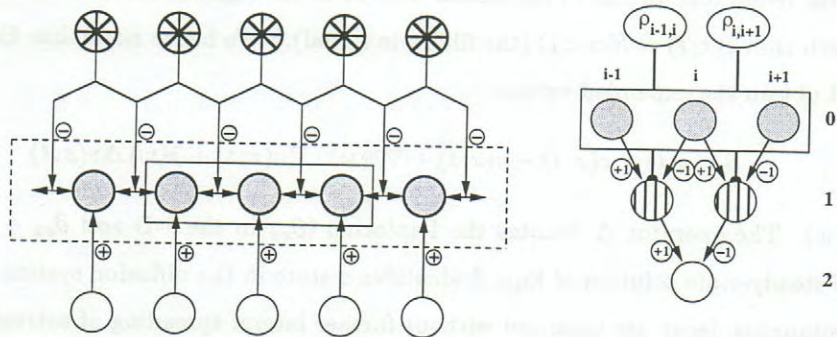


Figure 3: Sketch of the discretized network for filling-in. Left: Scheme of lateral interaction based on nearest neighbor coupling ($\mathcal{N}_i = \{i - 1, i, i + 1\}$, central rectangle). Each node in the filling-in network is fed by excitatory feedforward projections from contrast activation. Lateral coupling between neighboring lattice sites is modulated by inhibitory external inputs generated in the topographic boundary system (the wheel icon indicates pooled activation from filters tuned to different orientations). Right: Numerical first-order approximation of spatial derivatives (difference scheme for the central site coupled to its direct neighbors; efficacies are denoted by signed multipliers). The three-level cascade (0 – 1 – 2) realizes the inhomogeneous diffusion component $\text{div}(\rho \mathbf{grad} v)$ in the 1-D case where the hatching highlights those nodes whose contribution is modified by the spatially variant diffusivity (see text).

Filling-in in Continuous Form. The discretized filling-in equation denoted by Eqn. 1 contains a diffusion and a linear reaction term, the latter to bias the equilibrium solution towards the input data. The diffusion term is described by the sum of weighted differences between activations at neighboring network sites. Each component represents a numerical first-order difference term (or *computational molecule*; Terzopoulos, 1986) whose efficacy is modulated by the inhomogeneous diffusivity ρ_{ij} . Each difference can therefore be considered as a discretized version of the spatially continuous form $\rho(w) \mathbf{grad} v$, where $w = w(x)$. Summation of difference terms finally corresponds to the divergence of the gradient field. Formally, the relevant numerical terms can be

written as

$$\begin{aligned}\rho h^{-1}(v_i - v_{i-1}) &\approx \rho \mathbf{grad} v \equiv y_l, & \rho h^{-1}(v_{i+1} - v_i) &\approx \rho \mathbf{grad} v \equiv y_r, & \text{and} \\ h^{-1}(y_r - y_l) &\approx \text{div } y\end{aligned}$$

(compare with Fig. 3).³ In all, we can formulate the filling-in equation in *continuous* form as

$$\partial_t v(x; t) = \nabla \cdot (\rho(w) \nabla v(x; t)) + g(v, c), \quad (2)$$

with $g(v, c) = c(x; t) - Kv(x; t)$ ($\partial_t \equiv \partial/\partial t$). Here, K is the constant decay rate as in Eqn. 1, $\rho(\cdot)$ is the spatially inhomogeneous diffusivity function, and $\nabla \cdot$ is the div and ∇ the **grad** operator, respectively. By this mechanism the spatially inhomogeneous diffusion is biased by a source, $c(x; t)$, constituted by local contrast measurement signals from the ON or OFF channel, and a sink, $-Kv(x; t)$, that, in the absence of any input, produces a decay of activation at rate K .

In order to better relate this format to the results derived in the regularization framework, we will scale the input activation such that $\bar{c}(x; t) = Kc(x; t)$ (the filling-in signal).⁴ We hence normalize Eqn. 2 with respect to the decay rate and obtain the expanded version

$$\frac{1}{K} \partial_t v(x; t) = c(x; t) - v(x; t) + \nabla \bar{\rho}(w) \cdot \nabla v(x; t) + \bar{\rho}(w) \Delta v(x; t) \quad (3)$$

with $\bar{\rho}(w) = \frac{1}{K} \rho(w)$. The operator Δ denotes the Laplacian (∂_{xx} in the 1-D and $\partial_{xx} + \partial_{yy}$ in the 2-D case, respectively). The steady-state solution of Eqn. 3 identifies a state in the diffusion system in which the driving input and the spontaneous decay are balanced without further lateral spreading of activation.

Continuous Filling-in and Linear and Non-linear Diffusion. The continuous filling-in equation (Eqn. 3) reveals many similarities with mechanisms used in linear and non-linear diffusion models of computer vision (see Weickert (1997) for a recent review). Without the bias or (linear) reaction term $c(x; t) - v(x; t)$ in Eqn. 3, the filling-in mechanism results in a *spatially variant linear diffusion equation*. Fritsch (1992) proposed the use of the local gradient magnitude of the raw image intensity function to control the local lateral spreading in the diffusion process. In our filling-in framework contour signals are generated by the segregated topographically organized boundary system, namely $w(x)$. In the feature system the adaptation of the diffusivity is controlled by the activities generated in the boundary system to feed the function $\rho(w)$. The diffusive filling-in process is *isotropic* since the diffusivity $\rho(\cdot)$ is a scalar-valued function. Many non-linear diffusion models have been developed which enable directional (thus anisotropic) diffusion along oriented luminance edges (see, e.g., Alvarez *et al.*, 1992; Weickert, 1996).⁵ As pointed out, the filling-in equation contains

³The term $\sum_{j \in \mathcal{N}_i} (v_j - v_i) \rho_{ij}$ is thus a first-order discretization of $\text{div}(\rho(\cdot) \mathbf{grad} v(x))$ as pointed out by Weickert (personal communication).

⁴This scaling may be attained by the multiplication of the upper and lower saturation levels for the neural activity in the initial center-surround processing stage by the factor K (see Neumann, 1996).

⁵Since we concentrate on filling-in in 1-D domains only, spatially isotropic and anisotropic models are equivalent. We therefore do not discuss this issue in further detail.

a source and a sink component. Taken together they define an additional linear *reaction term* that guarantees the equilibrium solution being close to the input. Such reaction terms have been used in a number of models for non-linear diffusion and reaction-diffusion for image processing and the natural sciences (e.g. Cottet and Germain, 1993; Schnörr and Sprengel, 1994; Price *et al.*, 1989; Turing, 1952; Witkin and Kass, 1991). As said, the filling-in equation is of the spatially variant linear type. It combines reaction components to ensure an equilibrated activation that depends on the the structure of the input distribution.

ON and OFF Contrast Systems. Modelling brightness reconstruction from local contrast information must take into account the non-negativity of cell responses. Consider a luminance contrast: ON contrast cells signal the luminance increment at the brighter region whereas OFF contrast cells signal the luminance decrement for the darker region in the vicinity of the edge. For the filling-in mechanism to function properly the diffusion must be based on separate representations for ON and OFF contrast activation, respectively (see Gove *et al.*, 1995; Pessoa *et al.*, 1995; Arrington, 1996). Therefore, the computational architecture includes two segregated ON and OFF filling-in networks denoted by $\partial_t v^+(x; t)$ and $\partial_t v^-(x; t)$:

$$\frac{1}{K} \partial_t v^\pm(x; t) = c^\pm(x; t) - v^\pm(x; t) + \nabla \bar{\rho}(w) \cdot \nabla v^\pm(x; t) + \bar{\rho}(w) \Delta v^\pm(x; t).$$

In both systems the diffusivity is controlled by the activation of the boundary system. The corresponding discretized filling-in equations \dot{v}_i^+ and \dot{v}_i^- are defined in accordance to the scheme shown in Eqn. 1.

Inverse Problems and Regularization Theory. Brightness reconstruction from local (contrast) estimates can be described as the general problem of finding a solution for u ($u \in X$) given the data $d = Au$ (with $d \in Y$). In our case A lumps together the initial center-surround filtering and normalization such as described in Neumann (1996). The existence and uniqueness of a solution and its continuous dependence on the data cannot be guaranteed since the measurements may be noisy and are sparse. The inverse problem is therefore classified as ill-posed in the sense of Hadamard (Poggio *et al.*, 1985; Bertero *et al.*, 1988). The solution to the problem has to be regularized such that proper constraints are imposed on the possible set of candidates in the function space of solutions. We select the function \hat{u} as the solution that minimizes the norm $\|A\hat{u} - d\|_Y$ subject to the additional constraint of smoothness of \hat{u} , where smoothness is characterized by a minimized derivative (e.g., of first order). The overall goal is to minimize the constraint given by the local differences between the measured data and the reconstructed function values (data term) and the stabilizing functional imposed on the function (smoothness term). This results in the goal of minimizing the quadratic functional

$$\|Au - d\|^2 + \lambda \|Pu\|^2 \rightarrow \min, \quad (4)$$

where P denotes a “constraint operator” (a mapping $P : X \rightarrow Z$) that stabilizes the solution for the inverse problem and $\|\cdot\|$ represents a proper norm in the spaces Y and Z , respectively (Poggio *et al.*, 1985). For a detailed discussion of the formal mathematical background we refer to Bertero *et al.* (1988) and various textbooks such as Baumeister (1987).

From the calculus of variations (Gelfand and Fomin, 1963) the minimization of a functional, $E(u) = \int_{\Omega} E_d(u, d) + \lambda E_p(u) \rightarrow \min$, yields a solution function that realizes a compromise between the data term (“similarity” or “closeness” to the data) and the model term (“smoothness” of the solution). The model term represents the a priori model of the function searched for in the reconstruction process. A class of this set of functions which allows for a direct physical interpretation represents the behavior of elastic membranes and thin plates, which minimize the first- and second order variation, respectively (Courant and Hilbert, 1966).⁶

Continuous Filling-in and Regularization. The input to a stage of diffusion for filling-in of contrast signals is provided by the ON and OFF channel measures generated by an initial center-surround processing stage. The “painting” of brightness values in the interior of a region that depicts a continuous surface patch is driven by local contrast measures at region boundaries. Since such responses scale with edge luminance ratios they encode differences in relative reflectances. In this context, the generation of a distributed (or spatially dense) representation of filled-in signals may be viewed as one of reconstructing smooth perceptual surface quantities, such as brightness. This problem is ill-posed since there is no unique solution available on the basis of the initial contrast measurements. In order to solve that inverse problem we minimize a quadratic functional that considers first-order variations in the smoothness term. The resulting functional for the corresponding *membrane* regularization of reconstruction from sparse contrast signals, reads for the 1-D case⁷

$$\mathcal{E}(v) = \int_{\Omega} F[v(x), v_x(x), x] \rightarrow \min, \quad (5)$$

where $F = \kappa(x)(v(x) - c(x))^2 + \bar{\rho}(w)(v_x(x))^2$. In this function $c(\cdot)$ represents the local estimated contrast data,⁸ $v(\cdot)$ (and its first order derivative $v_x(\cdot)$) is the function we search for in the function space of the given inverse problem, and $\kappa(x)$ is a local confidence measure with $\kappa(x) = \varepsilon > 0$ where there is no input signal from the initial contrast processing stage (compare with Terzopoulos, 1986; Szeliski, 1990).⁹ The diffusivity $\bar{\rho}(w)$ is a space variant regularization parameter that allows the modulation of the magnitude of the contribution of the smoothness term.

The necessary condition for the existence of a solution for this minimization problem is denoted by the Euler equation, which for F results in

$$\frac{\partial}{\partial v} F - \frac{d}{dx} \frac{\partial}{\partial v_x} F = 0. \quad (6)$$

⁶Note that the quadratic functionals (Eqn. 4) always lead to Euler-Lagrange equations that are linear w.r.t. the unknown function (see Eqn. 3). An extension to non-quadratic convex functionals and nonlinear diffusion equations, respectively, has been proposed by Schnörr and Sprengel (1994).

⁷The model discussed in this article has been defined in 1-D. We therefore consider corresponding equations only. In general, however, the filling-in equation as well as the functionals subject to variations can be extended to higher dimensions in a straightforward manner.

⁸For simplicity we consider a general contrast input c without presenting all equations for the ON and OFF channel individually.

⁹In the original description of Terzopoulos (1986) and Szeliski (1990) the use of discrete confidence values κ_i generated by measurement functionals where data is available was suggested. This allows $\kappa_i = 0$ at locations with no input measurement. However, by locally zeroing out the data term, the well-posedness of the functional given in Eqn. 5 cannot be guaranteed (Schnörr, personal communication). In our further investigation we therefore incorporate a *continuous* confidence measure which approaches the minimal value $\varepsilon > 0$ for sites where no input is available.

In the functional defined for the given F we seek a solution close to the data whereas at the same time minimizing the first order derivative of the approximating function (see Eqn. 5). The corresponding Euler equation reads

$$\kappa(x)(v(x) - c(x)) - (\bar{\rho}_w(w) w_x v_x(x) + \bar{\rho}(w) v_{xx}(x)) = 0. \quad (7)$$

Using $\bar{\rho}(w) = \rho/(K(1 + f(w(x))))$ we expand this equation in order to directly include the activity of the boundary system and get

$$\kappa(x)(v(x) - c(x)) - \bar{\rho}(w) \left(v_{xx}(x) - \frac{f_w(w) w_x(x)}{1 + f(w)} v_x(x) \right) = 0. \quad (8)$$

Below we utilize the solution of membrane regularization for the problem of reconstruction from sparse data to investigate the filling-in equation for brightness perception.

3.3 Analysis and Predictions

Analysis. Above we have derived an equation for the continuous filling-in mechanism (see Eqn. 3) and the Euler equation for the membrane regularized solution of the problem of reconstructing surface quantities, e.g., brightness (see Eqn. 7). Since in 1-D $\nabla = d/dx$, we observe that both results appear almost identical. This indicates that the steady-state activity distribution in the filling-in network (compare Fig. 3) represents the regularized solution of the inverse problem of reconstructing from sparse initial data. In other words, the ill-posed problem of reconstructing a dense representation of surface quantities is regularized by the neural mechanism of filling-in. The formal frameworks of regularization and diffusion theory allow a more formal treatment of filling-in mechanisms and, furthermore, guide the modelling of more complex perceptual mechanisms within the filling-in framework.

An important difference between the above mentioned equations remains, however. In Eqn. 7 the data compatibility term $(v(x) - c(x))$ denoting the similarity between the reconstruction and the measured input data is multiplied by an explicit confidence measure, or “data availability” coefficient, $\kappa(x)$. In the task of reconstruction from sparse data the contribution of a data compatibility term should only be effective at those locations where input data is available. This problem falls into the more general category of visible surface reconstruction dealt with in computer vision tasks. Here one also starts from a sparse and noisy set of data points to end up with a smooth representation of continuous surface depth (see, e.g., Terzopoulos, 1986; Szeliski, 1990). If a confidence measure is omitted, any data point in the otherwise sparse input field would be used to drive the reconstruction. Thus a missing measurement – assuming an existing confidence measure – would be treated as a zero-strength signal data point that should be approached by the reconstructed function.

The confidence measure is used to weight the data compatibility term in the corresponding functional (see Eqn. 5), thus taking into account the availability of input data. At the same time, in the filling-in equation being analyzed, no provision is made for a confidence measure. In terms of the *regularization* framework, the

filling-in equation (Eqn. 3) may be interpreted as if a confidence measure of unit value, i.e., $\kappa(x) = 1$, exists at all spatial positions, irrespective of the distribution of input measurements. In terms of the *diffusion* framework, the additional reaction term provides a means to specify a nontrivial steady-state solution which is close to the original input (Cottet and Germain, 1993; Weickert, 1997). In filling-in used for *reconstruction* of surface quantities this is only a desirable solution at those locations where input data is available but not where no initial input is provided.

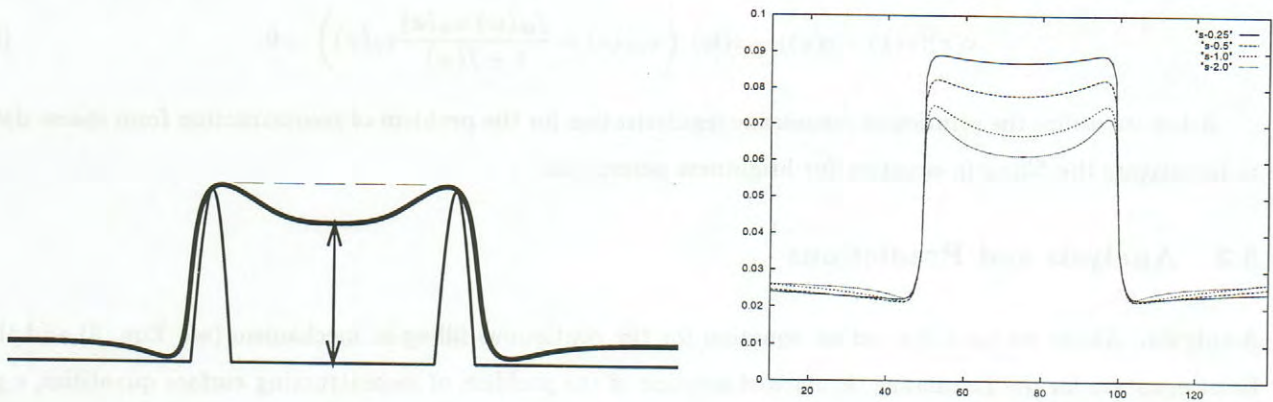


Figure 4: Left: Reconstruction process for standard filling-in. Given that a confidence value of one is assumed everywhere ($\kappa(x) = 1$), the zero contrast values for inner regions force the model terms to approach this value. Right: Simulations of standard filling-in. The reconstruction process produces a bending of the final activity distributions. See text for discussion.

Predictions. Filling-in does not include a mechanism that evaluates the validity or confidence of data contributions depending on the availability or not of such measures. As a result, filling-in treats locations of missing data as if there were regular *zero amplitude* input signals available at these positions. This implies that uniform regions must always, at equilibrium, exhibit “bowed” signal distributions. Figure 4 illustrates the situation. Associated with a luminance pedestal, contrast signals will be generated in the vicinity of the edges. These feeding input data terms provide the basis for the reconstruction process. Given that $\kappa(x)$ is assumed to be one everywhere, and that the local measures for inner regions value zero, a bow in the signal distribution is generated. This occurs since at inner positions the model term is driven to zero, the value of the feeding input data. Figure 4 (right) shows the results of computer simulations of a simple pedestal stimulus. The actual amount of bending is dependent on the spontaneous decay parameter K of the diffusion equation. For larger values the bending is quite considerable, although it can be reduced for smaller values (but it is always present).

4 Confidence-based Filling-in

We have shown above that under certain conditions the filling-in proposal relates to the regularization of the inverse problem of reconstructing a brightness representation from sparse data. Based on this formal analysis, we propose a new extended scheme of filling-in in which a confidence parameter is included to validate the contribution of the data compatibility term in the regularization functional. In terms of diffusion models the extended mechanism combines both types of biased and unbiased inhomogeneous diffusion: At locations of reliably available input data near luminance contrasts an effective reaction term forces the solution towards the level of the filling-in input signals, at locations of unreliable or no feeding contrast input (in the interior of homogeneous regions) filling-in effectively behaves as an unbiased linear diffusion system.

The details of this extended mechanism of *confidence-based filling-in* will be described in more detail below. We then show the application of the scheme to simulate selected perception data.

4.1 Specification of Confidence-based Filling-in

As shown above, within the standard filling-in framework, a lack of contrast measure is treated as a zero-strength signal data point that should be approached by the reconstruction process. In general this is clearly undesirable as it could lead to the incorrect recovery of information. Consider the reconstruction of depth information. In this case, a square in depth defined by local disparity information at the edges only would be reconstructed as a surface bent in depth, with the inner regions approaching the observer. The reason such a strategy has not proven disastrous in the case of brightness perception is that the tendency to approach zero-magnitude levels is controlled by the decay parameter of the filling-in equation. Therefore, this parameter has to be carefully chosen in order to produce more or less constant levels of activity. Incorrect settings can lead to the prediction of the perception of Mach bands – or brightness overshoots and undershoots – in pedestal stimuli such as in Fig. 4 (right), instead of constant brightness levels as found experimentally (Pessoa, 1996b).

Confidence values and the distribution of contrast input. Studies from computer vision suggest that a confidence measure can be exploited in the problem of brightness reconstruction through filling-in. How should confidence be defined in the domain of contrast measures and brightness reconstruction? The simplest approach would be to employ a two-level confidence value: $\kappa(x) = 1$ at locations where input measurements are available and $\kappa(x) = \varepsilon > 0$ where this is not the case. Its simplicity notwithstanding, this decision strategy suffers from several problems. For one, the inherent unreliability in the detection of contrast (Marr, 1982; Canny, 1986) indicates that incorrect confidence values could be assigned to false positive and false-negative responses. At false-positive responses, $\kappa(x) = 1$ would dictate that reconstruction approach a contrast measure that is spurious (where none should have been generated), while at false-negative responses, $\kappa(x) = \varepsilon$ would dictate that reconstruction be significantly reduced (where a non-zero level should have been generated and

used by reconstruction). A more robust alternative is to take into account the statistical distribution of the contrast measures. Szeliski (1990) suggests the use of a statistical measure $\kappa(x) = 1/\sigma^2$ in which σ denotes the spread of the corresponding density of the signal measurements. Thus the confidence is increased for reliable measurements with only minor noise. This principle of evaluating the reliability of input data has been applied in various computational methods in image processing, such as data similarity terms and image operator response (e.g., Sanger, 1988; Westelius *et al.*, 1995).

The measurement of local image structure during the first stages in the visual processing hierarchy may be conceptualized as a process of “template matching.” Local masks of finite aperture and specific selectivity (e.g., spatial profile, orientation) are utilized to match the presence of image structure which is conceptualized by a stage of vector projection. Similarity between mask profile and signal structure is encoded in the amplitude of response. Accordingly, we propose that confidence values used by filling-in be determined directly through contrast sensitive cells, and that its output strength determine confidence magnitude.

Within our neural computational framework, we suggest that model *complex cells* (Neumann and Pessoa, 1994; Pessoa *et al.*, 1995) be used to guide confidence measures. Complex cell responses are localized in the vicinity of boundaries. Initial responses from unoriented center-surround cells are integrated by the subfields of oriented polarity-sensitive model simple cells. Their responses in turn feed into polarity-insensitive complex cells. Due to the successive spatial pooling, complex cell responses at sharp boundaries span the spatial extent covered by the distribution of center-surround cells which are used as inputs (c in Eqn. 1) for the filling-in (Neumann, 1994). We therefore define $\kappa(x) = Z(x) + I^\epsilon$, where $Z(x)$ are complex cell responses and I^ϵ is a small tonic input with $I^\epsilon = \epsilon$. The resulting function thus defines a spatially continuous field of confidence values that weight the corresponding data similarity component in the filling-in equation. Consequently, the confidence weighting is close to zero for the inner regions of homogeneous surfaces, correctly disabling the data contributions. At the same time, confidence values are bounded by the saturation levels defined by the corresponding network layers (for details, see Neumann, 1993, 1996; Neumann and Pessoa, 1994). Between the lower and upper limits a continuous range of confidence values occurs.

Continuous Confidence-based Filling-in. With the above definition of a confidence measure $\kappa(x)$, we can now present the extended version of the non-conservative filling-in mechanism in continuous form. Utilizing the functional for the membrane regularization given in Eqn. 5 we now define $F = (Z(x) + I^\epsilon)(v(x) - c(x))^2 + \bar{\rho}(w)(v_x(x))^2$. The corresponding Euler equation for the minimization problem then finally reads

$$(Z(x) + I^\epsilon)(v(x) - c(x)) - (\bar{\rho}_w(w) w_x v_x(x) + \bar{\rho}(w) v_{xx}(x)) = 0. \quad (9)$$

This indicates that the filling-in equation can be simply extended by an external function to provide a mechanism for gain control of the reaction term. Thus the contribution of the source and sink component for the reconstruction is continuously controlled by the gain mechanism provided by model complex cell responses.

The corresponding continuous diffusion equation results in

$$\frac{1}{K} \partial_t v(x; t) = (Z(x) + I^\varepsilon) (c(x; t) - v(x; t)) + \nabla \cdot (\rho(w) \nabla v(x; t)). \quad (10)$$

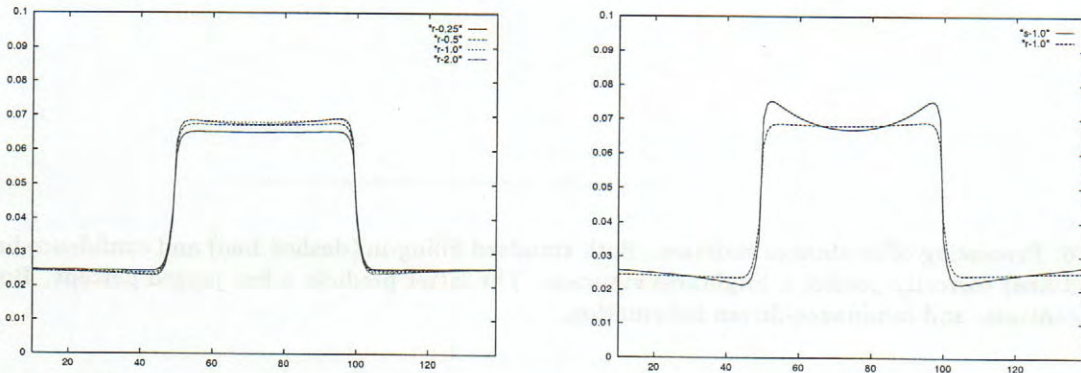


Figure 5: Processing of a luminance pedestal by confidence-based filling-in. Left: Reconstruction of constant brightness profiles (plateaus) for a set of increasing decay constants. Right: Comparison of reconstructed brightness profiles generated by standard and confidence-based filling-in (same parameter setting).

Discrete Confidence-based Filling-in. Similarly, we can also derive a discretized version of the new diffusion equation. We first sample the confidence activity $\kappa(x)$ to get $\mathbf{D}(x) (Z(x) + I^\varepsilon) = Z_i + I_i^\varepsilon \equiv \kappa_i$, with $\mathbf{D}(x)$ defining the sampling function. Corresponding to Eqn. 1 we then get

$$\dot{v}_i = K (c_i - v_i) \kappa_i + K \sum_{j \in \mathcal{N}_i} (v_j - v_i) \rho_{ij}. \quad (11)$$

It is instructive to observe the behavior of this new filling-in equation with the pedestal stimuli employed in Fig. 4. For the simulation of test data we utilize the architecture of two separate filling-in systems each of which is driven by segregated ON and OFF activations, respectively. In Fig. 5 (left) we show four simulations with increasing magnitudes of the spontaneous decay parameter K (as in Fig. 4, right). It can be observed that the reconstructed plateaus appear largely flat. Also, unlike the case with standard filling-in, the resulting graphs look almost identical. This behavior indicates the robustness of the proposed mechanism against parameter changes. For direct comparison Fig. 5 (right) shows the difference of reconstruction between the standard algorithm and the new version (for same parameter settings).

4.2 Simulations of Brightness Data

In this section we illustrate the behavior of the confidence filling-in scheme when applied to psychophysical stimuli. Arrington (1996) argued that an important class of stimuli that can be used to assess brightness models comprises brightness transitivity phenomena. One example is a luminance staircase in which luminance plateaus are separated by luminance steps (or edges). Such stimulus is perceived veridically, that is, as a brightness

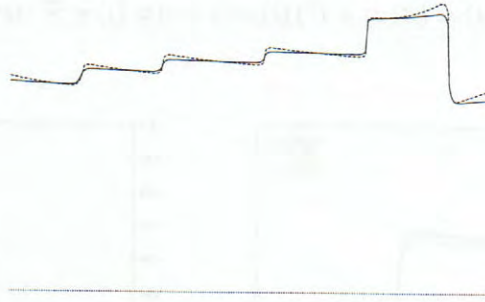


Figure 6: Processing of luminance staircase. Both standard filling-in (dashed line) and confidence-based filling-in (solid line) correctly predict a brightness staircase. The latter predicts a less jagged percept. Both schemes utilize contrast- and luminance-driven information.

staircase. As Arrington showed, the standard filling-in scheme cannot adequately handle them. Consider a luminance staircase in which all steps are matched for contrast. The contrast responses that are used for the reconstruction of the stimulus are identical at all steps. Consequently, the brightness levels are the same for all steps, directly violating the percept of a brightness staircase.¹⁰ Pessoa *et al.* (1995) introduced a version of brightness filling-in that takes into account both contrast- and luminance-driven information. Thus, although purely bounded contrast is insufficient to explain a luminance staircase stimulus, a luminance-channel is able to sense the increasing luminance along the staircase and add its contribution to the final percept. In all, a luminance staircase ensues (see Pessoa *et al.*, 1995). Confidence filling-in is also capable of generating valid predictions for a staircase stimulus when embedded in the contrast- and luminance-driven scheme. Figure 6 shows the results the model proposed by Pessoa *et al.* (dashed line) as well as confidence filling-in (solid line) produce. As we can see, both schemes correctly predict a brightness staircase. In line with our previous arguments (Section 3), confidence-based filling-in produces a less jagged brightness profile.

Two regions of uniform luminance separated by a “cusp edge” appear differently bright, the so-called Craik-O’Brien-Cornsweet effect (COCE) (Cornsweet, 1970; Todorović, 1987). Figure 7 (top) shows the input luminance distribution of a standard COC stimulus (see the form of a cusp edge in the center). The COCE constitutes yet another example of the crucial role edges have on brightness appearance. The lower-luminance side of the cusp is associated with a uniformly darker region, and the higher-luminance side of the cusp with a uniformly brighter region (Fig. 7 (bottom) shows the prediction generated by the confidence-based filling-in mechanism). What occurs when several cusps are viewed together, as in a “staircase COCE” stimulus? Although one cusp induces a brightness step, several cusps are not integrated, and the inner regions appear the same (see discussion in Grossberg and Todorović (1988), pp. 257-258). Only the outer regions appear of different

¹⁰Remember that contrast signals are only integrated within bounded regions since strong boundary signals block diffusion. In this way all (inner) plateaus of the staircase have identical contrast values associated with them (and hence the same output value).

Figure 7 shows the prediction of a brightness percept from a luminance distribution with a central cusp and two side lobes.

Figure 8 shows the prediction of a brightness percept from a staircase luminance distribution. The two inner regions have essentially the same brightness, while the leftmost region appears darker than the rightmost region, correctly matching the actual percept. For comparison we illustrate the output of a luminance staircase which is correctly predicted to appear as a brightness staircase.

Figure 7: Processing two regions of identical luminance level separated by a luminance cusp (standard Craik-O'Brien-Cornsweet stimulus). Top: Luminance distribution. Bottom: Model simulation utilizing the mechanism of confidence filling-in.

Figure 8: Processing of staircase Craik-O'Brien-Cornsweet stimulus. Top: Luminance distribution. Bottom: Model simulations (solid line). The two inner regions have essentially the same brightness, while the leftmost region appears darker than the rightmost region, correctly matching the actual percept. For comparison we illustrate the output of a luminance staircase which is correctly predicted to appear as a brightness staircase (dashed line).

Figure 7: Processing two regions of identical luminance level separated by a luminance cusp (standard Craik-O'Brien-Cornsweet stimulus). Top: Luminance distribution. Bottom: Model simulation utilizing the mechanism of confidence filling-in.

brightnesses. Figure 8 shows the predictions of confidence-based filling-in with contrast- and luminance-driven channels.

Brightness models that integrate all cusp edges in a scene, such as MIDAAS (Kingdom and Moulden, 1992) err for the staircase COC stimulus. This is also the case for Arrington’s (1996) proposal of directional filling-in. Although it correctly predicts the appearance of a luminance staircase, it errs for the stimulus shown in Fig. 8 (top). The staircase COC stimulus demonstrates that although local edge information is many times sufficient to induce brightness differences (such as with a single cusp, Fig. 7), sometimes direct luminance support is necessary for producing brightness changes.

5 Discussion

Confidence-based Filling-in. The solution of inverse problems through regularization theory is a powerful tool of computer vision for surface reconstruction. It addresses the fundamental problem of generating a more rich representation given only sparse and possibly conflicting samples of the input pattern. In particular, the computation of visible surface representation has been considered as a key step for integrating sparse data provided by separate modules for shape recovery, such as in stereopsis, motion, and monocular shape-from-X processing (Marr, 1982; Grimson, 1981; Terzopoulos, 1983, 1986; Blake and Zisserman, 1987).

Above we have demonstrated that diffusive filling-in as used in a number of perceptual models (e.g., Grossberg and Todorović, 1988; Gove *et al.*, 1995; Pessoa *et al.*, 1995; Grossberg and McLoughlin, 1997) can be understood in part as a reconstruction problem and directly linked to inverse problems and regularization theory. A fundamental constraint that applies to filling-in models, but not necessarily to computer vision applications in general, comprises the nature of *perceptual data*. Data must be used to assess perceptual models, and as such they should also be used to evaluate perceptual models that make use of regularization theory. For instance, the minimization of first-order information dictated by membrane regularization needs to meet the data on the perception of brightness. We therefore use the framework of regularization theory as a mathematical tool to predict perceptual data for the reconstruction of homogeneous surface qualities. In turn, data derived from psychophysical experiments should be used to test the model against stimuli, thus verifying or invalidating the suggested mechanisms involved in the generation of surface appearance.

Our analysis of filling-in in terms of regularization theory illustrated that standard filling-in does not utilize an important concept from this theoretical framework, namely, a confidence measure of the input contributions. This implies that filled-in regions will always contain a “bowed” distribution of signals. Accordingly, we introduced a variant of filling-in that we call *confidence-based filling-in* that makes explicit use of the validity (or availability) of the data terms. In this scheme boundary information is used to confirm the presence of input terms that can then fully drive reconstruction. This is possible since boundary responses always coincide

with regions of contrast input. We illustrated our proposal for input distributions such as pedestals, luminance staircases, and cusp staircases.

Nevertheless, confidence filling-in should be assessed by perceptual data. In this respect, the Chevreul illusion, namely, the perception of a jagged brightness distribution for luminance staircases, seems a suitable domain to assess whether confidence-based filling-in predicts the data more closely than standard filling-in. The interplay between plateau width and step magnitude on the strength of the illusion could be used to compare the two filling-in proposals. Irrespective of which scheme accounts for the data more accurately, one needs to face the issue of whether there is evidence for filling-in in the first place. Is filling-in a conceptual, or computational, construct, or does it occur in the brain (Pessoa *et al.*, 1998)?

Empirical Evidence for Neural Filling-in Mechanisms. Paradiso and Nakayama (1991) used a visual masking paradigm to investigate two issues: First, the role of edge information in determining the brightness of homogeneous regions, and second the temporal dynamics of perceptual filling-in. They reasoned that if the filling-in process involves some form of activity-spreading, it may be possible to demonstrate its existence by interrupting it. If boundaries interrupt filling-in, what happens when new borders are introduced? Is the filling-in process affected before it is complete?

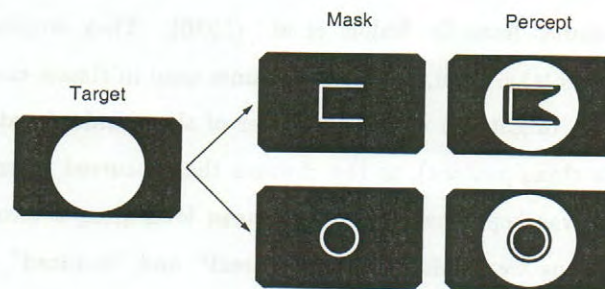


Figure 9: Masking paradigm in the temporal dynamics of brightness study by Paradiso and Nakayama (1991). Brightness suppression of a disk-shaped target by a mask. The target and mask are each presented for 16 msec. Optimizing the temporal delay between the stimuli yields a percept in which the brightness in a large central area of the disk is greatly suppressed. Brightness suppression is highly dependent on the arrangement of the contours in the mask.

Figure 9 shows the paradigm they used as well as the basic result. The target is presented first and is followed at variable intervals by a mask. For intervals on the order of 50-100 msec the brightness of the central area is highly dependent on the shape of the mask. For example, for a C-shaped mask, a darkening of the middle region is observed, with the bright region “protruding” inside the C. For a circular-shaped mask, an inner dark disk is perceived. Both these results are consistent with the hypothesis that brightness signals are generated at the borders of their target stimuli and propagate inward. Moreover, contours interrupt the propagation. Thus for a circular-shaped mask, brightness signals originating from the target border seem to be entirely “blocked” (hence a dark middle disk), while for the C-shaped mask, the brightness signals are only partially

blocked (hence the protrusion of brightness inside the C shape). An altogether different outcome results for larger delays between target and mask stimuli. If the mask is presented after 100msec, the brightness of the central region is largely unaffected. Corroborating the hypothesis that the propagation of brightness signals is involved, Paradiso and Nakayama (1991) showed that brightness suppression depended on the distance between target and mask. In particular, for larger distances maximal suppression occurred at later times, revealing a propagation rate of 110-150 deg/sec (6.7-9.2 msec/deg).

Some of these results were anticipated in an earlier study by Stoper and Mansfield (1978). They employed a masking paradigm in which the masks were varied systematically in time. They interpreted their "area suppression" effects as resulting from the interference of a mask with the process of filling-in of target brightness. Their paradigm enabled them to show that brightness suppression could not be due simply to contour suppression, thereby indicating that brightness and contour processes are subserved by independent systems.

The filling-in model of brightness perception proposed by Grossberg and Todorović (1988) has been shown by Arrington (1994) to produce excellent fits to the data from both Stoper and Mansfield (1978) and Paradiso and Nakayama (1991). This lends further support to the interpretation of these data as due to active filling-in mechanisms.

Another relevant study comes from De Valois *et al.* (1986). They employed center-surround standard (reference) and matching (variable) stimuli, similar to the ones used in classic contrast studies. They compared the results of direct changes in brightness where the center of the standard pattern was explicitly modulated in luminance (as was the matching pattern), to the changes that occurred when the surround was modulated sinusoidally while the center was kept constant at the mean level (i.e., a temporal version of simultaneous contrast). These two conditions were referred to as "direct" and "induced" respectively. The purpose of the experiments was to measure brightness changes produced by oscillations at various temporal frequencies between 0.5 and 8 Hz. Their studies revealed two main findings: (1) The temporal frequencies studied had little effect on the apparent brightness change in the direct condition; and (2) in the induced condition, the amount of brightness change fell drastically as the temporal frequency increased (around 2.5 Hz).¹¹ These results can be interpreted in terms of a spreading mechanism of induction that occurs over time, one that would provide a spatially continuous representation for filling-in. Brightness and color signals would be generated at the edges between center and surround, and would propagate inside the center region determining the appearance. An optimal temporal frequency would reflect the time interval necessary for the signal to propagate from the edges. The drastic fall-off found by De Valois *et al.* would result from a change in the surround before the edge signal was able to reach the middle of the center region.

Rossi and Paradiso (1996) replicated the brightness induction results of De Valois *et al.* (1986) and studied

¹¹Note that these frequencies are much lower than the ones usually revealed in flicker studies, which have cut-off frequencies of more than 30 Hz and peak around 4-6 Hz.

the role of pattern size on the effect by varying the spatial frequency of the inducing pattern. The correlation found between spatial scale, degree of induction, and cut-off frequency indicates that there is a limited speed at which induction proceeds and that larger areas take more time to induce. Rossi and Paradiso conclude that the limits on the rate of induction are consistent with an active filling-in mechanism initiated at the edges and propagated inward.

In a remarkable study, Rossi *et al.* (1996) showed that a significant percentage of neurons in cat primary visual cortex respond in a manner that correlates with *perceived brightness*, rather than responding strictly to the light level in the receptive field of the cells. Rossi *et al.* studied cell responses in conditions analogous to the direct and induced conditions studied psychophysically by De Valois *et al.* (1986) and Rossi and Paradiso (1996). In the induced condition, neural responses were largest at low temporal frequencies and decreased as the rate of modulation increased over 1.0Hz. In the direct condition, however, response amplitudes progressively increased with increasing temporal frequencies. These results, as well as other findings, are closely paralleled by psychophysical findings, suggesting that such cell responses contribute to the perception of brightness. Most important in the present context is the possibility that cell responses are the product of filling-in. This possibility deserves attention if we note that the psychophysical results strongly indicate a temporal filling-in mechanism.

The studies discussed above provide strong evidence for active filling-in. In brightness filling-in the brain seems to be generating a spatially organized representation, and it seems to be doing so through a roughly continuous propagation of signals, a process that takes time (see Pessoa *et al.*, 1998). The data above indicate that, at least in the case of brightness, the brain may be reconstructing information through a diffusive process. In this respect, it may prove useful to employ formal techniques for the solution of inverse problems, such as was done with regularization theory above, to further our understanding of the associated brain mechanisms.

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References

- Alvarez, L., Lions, J.-P., and Morel, J.-M. (1992). Image selective smoothing and edge detection by non-linear diffusion, II. *SIAM Journal of Numerical Analysis*, **29**, 845-866.
- Anstis, S.M. and Howard, I.P. (1978). A Craik-O'Brien-Cornsweet illusion for visual depth. *Vision Research*, **18**, 213-217.
- Arrington, K.F. (1994). The temporal dynamics of brightness filling-in. *Vision Research*, **34**, 3371-3387.
- Arrington, K.F. (1996). Directional filling-in. *Neural Computation*, **8**, 300-318.
- Baumeister, J. (1987). *Stable solution of inverse problems*, Braunschweig, Germany, F. Vieweg & Sohn.
- Bertero, M., Poggio, T., and Torre, V. (1988). Ill-posed problems in early vision. *Proc. of the IEEE*, **76**, 869-889.

- Blake, A. and Zisserman, A. (1987). *Visual reconstruction*, Cambridge, MA, MIT Press.
- Brookes, A. and Stevens, K.A. (1989). The analogy between stereo depth and brightness. *Perception*, **18**, 601-614.
- Canny, J.F. (1986). A computational approach to edge detection. *IEEE Trans. on PAMI*, **8**, 679-698.
- Cohen, M. and Grossberg, S. (1984). Neural dynamics of brightness perception: Features, boundaries, diffusion, and resonance. *Perception & Psychophysics*, **36**, 428-456.
- Cornsweet, T.N. (1970). *Visual Perception*, New York, Academic Press.
- Cottet, G.-H. and Germain, L. (1993). Image processing through reaction combined with nonlinear diffusion. *Mathematics of Computation*, **61**, 659-673.
- Courant, R. and Hilbert, D. (1966). *Methods of mathematical physics*, New York, Interscience Publ.
- Davidson, M. and Whiteside, J. (1971). Human brightness perception near sharp contours. *J. of the Opt. Soc. of Am.*, **A**, **2**, 1160-1169.
- De Valois, R.L., Webster, M.A., De Valois, K.K., and Lingelbach, B. (1986). Temporal properties of brightness and color induction. *Vision Research*, **26**, 887-897.
- Elder, R.L. and Zucker, S.W. (1998). Evidence for boundary specific grouping. *Vision Research*, **38**, 143-152.
- Fritsch, D.S. (1992). A medial description of greyscale image structure by gradient-limited diffusion. R.A. Robb (Ed.). *Visualization in biomedical computing '92*, SPIE Vol.1808, 105-117.
- Gelfand, I.M. and Fomin, S.V. (1963). *Calculus of variations*, Englewood Cliffs, Prentice-Hall.
- Gerrits, H. and Vendrik, A. (1970). Simultaneous contrast, filling-in process and information processing in man's visual system. *Exp. Brain Res.*, **11**, 411-430.
- Gove, A., Grossberg, S., and Mingolla, E. (1995). Brightness perception, illusory contours, and corticogeniculate feedback. *Vis. Neurosc.*, **12**, 1027-1052.
- Grimson, W.E.L. (1981). *From images to surfaces: A computational study of the human visual system*, Cambridge, MA, MIT Press.
- Grossberg, S. and McLoughlin, N. (1997). Cortical dynamics of three-dimensional surface perception: Binocular and half-occluded scenic images. *Neural Networks*, **10**, 1583-1605.
- Grossberg, S. and Mingolla, E. (1985). Neural dynamics of form perception: Boundary completion, illusory figures, and neon color spreading. *Psych. Rev.*, **92**, 173-211.
- Grossberg, S. and Todorović, D. (1988). Neural dynamics of 1-D and 2-D brightness perception: A unified model of classical and recent phenomena. *Perception & Psychophysics*, **43**, 723-742.
- Hamada, J. (1988). A multistage model for border contrast. *Biol. Cybernetics*, **39**, 81-86.
- Horn, B.K.P. (1986). *Robot vision*, Cambridge, MA, MIT Press.
- Hubel, D. and Wiesel, T. (1962). Receptive fields, binocular interaction and functional architecture in the cat's visual cortex. *J. of Physiology*, **160**, 106-154.
- Hubel, D. and Wiesel, T. (1968). Receptive fields and functional architecture of monkey striate cortex. *J. of Physiology*, **195**, 215-243.
- Julesz, B. (1971). *Foundations of cyclopean perception*, Chicago, Univ. of Chicago Press.
- Kingdom, F. and Moulden, B. (1992). A multi-channel approach to brightness coding. *Vision Research*, **32**, 1565-1582.
- Krauskopf, J. (1963). Effect of retinal image stabilization on the appearance of heterochromatic targets. *J. of the Opt. Soc. of Am.*, **A**, **53**, 741-744.

- Land, E. (1983). Recent advances in retinex theory and some implications for cortical computations: Color vision and the natural image. *Proc. Natl. Acad. Sci. USA*, **80**, 5163-5169.
- Lesher, G.W. (1995). Illusory contours: Toward a neurally based perceptual theory. *Psychonomic Bulletin & Review*, **2**, 279-321.
- Marr, D. (1982). *Vision*, San Francisco, W.H. Freeman & Co.
- Mumford, D. (1994). Neuronal architectures for pattern-theoretic problems. C. Koch, J.L. Davis (Eds.). *Large-scale neuronal theories of the brain*. Cambridge, MA, MIT Press.
- Mumford, D. and Shah, J. (1989). Optimal approximations by piecewise smooth functions and associated variational problems. *Comm. Pure and Appl. Math.*, **XLII**, 577-685.
- Nakayama, K. and Shimojo, S. (1992). Experiencing and perceiving visual surfaces. *Science*, **257**, 1357-1363.
- Neumann, H. (1993). Toward a computational architecture for unified visual contrast and brightness perception: I. Theory and model. *Proc. of the World Congress on Neural Networks (WCNN-93)*, Portland, Oregon, Vol. I-IV, (I)84-91. LEA.
- Neumann, H. (1994). An outline of a neural architecture for unified visual contrast and brightness perception. Boston University, Dept. of Cognitive and Neural Systems, Technical Report CAS/CNS-94-003.
- Neumann, H. (1996). Mechanisms of neural architecture for visual contrast and brightness perception. *Neural Networks*, **9**, 921-936.
- Neumann, H. and Pessoa, L. (1994). A simple cell model with multiple spatial frequency selectivity and linear/non-linear response properties. *Proc. of the World Congress on Neural Networks (WCNN-94)*, San Diego, Vol. I-IV, (IV)290-298. LEA.
- Paradiso, M. and Nakayama, K. (1991). Brightness perception and filling-in. *Vision Research*, **31**, 1221-1236.
- Pessoa, L. (1996a). Mach band attenuation by adjacent stimuli: Experiment and filling-in simulations. *Perception*, **25**, 425-442.
- Pessoa, L. (1996b). Mach bands: How many models are possible? Recent experimental findings and modeling attempts. *Vision Research*, **36**, 3205-3227.
- Pessoa, L. and Ross, W. (1996a). A contrast/filling-in model of 3-D lightness perception. Proc. 9th Ann. Conf. on Neural Information Processing Systems (NIPS'95), Denver, *Advances in Neural Information Processing Systems 8*, D. Touretzky, M. Mozer, M. Hasselmo (Eds.). Cambridge, MA, MIT Press.
- Pessoa, L., Mingolla, E., and Neumann, H. (1995). A contrast- and luminance-driven multiscale network model of brightness perception. *Vision Research*, **35**, 2201-2223.
- Pessoa, L., Thompson, E., and Noë, A. (1998). Finding out about filling-in: A guide to perceptual completion for visual science and the philosophy of perception. To appear in *Behavioral and Brain Sciences*.
- Poggio, T., Torre, V., and Koch, C. (1985). Computational vision and regularization theory. *Nature*, **317**, 314-319.
- Price, C.B., Wambacq, P., and Oosterlinck, A. (1989). Applications of reaction-diffusion equations to image processing. *Proc. 3rd Int. Conf. on Image Processing and its Applications*, Univ. of Warwick, July 18-20, 49-53.
- Rogers-Ramachandran, D.C. and Ramachandran, V.S. (1998). Psychophysical evidence for boundary and surface systems in human vision. *Vision Research*, **38**, 71-77.
- Ross, W. D., and Pessoa, L. (1996). Lightness from contrast: A network model. Submitted to *Perception & Psychophysics*.
- Rossi, A.F. and Paradiso, M.A. (1996). Temporal limits of brightness induction and mechanisms of brightness perception. *Vision Research*, **36**, 1391-1398.
- Rossi, A.F., Rittenhouse, C.D., and Paradiso, M.A. (1996). The representation of brightness in primary visual cortex. *Science*, **273**, 1104-1107.

- Sanger, T.D. (1988). Stereo disparity computation using Gabor filters. *Biol. Cybernetics*, **59**, 404-418.
- Schnörr, C. and Sprengel, R. (1994). A nonlinear regularization approach to early vision. *Biol. Cybernetics*, **72**, 141-149.
- Szeliski, R. (1990). Bayesian modeling of uncertainty in low-level-vision. *Int. J. of Computer Vision*, **5**(3), 271-301.
- Stoper, A. and Mansfield, J. (1978). Metacontrast and paracontrast suppression of a contourless area. *Vision Research*, **18**, 1669-1674.
- Terzopoulos, D. (1983). Multilevel computational processes for visual surface reconstruction. *Computer Vision, Graphics, and Image Processing*, **24**, 52-96.
- Terzopoulos, D. (1986). Regularization of inverse visual problems involving discontinuities. *IEEE Trans. on PAMI*, **8**(4), 413-424.
- Todorović, D. (1987). The Craik-O'Brien Cornsweet effect: New varieties and their theoretical implications. *Perception & Psychophysics*, **42**, 52-96.
- Turing, A.M. (1952). The chemical basis of morphogenesis. *Phil. Trans. Roy. Soc. of London, B*, **237**, 37-72.
- Weickert, J. (1996). Theoretical foundations of anisotropic diffusion in image processing. W. Kropatsch, R. Klette, F. Solina (Eds.). *Theoretical Foundations of Computer Vision*, Computing Suppl. 11, 221-236. Wien, Springer.
- Weickert, J. (1997). A review of nonlinear diffusion filtering. B. ter Haar Romeny, L. Florack, J. Koenderink, M. Viergever (Eds.). *Scale-Space Theory in Computer Vision* (Proc. 1st Int. Conf., Scale-Space '97, Utrecht, NL, July 2-4, 1997). Berlin, Springer.
- Westelius, C.-J., Knutsson, H., Wiklund, J., and Westin, C.-F. (1995). Phase-based disparity estimation. J.L. Crowley and H.I. Christensen (Eds.). *Vision as process*. Berlin, Springer.
- Witkin, A. and Kass, M. (1991). Reaction-diffusion textures. *Computer Graphics*, **25**, 299-308.
- Yarbus, A. (1967). *Eye movements and vision*, New York, Plenum Press.

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