Theorems from Compiler Verification: A Problem Set for Automated Theorem Provers*

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1 Introduction

This paper describes a problem set for automated theorem provers taken from a KIV case study in software verification. The goal is to prove 733 theorems, which were used in the case study on compilation of Prolog to the Warren Abstract Machine (WAM). The case study itself is based on [BR95] and described in [SA97] and [SA98]. We present

- a structured algebraic specification of the data types in question with 456 axioms.
- 580 theorems. Some of these were proved using induction in KIV. In this case additional goals were generated for the induction base and the induction step (further goals were created if several induction principles were used in the proof), giving a total of 1059 goals. The additional goals are sometimes irrelevant, since the KIV proof may have used induction, although the goal is provable without induction (e.g. induction may have been used to do case distinction on the constructors). We have removing all induction steps and bases where one of the provers Otter, Setheo or Spass can prove the original goal. This gives 733 noninductive goals.
- results for the 3 provers Otter, Setheoand Spass in two versions, with and without an axiom reduction preprocessing step ([RS98]). For Otter and Spass two settings were tested, the second one defines an additional precedence order (for an LPO) generated from the specification structure.

Test files are available for Otter, Setheoand Spass(in the common syntax of the DFG-Schwerpunkt "Deduktion" [RH96]). Section 2 describes the specification of the datatype. Section 4 gives a listing of the available axioms and the theorems to prove. Section 5 describes the test scenario, and Section 6 gives our experimental results.

2 The Datatypes of the WAM case study

Specifications in KIV ([RSS95],[Rei95],[RSS97]) are *structured* algebraic specifications. They are built up from elementary first-order theories with the usual operations known in algebraic specification: union, enrichment, parameterization, actualization and renaming. Their semantics is the class of all models (loose semantics). Reachability constraints like "nat generated by 0, +1" or "list generated by nil, cons" restrict the semantics to term-generated models. The constraints are reflected by induction principles in the calculus for theorem proving used in KIV. The structure of a specification is visualized as a specification graph. Roughly, each arrow in such a specification graph indicates that one specification is based upon the other (for formal details see [Rei95]).

For the WAM the specifications can be partitioned into seven parts:

- Natural numbers: The specifications *nat-basic1*, *nat-basic2* and *nat-sub* describe natural numbers with the usual operations. Note that successor and predecessor are written as postfix $+1$ and -1.

- Finite Sets: Finite sets as described by specification *set* are built up from the empty set ($\emptyset$) by adding elements with an insert operation ($++$). Operations to delete an element ($--$), to construct a set of one element (postfix $'$), to compute the cardinality ($\#$), to test for membership ($\in$), and to test for one set being subset of another ($\subseteq$) are provided. Specification *enrset* defines the operation $new(s)$, which gives an element, which is not already in the set s. Instances of the parameter of elements are required to be infinite (otherwise the instance would be inconsistent). The specifications is used with two parameters: *node*s (of a Prolog search tree) and *state*s of a stack (of Prolog choicepoints).

- Lists: The specification *list-data* specifies lists as a data type with constructors @ (empty list) and $\oplus$ (adding an element to a list). *.first* selects the first element, *.rest* the rest of the list (this operation is left unspecified for the empty list). $\#$ computes the length of a list an $<<$ test for one list being a sublist of another. *list* additionally defines membership ($\in$), and append ($\odot$). Lists are used in various instances: lists of states (*statelist*), lists of nodes (*nodelist*), lists of decorated goals (*decgoallist* and *sdecgoallist*), lists of code adresses (*codelist* and *codealist*), lists of Prolog clauses (*clauselist*) and lists of literals (*goal*)

- Pairs: Pairs as specified in *pair* are also used in various instances: pairs of head and body of a Prolog clause (*clause*), pairs of goals and cutpoints, where a cutpoint is either a node or a state (*decgoal* and *sdecgoal*) pairs of operator symbol and arity (*ident*), and pairs of compiled Program and a table of start adresses (the result of the third compilation step of Prolog in *comp3result*).

- Dynamic Functions: *DynFun* specifies dynamic functions, which are the basic data type used in the formalism of ASMs. This data type is used in various instances to specify mappings from adresses (nodes or states) to values, i.e. pieces of memory.

- instructions: *instr+clau* specifies the instructions which are used in code which results from the first 3 compilation steps of Prolog. These instructions contain WAM-instructions (like *try*), but also still uncompiled clauses.

- Application specific data types: Specifications *PrologTree*, *PrologStack*, *PrologStack+F*, *PrologStack+H*, *Tree+Stack+F*, *Switching* specify additional functions and predicates on the basic data types described above that are specific to the application

- Parameters and specifications used for structuring purposes: These contain no theorems.

The first four parts of the specification are from the library, the three last ones were defined during the case study. They form the kernel of the specification, and contain the most theorems. Automating their proofs is more important than automating the proofs for the library theorems.

3 The Specifications

The following listing gives an overview of the specification structure. In signature definitions, dots around/before/after declare the symbol to be an infix/postfix/praefix symbol. Regular application of a function symbol binds more tight than postfix application, which binds more tight than prefix application. Prefix function applications bind more tight than infix applications. As an example, # l .rest + 0 over the list specificatrion is the same as (# (l .rest)) + 0. Infix applications bind according to the absolute value (between 1 and 9) of their priority. A negative priority means left associative binding. E.g. func $+_{fun}$ a' / a" $+_{fun}$ a'$_0$ / a"$_0$ ^ a' over the *Dynfun* specification is the same ((func $+_{fun}$ (a' / a")) $+_{fun}$ (a'$_0$ / a"$_0$)) ^ a'

Dynfun =
generic specification
parameter elemI + elemII **target**
sorts dynfun, pairdomcod;
functions

constfun	:	elem"	→	dynfun	;
. ^ .	:	dynfun × elem'	→	elem"	**prio** 1;
. $+_{fun}$.	:	dynfun × pairdomcod	→	dynfun	;
. / .	:	elem' × elem"	→	pairdomcod	**prio** 9;

variables func$_2$, func$_1$, func: dynfun; pdc: pairdomcod;

axioms

dynfun **generated by** constfun, $+_{fun}$;
pairdomcod **generated by** /;
constfun(a") ^ a' = a",
func $+_{fun}$ a' / a" ^ a' = a",
a' ≠ b' → func $+_{fun}$ a' / a" ^ b' = func ^ b'

end generic specification

f-st-no =
actualize Dynfun **with parameter** node, **parameter** state **by morphism**
elem' → statesort, elem" → nodesort, pairdomcod → pairstatenode, dynfun → funstatenode, constfun → cfn, ^ → $^{}_{fn}$, / → $/_{fn}$, $+_{fun}$ → $+_{fn}$, func → fn
end actualize

f-st-st =
actualize Dynfun **with parameter** state **by morphism**
elem' → statesort, elem" → statesort, pairdomcod → pairstatestate, dynfun → funstatestate, constfun → cf, ^ → $^{}_{f}$, / → $/_{f}$, $+_{fun}$ → $+_{f}$, func → f
end actualize

PrologStack+F =
enrich F-st-st, PrologStack **with**
functions

fd	:	funstatestate × sdecgoallist	→	sdecgoallist	;
fs	:	funstatestate × stateset	→	stateset	;
fl	:	funstatestate × statelist	→	statelist	;

predicates . injon . : funstatestate × statelist;

axioms

fd(f, sdnil) = sdnil,
fd(f, mksdecgoal(go, st) $+_{sdl}$ sdgl) = mksdecgoal(go, f $\hat{}_f$ st) $+_{sdl}$ fd(f, sdgl),
fs(f, $@_s$) = $@_s$,
fs(f, s $+_s$ st) = fs(f, s) $+_s$ (f $\hat{}_f$ st),
fl(f, snil) = snil,
fl(f, st $+_{sl}$ stl) = (f $\hat{}_f$ st) $+_{sl}$ fl(f, stl),
f injon stl
$\leftrightarrow \forall$ st, st_1. st $\in_{sl}$ stl $\wedge$ st_1 $\in_{sl}$ bottom $+_{sl}$ stl $\wedge$ st $\neq$ st_1 $\rightarrow$ f $\hat{}_f$ st $\neq$ f $\hat{}_f$ st_1

end enrich

PrologStack =
enrich union2 **with**
functions

mapclause'	:	codealist × program	→	clauselist	;
. from .	:	statelist × statesort	→	statelist	**prio** 3;
cdr	:	statelist	→	statelist	;

predicates

. cutptsin .	:	sdecgoallist × statelist;
. ctpelem .	:	sdecgoallist × stateset;
. sl$\subseteq$s .	:	statelist × stateset;

axioms

mapclause'(canil, db) = clnil,
mapclause'(coa $+_{cal}$ cal, db) = the_clau(clause'(coa, db)) $+_{cli}$ mapclause'(cal, db),
sdnil cutptsin stl,
mksdecgoal(go, st) $+_{sdl}$ sdgl cutptsin stl
$\leftrightarrow$ (st = bottom $\vee$ st $\in_{sl}$ stl) $\wedge$ sdgl cutptsin stl from st,
snil from st = snil,
st $+_{sl}$ stl from st = st $+_{sl}$ stl,
st_1 $\neq$ st $\rightarrow$ st_1 $+_{sl}$ stl from st = stl from st,
sdnil ctpelem s,
mksdecgoal(go, st) $+_{sdl}$ sdgl ctpelem s $\leftrightarrow$ st $\in_s$ s $\wedge$ sdgl ctpelem s,
stl sl$\subseteq$s s $\leftrightarrow$ ($\forall$ st. st $\in_{sl}$ stl $\rightarrow$ st $\in_s$ s),
cdr(snil) = snil,
cdr(st $+_{sl}$ stl) = stl

end enrich

PrologTree =
enrich union1 **with**
functions

mapclause	:	codelist × program	→	clauselist	;
map	:	cllfun × nodelist	→	codelist	;

predicates

every	:	fatherfun × nodelist × nodesort;
nodups	:	nodelist;
. nl$\subseteq$s .	:	nodelist × nodeset;
disjoint	:	nodelist × nodelist;
disjointls	:	nodelist × nodeset;

axioms

mapclause(cnil, db) = clnil,
mapclause(co $+_{col}$ col, db) = clause(co, db) $+_{cli}$ mapclause(col, db),
every(father, nnil, no),
every(father, no_1 $+_{nl}$ nol, no) $\leftrightarrow$ father $\hat{}_{fa}$ no_1 = no $\wedge$ every(father, nol, no),
map(cll, nnil) = cnil,
map(cll, no $+_{nl}$ nol) = (cll $\hat{}_{cll}$ no) $+_{col}$ map(cll, nol),
nodups(nnil),
nodups(no $+_{nl}$ nol) $\leftrightarrow$ $\neg$ no $\in_{nl}$ nol $\wedge$ nodups(nol),
nol nl$\subseteq$s ns $\leftrightarrow$ ($\forall$ no. no $\in_{nl}$ nol $\rightarrow$ no $\in_n$ ns),
disjoint(nol, nol_0) $\leftrightarrow$ ($\forall$ no. no $\in_{nl}$ nol $\rightarrow$ $\neg$ no $\in_{nl}$ nol_0),
disjointls(nol, ns) $\leftrightarrow$ ($\forall$ no. no $\in_{nl}$ nol $\rightarrow$ $\neg$ no $\in_n$ ns)

end enrich

Tree+Stack+F =
enrich F-st-no, PrologTree, PrologStack **with**
functions

fnd	:	funstatenode $\times$ sdecgoallist	$\rightarrow$	decgoallist	;
fns	:	funstatenode $\times$ stateset	$\rightarrow$	nodeset	;

predicates

candsdisjoint	:	funstatenode $\times$ candsfun $\times$ statelist;
. injonn .	:	funstatenode $\times$ statelist;
nocands	:	funstatenode $\times$ candsfun $\times$ statelist;

axioms

fnd(fn, sdnil) = dnil,
fnd(fn, mksdecgoal(go, st) $+_{sdl}$ sdgl) = mkdecgoal(go, fn $\hat{}_{fn}$ st) $+_{dl}$ fnd(fn, sdgl),
fns(fn, $@_s$) = $@_{ns}$,
fns(fn, s $+_s$ st) = fns(fn, s) $+_{ns}$ (fn $\hat{}_{fn}$ st),
candsdisjoint(fn, cands, stl)
$\leftrightarrow$ $\forall$ st, st_1. st $\in_{sl}$ stl $\wedge$ st_1 $\in_{sl}$ stl $\wedge$ st $\neq$ st_1
$\rightarrow$ disjoint(cands $\hat{}_n$ fn $\hat{}_{fn}$ st_1, cands $\hat{}_n$ fn $\hat{}_{fn}$ st),
fn injonn stl
$\leftrightarrow$ $\forall$ st, st_1. st $\in_{sl}$ stl $\wedge$ st_1 $\in_{sl}$ bottom $+_{sl}$ stl $\wedge$ st $\neq$ st_1 $\rightarrow$ fn $\hat{}_{fn}$ st $\neq$ fn $\hat{}_{fn}$ st_1,
nocands(fn, cands, stl)
$\leftrightarrow$ $\forall$ st, st_1. st $\in_{sl}$ stl $\wedge$ st_1 $\in_{sl}$ bottom $+_{sl}$ stl $\rightarrow$ $\neg$ (fn $\hat{}_{fn}$ st_1) $\in_{nl}$ cands $\hat{}_n$ fn $\hat{}_{fn}$ st

end enrich

atom =
specification
sorts atomsort;
variables at: atomsort;
end specification

b =
actualize Dynfun **with parameter** state **by morphism**
elem' $\rightarrow$ statesort, elem" $\rightarrow$ statesort, pairdomcod $\rightarrow$ pairstatestate, dynfun $\rightarrow$ bfun, constfun $\rightarrow$ cb, $\hat{}$ $\rightarrow$ $\hat{}_b$, / $\rightarrow$ $/_b$, $+_{fun}$ $\rightarrow$ $+_b$, func $\rightarrow$ b
end actualize

cands =
actualize Dynfun **with parameter** node, nodelist **by morphism**

elem' → nodesort, elem" → nodelist, pairdomcod → pairnodenodelist, dynfun → candsfun, constfun → ccands, ˆ → $\hat{}_n$, / → $/_n$, $+_{fun}$ → $+_n$, func → cands
end actualize

clause =
actualize pair **with** goal **by morphism**
elem' → paramterm, elem" → goalsort, pair → clausesort, mkpair → mkclause, .p1 → hd **prio** 0, .p2 → bdy **prio** 0, p → cl
end actualize

clause'fun =
enrich parameter codearea, clauseornull, **parameter** program **with**
functions clause' : codearea × program → clauseornull ;

axioms

clause'(failcode, db) = null

end enrich

clausefun =
enrich parameter code, **parameter** program, clause **with**
functions clause : codesort × program → clausesort ;

end enrich

clauselist =
actualize sublist **with** clause **by morphism**
elem → clausesort, list → clauselist, @ → clnil, ⊕ → $+_{cli}$, .first → clcar **prio** 0, .rest → clcdr **prio** 0, # → $\#_{cli}$, ⊙ → $\odot_{cli}$, ≪ → $\ll_{cl}$, subli → subli_of, ∈ → $\in_{cli}$, $@_p$ → clnilp, a → cl, x → cli
end actualize

clauseornull =
data specification
using clause
clauseornull = mkclau (the_clau : clausesort)
| null
;
variables cln: clauseornull;
end data specification

cll =
actualize Dynfun **with parameter** node, **parameter** code **by morphism**
elem' → nodesort, elem" → codesort, pairdomcod → pairnodecode, dynfun → cllfun, constfun → ccll, ˆ → $\hat{}_{cll}$, / → $/_{cll}$, $+_{fun}$ → $+_{cll}$, func → cll
end actualize

code =
specification
sorts codesort;
constants undefcode : codesort;
variables co: codesort;
end specification

codealist =
actualize list **with parameter** codearea **by morphism**
elem → codearea, list → codealist, @ → canil, ⊕ → $+_{cal}$, .first → cacar **prio** 0, .rest → cacdr **prio** 0, # → $\#_{cal}$, ⊙ → $\odot_{cal}$, ≪ → $\ll_{ca}$, ∈ → $\in_{cal}$, $@_p$ → canilp, a → coa, x → cal
end actualize

codearea =
specification
sorts codearea;
constants failcode : codearea;
functions next : codearea → codearea ;
variables coa: codearea;
end specification

codefun =
enrich parameter codearea, **parameter** prorgram“, instr+clau **with**
constants start : codearea;
functions code : codearea × program” → instr-or-cl ;

axioms

coa = start ↔ code(coa, db”) = code_of_start,
code(failcode, db”) = nil’

end enrich

codelist =
actualize list **with parameter** code **by morphism**
elem → codesort, list → codelist, @ → cnil, # → $\#_{col}$, ⊕ → $+_{col}$, .first → ccar **prio** 0, .rest → ccdr **prio** 0, ⊙ → $\odot_{col}$, $@_p$ → cnilp, ≪ → $\ll_c$, ∈ → $\in_{col}$, a → co, x → col
end actualize

ct =
actualize Dynfun **with parameter** state **by morphism**
elem’ → statesort, elem” → statesort, pairdomcod → pairstatestate, dynfun → ctfun, constfun → cct, ^ → $\hat{}_{ct}$, / → $/_{ct}$, $+_{fun}$ → $+_{ct}$, func → ct
end actualize

decglseq =
actualize Dynfun **with parameter** node, decgoallist **by morphism**
elem’ → nodesort, elem” → decgoallist, pairdomcod → pairnodedecgoallist, dynfun → decglseqfun, constfun → cdecglseq, ^ → $\hat{}_d$, / → $/_d$, $+_{fun}$ → $+_d$, func → decglseq
end actualize

decgoal =
actualize pair **with parameter** node, goal **by morphism**
elem” → nodesort, elem’ → goalsort, pair → decgoal, mkpair → mkdecgoal, .p1 →.1, .p2 →.2, p → dg
end actualize

decgoallist =
actualize list **with** decgoal **by morphism**
elem → decgoal, list → decgoallist, @ → dnil, ⊕ → $+_{dl}$, .first → dcar **prio** 0, .rest → dcdr **prio** 0, # → $\#_{dgl}$, ⊙ → $\odot_{dgl}$, ≪ → $\ll_d$, ∈ → $\in_{dgl}$, $@_p$ → dnilp, a → dg, x → dgl
end actualize

elem =
specification
sorts elem;
variables c, b, a: elem;
end specification

elemI =
rename elem **by morphism**
elem → elem', a → a'
end rename

elemII =
rename elem **by morphism**
elem → elem", a → a"
end rename

enrnodeset =
enrich nodeset **with**
constants root : nodesort;

axioms

new($@_{ns}$) = root

end enrich

enrset =
enrich set **with**
functions new : set → elem ;

axioms

¬ new(s) ∈ s

end enrich

enrstateset =
enrich stateset **with**
constants bottom : statesort;

axioms

snew($@_s$) = bottom

end enrich

enrterm =
enrich term **with**
functions
arity : term → nat ;
arg : term × nat → term ;
predicates is_user_defined : term;

axioms

is_user_defined(trm) $\leftrightarrow$ is_struct(trm) $\vee$ is_const(trm) $\vee$ is_var(trm) $\vee$ is_list(trm),
arity(trm) = tlen(args(trm))+1,
args(trm) = the_one(trm_1) $\rightarrow$ arg(trm, 0 +1) = trm_1,
args(trm) = tcons(trm_1, trmli)
$\rightarrow$ arg(trm, 0 +1) = trm_1 $\wedge$ (0 < n $\rightarrow$ arg(trm, n +1) = arg(struct(funct(trm), trmli), n))

end enrich

father =
actualize Dynfun **with parameter** node **by morphism**
elem' $\rightarrow$ nodesort, elem" $\rightarrow$ nodesort, pairdomcod $\rightarrow$ pairnodenode, dynfun $\rightarrow$ fatherfun, constfun $\rightarrow$ cfather, ^ $\rightarrow$ $\hat{}_{fa}$, / $\rightarrow$ $/_{fa}$, $+_{fun}$ $\rightarrow$ $+_{fa}$, func $\rightarrow$ father
end actualize

finitefun =
generic specification
parameter elemI + elemII **target**
sorts finitefun, pairdomcod;
functions

constfun	:	elem"	$\rightarrow$	finitefun	;
. ^ .	:	finitefun $\times$ elem'	$\rightarrow$	elem"	**prio** 1;
. $+_{fun}$.	:	finitefun $\times$ pairdomcod	$\rightarrow$	finitefun	;
. / .	:	elem' $\times$ elem"	$\rightarrow$	pairdomcod	**prio** 9;

variables $func_2$, $func_1$, func: finitefun; pdc: pairdomcod;

axioms

finitefun **generated by** constfun, $+_{fun}$;
pairdomcod **generated by** /;
constfun(a") ^ a' = a",
func $+_{fun}$ a' / a" ^ a' = a",
a' $\neq$ b' $\rightarrow$ func $+_{fun}$ a' / a" ^ b' = func ^ b'

end generic specification
goal =
actualize list **with parameter** paramterm **by morphism**
elem $\rightarrow$ paramterm, list $\rightarrow$ goalsort, @ $\rightarrow$ gnil, $\oplus$ $\rightarrow$ $+_g$, .first $\rightarrow$ gcar **prio** 0, .rest $\rightarrow$ gcdr **prio** 0, # $\rightarrow$ $\#_{goal}$, $\odot$ $\rightarrow$ $\odot_{goal}$, $\ll$ $\rightarrow$ $\ll_g$, $\in$ $\rightarrow$ $\in_{goal}$, $@_p$ $\rightarrow$ gnilp, a $\rightarrow$ trm, x $\rightarrow$ go
end actualize

ident =
actualize pair **with** nat, **parameter** atom **by morphism**
elem' $\rightarrow$ atomsort, elem" $\rightarrow$ nat, pair $\rightarrow$ ident, mkpair $\rightarrow$ mkident, .p1 $\rightarrow$ atom, .p2 $\rightarrow$ ari, p $\rightarrow$ vid
end actualize

idfun =
enrich enrterm, ident **with**
functions id : term $\rightarrow$ ident ;

axioms

is_struct(trm) $\rightarrow$ id(trm) = mkident(funct(trm), arity(trm)),
is_const(trm) $\rightarrow$ id(trm) = mkident(constsym(trm), 0)

end enrich

instr+clau =
data specification
using nat, clause, **parameter** codearea
instr-or-cl = try_me_else (where : codearea) **with** is_try_me
| retry_me_else (where : codearea) **with** is_retry_me
| trust_me **with** is_trust_me
| try' (what : codearea) **with** is_try
| retry' (what : codearea) **with** is_retry
| trust (what : codearea) **with** is_trust
| switch_on_term (argindex : nat, vlabel : codearea,
clabel : codearea, llabel : codearea,
slabel : codearea) **with** is_sw_term
| switch_on_constant (argindex : nat, tabsize : nat,
table : codearea) **with** is_sw_const
| switch_on_structure (argindex : nat, tabsize : nat,
table : codearea) **with** is_sw_struct
| mkcl (the_cl : clausesort) **with** is_clause
| nil'
| code_of_start
;
variables ioc: instr-or-cl;
end data specification

list-data =
generic data specification
parameter elem **using** nat
list = @ **with** $@_p$
| . $\oplus$. (. .first : elem, . .rest : list)
;
variables z, y, x: list;
size functions # : list $\to$ nat ;
order predicates . $\ll$. : list $\times$ list;
end generic data specification

list =
enrich list-data **with**
functions . $\odot$. : list $\times$ list $\to$ list **prio** 4;
predicates . $\in$. : elem $\times$ list;

axioms

@ $\odot$ x = x,
a $\oplus$ x $\odot$ y = a $\oplus$ (x $\odot$ y),
a $\in$ x $\leftrightarrow$ ($\exists$ y, z. x = y $\odot$ a $\oplus$ z)

end enrich

mode =
data specification
modesort = select | call ;
variables mode: modesort;
end data specification

nat-basic1 =
data specification
nat = 0
| . +1 (. −1 : nat)
;
variables n: nat;
order predicates . < . : nat × nat;
end data specification

nat-basic2 =
enrich nat-basic1 **with**
functions . + . : nat × nat → nat ;
variables n_0, m: nat;

axioms

$n + 0 = n$,
$m + n + 1 = (m + n) + 1$,
$n < n_0 \vee n = n_0 \vee n_0 < n$

end enrich

nat-lec =
enrich nat-basic2 **with**
constants 1 : nat; 2 : nat;
predicates
$. \leq .$: nat × nat;
$. > .$: nat × nat;
$. \geq .$: nat × nat;

axioms

$1 = 0 + 1$,
$0 \neq 1$,
$2 = 0 + 1 + 1$,
$2 \neq 0$,
$2 \neq 1$,
$m \leq n \leftrightarrow \neg\, n < m$,
$m > n \leftrightarrow n < m$,
$m \geq n \leftrightarrow \neg\, m < n$

end enrich

nat = nat-lec + nat-sub

nat-sub =
enrich nat-basic2 **with**
functions . − . : nat × nat → nat **prio** 4 **left**;

axioms

$m - 0 = m$,
$m - n + 1 = (m - n) - 1$

end enrich

node =
specification
sorts nodesort;
variables no: nodesort;
end specification

nodelist =
actualize list **with parameter** node **by morphism**
elem → nodesort, list → nodelist, @ → nnil, .first → ncar **prio** 0, .rest → ncdr **prio** 0, # → $\#_{nl}$, ⊙ → $\odot_{nl}$, ⊕ → $+_{nl}$, ≪ → $\ll_n$, ∈ → $\in_{nl}$, $@_p$ → nnilp, a → no, x → nol
end actualize

nodeset =
actualize enrset **with parameter** node **by morphism**
elem → nodesort, set → nodeset, ∅ → $@_{ns}$, ++ → $+_{ns}$, −− → $-_{ns}$, # → $\#_{ns}$, ' → $'_{ns}$, new → new, ∈ → $\in_n$, ⊆ → $\subseteq_{ns}$, a → no, s → ns
end actualize

p =
actualize Dynfun **with parameter** state, **parameter** codearea **by morphism**
elem' → statesort, elem" → codearea, pairdomcod → pairstatecoa, dynfun → pfun, constfun → cp, ^ → $\hat{}_p$, / → $/_p$, $+_{fun}$ → $+_p$, func → p
end actualize

pair =
generic data specification
parameter elemI + elemII
pair = mkpair (. .p1 : elem', . .p2 : elem");
variables p_1, p_0, p: pair;
end generic data specification

paramterm =
specification
sorts paramterm;
constants ! : paramterm; true' : paramterm; fail' : paramterm;
predicates is_user_defined : paramterm;
variables trm: paramterm;

axioms

! ≠ true',
! ≠ fail',
true' ≠ fail',
is_user_defined(trm) ↔ trm ≠ true' ∧ trm ≠ fail' ∧ trm ≠ !

end specification

procdef =
enrich parameter paramterm, **parameter** program, codelist **with**
functions procdef : paramterm × program → codelist ;

end enrich

procdef1 =
enrich parameter codearea, **parameter** program, **parameter** paramterm **with**
functions procdef' : paramterm × program → codearea ;

end enrich

procdef2 =
enrich parameter codearea, **parameter** prorgram“, **parameter** paramterm **with**
functions procdef” : paramterm × program” → codearea ;

end enrich

program =
specification
sorts program;
variables db: program;
end specification

program‘ =
specification
sorts program';
variables db': program';
end specification

program“ =
specification
sorts program”;
variables db”: program”;
end specification

rename =
enrich nat, clause **with**
functions ren : clausesort × nat → clausesort ;

end enrich

rmode =
data specification
rmodesort = try | retry | enter | call';
variables rmode: rmodesort;
end data specification

scll =
actualize Dynfun **with parameter** state, **parameter** codearea **by morphism**
elem' → statesort, elem” → codearea, pairdomcod → pairstatecoa, dynfun → scllfun, constfun → cscll, ^ → $\hat{}_{sc}$, / → $/_{sc}$, $+_{fun}$ → $+_{sc}$, func → scll
end actualize

sdecglseq =
actualize Dynfun **with parameter** state, sdecgoallist **by morphism**
elem' → statesort, elem” → sdecgoallist, pairdomcod → pairstatesdecgoallist, dynfun → sdecglseqfun, constfun → csdecglseq, ^ → $\hat{}_{sd}$, / → $/_{sd}$, $+_{fun}$ → $+_{sd}$, func → sdecglseq
end actualize

sdecgoal =
actualize pair **with parameter** state, goal **by morphism**
elem" → statesort, elem' → goalsort, pair → sdecgoal, mkpair → mksdecgoal, .p1 →.s1, .p2 →.s2, p → sdg
end actualize

sdecgoallist =
actualize list **with** sdecgoal **by morphism**
elem → sdecgoal, list → sdecgoallist, @ → sdnil, $\oplus \rightarrow +_{sdl}$, .first → sdcar **prio** 0, .rest → sdcdr **prio** 0, # → $\#_{sdl}$, $\odot \rightarrow \odot_{sdl}$, $\ll \rightarrow \ll_{sd}$, $\in \rightarrow \in_{sdl}$, $@_p \rightarrow$ sdnilp, a → sdg, x → sdgl
end actualize

set =
generic specification
parameter elem **using** nat **target**
sorts set;
constants $\emptyset$: set;
functions

. ++ .	:	set × elem	→	set	**prio** 5 **left**;
. '	:	elem	→	set	;
#	:	set	→	nat	;
. −− .	:	set × elem	→	set	**prio** 5 **left**;

predicates

. ∈ .	:	elem × set;
. ⊆ .	:	set × set;

variables s', s: set;

axioms

set **generated by** $\emptyset$, ++;
s = s' ↔ (∀ a. a ∈ s ↔ a ∈ s'),
¬ a ∈ $\emptyset$,
a ∈ s ++ b ↔ a = b ∨ a ∈ s,
a ' = $\emptyset$ ++ a,
#($\emptyset$) = 0,
¬ a ∈ s → #(s ++ a) = #(s)+1,
a ∈ s −− b ↔ a ≠ b ∧ a ∈ s,
s ⊆ s' ↔ (∀ a. a ∈ s → a ∈ s')

end generic specification

ssub =
actualize Dynfun **with parameter** subst, **parameter** state **by morphism**
elem' → statesort, elem" → substitution, pairdomcod → pairstatesubst, dynfun → ssubfun, constfun → cssub, ^ → $\hat{}_{su}$, / → $/_{su}$, $+_{fun} \rightarrow +_{su}$, func → ssub
end actualize

ssubres =
enrich sdecgoallist, substgoal **with**
functions ssubres : sdecgoallist × substitution → sdecgoallist ;

axioms

ssubres(sdnil, su) = sdnil,
ssubres(mksdecgoal(go, st) $+_{sdl}$ sdgl, su)
= mksdecgoal(su $\hat{}_{sg}$ go, st) $+_{sdl}$ ssubres(sdgl, su)

end enrich

state =
specification
sorts statesort;
variables st: statesort;
end specification

statelist =
actualize list **with parameter** state **by morphism**
elem → statesort, list → statelist, @ → snil, ⊕ → $+_{sl}$, .first → scar **prio** 0, .rest → scdr **prio** 0, ⊙ → $\odot_{stl}$, # → $\#_{stl}$, ≪ → $\ll_s$, ∈ → $\in_{sl}$, $@_p$ → snilp, a → st, x → stl
end actualize

stateset =
actualize enrset **with parameter** state **by morphism**
elem → statesort, set → stateset, ∅ → $@_s$, ++ → $+_s$, −− → $-_s$, # → $\#_s$, ' → $'_s$, new → snew, ∈ → $\in_s$, ⊆ → $\subseteq_s$, a → st, s → s
end actualize

stopmode =
data specification
stopmodesort = success | failure | run;
variables stop: stopmodesort;
end data specification

sub =
actualize Dynfun **with parameter** node, **parameter** subst **by morphism**
elem' → nodesort, elem" → substitution, pairdomcod → pairnodesubst, dynfun → subfun, constfun → csub, ^ → $\hat{}_u$, / → $/_u$, $+_{fun}$ → $+_u$, func → sub
end actualize

sublist =
enrich list **with**
predicates . subli . : list × list;

axioms

@ subli x,
¬ a ⊕ x subli @,
a ⊕ x subli b ⊕ y ↔ a = b ∧ x subli y ∨ a ≠ b ∧ a ⊕ x subli y

end enrich

subres =
enrich decgoallist, substgoal **with**
functions subres : decgoallist × substitution → decgoallist ;

axioms

subres(dnil, su) = dnil,
subres(mkdecgoal(go, no) $+_{dl}$ dgl, su) = mkdecgoal(su $\hat{}_{sg}$ go, no) $+_{dl}$ subres(dgl, su)

end enrich

```
 subst =
specification
    sorts substitution;
    constants @_su : substitution;
    functions . o . : substitution × substitution → substitution ;
    variables su_2, su_1, su: substitution;

axioms

      (su o su_1) o su_2 = su o su_1 o su_2,
      su o @_su = su,
      @_su o su = su

end specification

 substgoal =
enrich substterm, goal with
    functions . ^_sg . : substitution × goalsort → goalsort ;

axioms

      su ^_sg gnil = gnil,
      su ^_sg trm +_g go = (su ^_t trm) +_g su ^_sg go

end enrich

 substorfail =
data specification
    using parameter subst
    substorfail =  oksubst (the_subst : substitution)
                 | fail
                 ;
    variables subst: substorfail;
end data specification

 substterm =
enrich parameter subst, parameter paramterm with
    functions . ^_t . : substitution × paramterm → paramterm ;

   end enrich

 term =
data specification
    using nat, parameter atom, parameter var
    term =  struct (funct : atomsort, args : termlist) with is_struct
          | mkconst (constsym : atomsort) with is_const
          | mkvar (varsym : varsort) with is_var
          | mklist (thelist : termlist) with is_list
          | !
          | true'
          | fail'
          ;
    termlist =  the_one (and_only : term)
              | tcons (tcar : term, tcdr : termlist)
              ;
    variables trm_1, trm: term; trmli: termlist;
    size functions tlen : termlist → nat ;
end data specification
```

unify =
enrich substorfail, **parameter** paramterm **with**
functions unify : paramterm × paramterm → substorfail ;

end enrich

union1 = enrnodeset + sub + father + cands + mode + stopmode + cll + unify + subres + decglseq + rename + procdef + clausefun + clauselist

union2 = mode + stopmode + unify + rename + clauselist + ssubres + sdecglseq + procdef1 + procdef2 + enrstateset + codealist + b + ssub + scll + statelist + clause'fun

var =
specification
sorts varsort;
variables va: varsort;
end specification

4 The Axioms and Theorems

This section gives the axioms and theorems from the case study. Note that according to the specification structure, that one axiom from a generic specification (e.g. a list axiom) may be listed with several instances (e.g. lists of clauses and lists of decorated goals) in the axioms available for one goalfile. The comment for an axiom or lemma in a goalfile gives the specification name, the instance name (i.e. the name of the specification, where the actualization was done) and the axiom or theorem name, separated by underscores. Additionally the goalfiles contain case distinction axioms for the constructors of generated by clauses, except those generated by clauses, which were constructed from data type specifications (the automatically generated axioms for data type specifications already contain such an axiom). As an example the generated by clause for sets set **generated by** $\emptyset$, ++ implies the following axiom:

set_genax: s = $\emptyset$ ∨ ∃ e, s_0. s = s_0 ++ e.

Axioms derived from generated by clauses are given names (in the comments of the formulas) in the goalfiles which start with the specification name of the theorem that is considered, then comes the string '_genaxjsub', and finally a number.

4.1 Axioms and Theorems from CompAssum2

ax-01 : mapcode(canil, db") = clnil
ax-02 : mapcode(coa $+_{cal}$ cal, db")
= the_cl(code(coa, db")) $+_{cli}$ mapcode(cal, db")
th-1 (lem-01): mapcode(cal, db") = clnil ↔ cal = canil
th-2 (lem-02): mapcode(cal, db") = cl $+_{cli}$ clnil
↔ cal ≠ canil ∧ the_cl(code(cacar(cal), db")) = cl
∧ cacdr(cal) = canil
th-3 (lem-03): cal_1 ≠ canil ∧ cal_2 ≠ canil
→ mapcode(cal_1 $\odot_{cal}$ cal_2, db") ≠ cl $+_{cli}$ clnil

4.2 Axioms and Theorems from Dynfun

ax-01 : constfun(a") ^ a' = a"
ax-02 : func $+_{fun}$ a' / a" ^ a' = a"
ax-03 : a' ≠ b' → func $+_{fun}$ a' / a" ^ b' = func ^ b'

4.3 Axioms and Theorems from PrologStack+F

ax-01 : $fd(f, sdnil) = sdnil$

ax-02 : $fd(f, mksdecgoal(go, st) +_{sdl} sdgl) = mksdecgoal(go, f \hat{}_f st) +_{sdl} fd(f, sdgl)$

ax-03 : $fs(f, @_s) = @_s$

ax-04 : $fs(f, s +_s st) = fs(f, s) +_s (f \hat{}_f st)$

ax-05 : $fl(f, snil) = snil$

ax-06 : $fl(f, st +_{sl} stl) = (f \hat{}_f st) +_{sl} fl(f, stl)$

ax-07 : $f\ injon\ stl \leftrightarrow (\forall\ st, st_1.\quad st \in_{sl} stl \wedge st_1 \in_{sl} bottom +_{sl} stl \wedge st \neq st_1 \rightarrow f \hat{}_f st \neq f \hat{}_f st_1)$

th-1 (lem-14): $f\ injon\ snil$

th-10 (lem-09): $fd(f, sdgl) = mksdecgoal(go, st) +_{sdl} sdgl_0 \leftrightarrow \neg\ \neg\ (\quad sdgl \neq sdnil \wedge sdcar(sdgl).s1 = go \wedge f \hat{}_f sdcar(sdgl).s2 = st \wedge fd(f, sdcdr(sdgl)) = sdgl_0)$

th-11 (lem-10): $stl\ sl{\subseteq}s\ s \rightarrow fl(f +_f snew(s) /_f st, stl) = fl(f, stl)$

th-12 (lem-17): $fl(f, stack)\ sl{\subseteq}s\ s \wedge st \in_{sl} stack \rightarrow f \hat{}_f st \neq snew(s)$

th-13 (lem-25): $\neg\ st \in_{sl} stl \wedge sdgl\ cutptsin\ stl \wedge \neg\ bottom \in_{sl} stl \wedge st \neq bottom \wedge f \hat{}_f bottom = bottom \rightarrow fd(f +_f st /_f st_1, sdgl) = fd(f, sdgl)$

th-14 (lem-02): $fd(f, sdgl) = sdnil \leftrightarrow sdgl = sdnil$

th-15 (lem-03): $fl(f, stl) = snil \leftrightarrow stl = snil$

th-16 (lem-05): $f\ injon\ stl \rightarrow (st \in_{sl} fl(f, stl) \leftrightarrow (\exists\ st_0.\ st = f \hat{}_f st_0 \wedge st_0 \in_{sl} stl))$

th-17 (lem-06): $f\ injon\ stl \rightarrow (\quad stl_0 \ll_s fl(f, stl) \leftrightarrow (\exists\ stl_1.\ stl_0 = fl(f, stl_1) \wedge stl_1 \ll_s stl))$

th-18 (lem-11): $stl\ sl{\subseteq}s\ s \wedge sdgl\ cutptsin\ stl\ from\ st_0 \wedge bottom \in_s s \rightarrow fd(f +_f snew(s) /_f st_1, sdgl) = fd(f, sdgl)$

th-19 (lem-19): $f\ injon\ stack \wedge stack\ sl{\subseteq}s\ s \wedge fl(f, stack)\ sl{\subseteq}s\ s_1 \wedge bottom \in_s s \wedge bottom \in_s s_1 \wedge f \hat{}_f bottom = bottom \rightarrow f +_f snew(s) /_f snew(s_1)\ injon\ snew(s) +_{sl} stack$

th-2 (lem-15): $f\ injon\ st +_{sl} stl \rightarrow f\ injon\ stl$

th-20 (lem-21): $f\ injon\ stack \wedge f \hat{}_f bottom = bottom \wedge sdgl\ cutptsin\ stack\ from\ st_1 \wedge \neg\ bottom \in_{sl} stack \rightarrow fd(f, sdgl)\ cutptsin\ fl(f, stack\ from\ st_1)$

th-21 (lem-22): $stl\ sl{\subseteq}s\ s \rightarrow fl(f +_f snew(s) /_f st_1, stl\ from\ st) = fl(f, stl\ from\ st)$

th-22 (lem-23): $fd(f, ssubres(sdgl, su)) = ssubres(fd(f, sdgl), su)$

th-23 (lem-24): $\neg\ st \in_{sl} stl \rightarrow fl(f +_f st /_f st_0, stl) = fl(f, stl)$

th-24 (lem-26): $\neg\ st \in_{sl} stl \wedge sdgl\ cutptsin\ stl \wedge \neg\ bottom \in_{sl} stl \wedge st \neq bottom \wedge sdgl \neq sdnil \wedge f \hat{}_f bottom = bottom \rightarrow fd(f +_f st /_f st_1, sdcdr(sdgl)) = fd(f, sdcdr(sdgl))$

th-25 (rew-1): $sdgl \neq sdnil \rightarrow \quad fd(f, sdgl) = \quad mksdecgoal(sdcar(sdgl).s1, f \hat{}_f sdcar(sdgl).s2) +_{sdl} fd(f, sdcdr(sdgl))$

th-26 (lem-12): $stl\ sl{\subseteq}s\ s \wedge sdgl\ cutptsin\ stl\ from\ st \rightarrow sdgl\ cutptsin\ snew(s) +_{sl} (stl\ from\ st)$

th-3 (lem-04): $f\ injon\ stl \wedge st \in_{sl} stl \rightarrow (f \hat{}_f st) \in_{sl} fl(f, stl)$

th-4 (lem-08): f injon stack $\wedge$ st $\in_{sl}$ stack
$\rightarrow$ fl(f, stack) from (f $\hat{}_f$ st) = fl(f, stack from st)

th-5 (lem-13): f injon stack $\rightarrow$ f injon stack from st

th-6 (lem-16): f injon stack $\wedge$ f $\hat{}_f$ bottom = bottom
$\wedge \neg$ bottom $\in_{sl}$ stack
$\rightarrow$ fl(f, stack) from bottom = snil

th-7 (lem-20): f $\hat{}_f$ bottom = bottom $\wedge$ sdgl cutptsin snil
$\rightarrow$ fd(f, sdgl) cutptsin snil

th-8 (lem-18): st $\in_s$ s $\rightarrow$ f $+_f$ snew(s) $/_f$ st$_0$ $\hat{}_f$ st = f $\hat{}_f$ st

th-9 (lem-01): f injon stack $\wedge$ f $\hat{}_f$ bottom = bottom
$\wedge$ sdgl cutptsin stack $\wedge \neg$ bottom $\in_{sl}$ stack
$\rightarrow$ fd(f, sdgl) cutptsin fl(f, stack)

4.4 Axioms and Theorems from PrologStack+H

ax-01 : hdg(h, sdnil) = sdnil

ax-02 : hdg(h, mksdecgoal(go, st) $+_{sdl}$ sdgl)
= mksdecgoal(go, car(h $\hat{}_h$ st)) $+_{sdl}$ hdg(h, sdgl)

ax-03 : hl(h, snil) = snil

ax-04 : hl(h, st $+_{sl}$ stl) = (h $\hat{}_h$ st) $\odot_{sl}$ hl(h, stl)

ax-05 : car(snil) = bottom

ax-06 : car(st $+_{sl}$ stl) = st

th-1 (lem-06): st $\in_{sl}$ stl $\wedge$ hl(h, stl) sl$\subseteq$s s $\rightarrow \neg$ snew(s) $\in_{sl}$ h $\hat{}_h$ st

th-10 (lem-08): hdg(h, ssubres(sdgl, su)) = ssubres(hdg(h, sdgl), su)

th-11 (lem-09): hdg(h $+_h$ st $/_h$ (h $\hat{}_h$ st), sdgl) = hdg(h, sdgl)

th-12 (lem-11): st $\in_{sl}$ hl(h, stl)
$\wedge$ ($\forall$ st$_0$. st$_0$ $\in_{sl}$ stl
$\rightarrow$ h $\hat{}_h$ st$_0$ = h$_0$ $\hat{}_h$ st$_0$
$\vee$ (h $\hat{}_h$ st$_0$) $\ll_s$ h$_0$ $\hat{}_h$ st$_0$)
$\rightarrow$ st $\in_{sl}$ hl(h$_0$, stl)

th-13 (lem-12): st$_1$ $\in_{sl}$ h $\hat{}_h$ st$_0$ $\wedge$ st$_0$ $\in_{sl}$ stl $\wedge$ hl(h, stl) sl$\subseteq$s s
$\rightarrow$ st$_1$ $\in_s$ s

th-14 (lem-13): hl(h, stack) = snil $\rightarrow$ hl(h, stack from st) = snil

th-15 (lem-14): st $\in_{sl}$ stl $\wedge$ st$_1$ $\in_{sl}$ h $\hat{}_h$ st $\wedge$ (h $\hat{}_h$ st) $\ll_s$ h$_0$ $\hat{}_h$ st
$\rightarrow$ st$_1$ $\in_{sl}$ hl(h$_0$, stl)

th-16 (lem-16): st$_0$ $\in_{sl}$ stl $\wedge$ stl$_0$ $\ll_s$ h $\hat{}_h$ st$_0$ $\wedge$ hl(h, stl) sl$\subseteq$s s
$\rightarrow \neg$ snew(s) $\in_{sl}$ stl$_0$

th-2 (lem-01): hdg(h, sdg $+_{sdl}$ sdgl)
= mksdecgoal(sdg.s1, car(h $\hat{}_h$ sdg.s2)) $+_{sdl}$ hdg(h, sdgl)

th-3 (lem-04): st $\in_{sl}$ stl $\rightarrow$ ($\forall$ st$_0$. st$_0$ $\in_{sl}$ h $\hat{}_h$ st $\rightarrow$ st$_0$ $\in_{sl}$ hl(h, stl))

th-4 (lem-10): st $\in_{sl}$ stl $\wedge$ st$_1$ $\in_{sl}$ h $\hat{}_h$ st $\rightarrow$ st$_1$ $\in_{sl}$ hl(h, stl)

th-5 (lem-15): st$_0$ $\in_{sl}$ stl $\wedge$ hl(h, stl) sl$\subseteq$s s
$\rightarrow \neg$ snew(s) $\in_{sl}$ h $\hat{}_h$ st$_0$

th-6 (lem-02): $\neg$ st $\in_{sl}$ stl $\rightarrow$ hl(h $+_h$ st $/_h$ stl$_0$, stl) = hl(h, stl)

th-7 (lem-03): $\neg$ st $\in_{sl}$ stl $\wedge$ sdgl cutptsin stl
$\wedge \neg$ bottom $\in_{sl}$ stl $\wedge$ st $\neq$ bottom
$\wedge$ h $\hat{}_h$ bottom = bottom $+_{sl}$ snil
$\rightarrow$ hdg(h $+_h$ st $/_h$ stl$_0$, sdgl) = hdg(h, sdgl)

th-8 (lem-05): st $\in_{sl}$ stl $\wedge$ h $\hat{}_h$ st = st$_0$ $+_{sl}$ stl$_0$ $\rightarrow$ st$_0$ $\in_{sl}$ hl(h, stl)

th-9 (lem-07): hl(h, stl) sl$\subseteq$s s $\rightarrow$ hl(h, stl from st) sl$\subseteq$s s

4.5 Axioms and Theorems from PrologStack

ax-01 : mapclause'(canil, db) = clnil

ax-02 : mapclause'(coa $+_{cal}$ cal, db)
= the_clau(clause'(coa, db)) $+_{cli}$ mapclause'(cal, db)

ax-03 : sdnil cutptsin stl

ax-04 : mksdecgoal(go, st) $+_{sdl}$ sdgl cutptsin stl
$\leftrightarrow$ (st = bottom $\vee$ st $\in_{sl}$ stl) $\wedge$ sdgl cutptsin stl from st

ax-05 : snil from st = snil

ax-06 : st $+_{sl}$ stl from st = st $+_{sl}$ stl

ax-07 : $st_1 \neq st \rightarrow st_1 +_{sl}$ stl from st = stl from st

ax-08 : sdnil ctpelem s

ax-09 : mksdecgoal(go, st) $+_{sdl}$ sdgl ctpelem s
$\leftrightarrow$ st $\in_s$ s $\wedge$ sdgl ctpelem s

ax-10 : stl sl$\subseteq$s s $\leftrightarrow$ ($\forall$ st. st $\in_{sl}$ stl $\rightarrow$ st $\in_s$ s)

ax-11 : cdr(snil) = snil

ax-12 : cdr(st $+_{sl}$ stl) = stl

th-1 (lem-10): stl sl$\subseteq$s s $\wedge$ st $\in_{sl}$ stl $\rightarrow$ st $\in_s$ s

th-10 (lem-06): stl sl$\subseteq$s s $\rightarrow \neg$ snew(s) $\in_{sl}$ stl

th-11 (lem-21): stl $\ll_s$ $stl_1 \wedge$ sdgl cutptsin stl $\rightarrow$ sdgl cutptsin stl_1

th-12 (cdr-1): stl $\neq$ snil $\rightarrow$ cdr(stl) = scdr(stl)

th-13 (lem-07): stl sl$\subseteq$s s $\rightarrow \neg$ snew(s) $\in_{sl}$ stl from st_0

th-14 (lem-40): sdgl cutptsin stack $\rightarrow$ sdgl cutptsin st $+_{sl}$ stack

th-15 (lem-03): $\neg$ st $\in_{sl}$ stl $\rightarrow \neg$ st $\in_{sl}$ stl from st_1

th-16 (lem-16): (stl from st) from st = stl from st

th-17 (lem-17): sdgl cutptsin stl from st $\rightarrow$ sdgl cutptsin stl

th-18 (lem-24): st $\in_{sl}$ stl $\rightarrow$ st $\in_{sl}$ stl from st

th-19 (lem-27): stl from st = $st_1 +_{sl}$ $stl_0 \wedge$ stl sl$\subseteq$s s $\rightarrow \neg$ snew(s) $\in_{sl}$ stl_0

th-2 (lem-19): stl from st $\ll_s$ stl $\leftrightarrow$ stl $\neq$ snil $\wedge$ scar(stl) $\neq$ st

th-20 (lem-33): stl $\odot_{sl}$ stl_0 sl$\subseteq$s s $\leftrightarrow$ stl sl$\subseteq$s s $\wedge$ stl_0 sl$\subseteq$s s

th-21 (lem-35): $\neg$ st $\in_{sl}$ $stl_0 \rightarrow stl_0 \odot_{sl} stl_1$ from st = stl_1 from st

th-22 (lem-36): st $\in_{sl}$ $stl_0 \rightarrow stl_0 \odot_{sl} stl_1$ from st = (stl_0 from st) $\odot_{sl}$ stl_1

th-23 (lem-37): st $\in_{sl}$ stl $\rightarrow$ stl from st $\neq$ snil

th-24 (lem-39): sdgl cutptsin cdr(stack from st) $\rightarrow$ sdgl cutptsin stack

th-25 (lem-43): stl from st = $st_0 +_{sl}$ $stl_1 \rightarrow$ st = st_0

th-26 (lem-45): stl $\ll_s$ $stl_0 \wedge \neg$ st $\in_{sl}$ stl $\wedge$ st $\in_{sl}$ $stl_0 \rightarrow$ stl $\ll_s$ stl_0 from st

th-27 (lem-ax-09): mksdecgoal(go, st) $+_{sdl}$ sdgl ctpelem s
$\leftrightarrow \neg \neg$ (st $\in_s$ s $\wedge$ sdgl ctpelem s)

th-28 (lem-05): st $+_{sl}$ stl sl$\subseteq$s s $\rightarrow$ stl sl$\subseteq$s s

th-29 (lem-09): $\neg$ snew(s) $+_{sl}$ stl sl$\subseteq$s s

th-3 (lem-01): $\neg$ st $\in_{sl}$ stl $\rightarrow$ stl from st = snil

th-30 (lem-11): stl sl$\subseteq$s s $\rightarrow$ st $+_{sl}$ stl sl$\subseteq$s s $+_s$ st

th-31 (lem-13): mksdecgoal(su $\hat{}_{sg}$ go, st)
$+_{sdl}$ ssubres(sdgl, su) cutptsin stl
$\leftrightarrow$ mksdecgoal(go, st) $+_{sdl}$ sdgl cutptsin stl

th-32 (lem-14): st $\neq$ bottom $\wedge \neg$ st $\in_{sl}$ stl $\wedge$ sdgl cutptsin stl
$\rightarrow$ sdgl cutptsin st $+_{sl}$ stl

th-33 (lem-18): $\neg$ stl $\ll_s$ stl from st

th-34 (lem-23): st $\in_{sl}$ stl from $st_0 \rightarrow$ st $\in_{sl}$ stl

th-35 (lem-25): sdgl ctpelem s $\rightarrow$ sdgl ctpelem s $+_s$ st

th-36 (lem-26): ssubres(sdgl, su) ctpelem s $\leftrightarrow$ sdgl ctpelem s

th-37 (lem-28): stl sl$\subseteq$s s $\rightarrow \neg$ snew(s) $\in_{sl}$ cdr(stl from st)

th-38 (lem-29): stl from st = $st_1 +_{sl}$ $stl_0 \wedge$ $st_0 \in_{sl}$ $stl_0 \wedge$ stl sl$\subseteq$s s
$\rightarrow$ $st_0 \in_s$ s

th-39 (lem-30): $\mathrm{stl}\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} \rightarrow \mathrm{stl}\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} +_s \mathrm{st}$

th-4 (lem-02): $\mathrm{sdgl\ cutptsin\ snil} \rightarrow \mathrm{sdgl\ cutptsin\ stl}$

th-40 (lem-31): $\neg\ \mathrm{st} \in_{sl} \mathrm{stl}_0 \rightarrow \neg\ \mathrm{st} \in_{sl} \mathrm{cdr}(\mathrm{stl}_0\ \mathrm{from}\ \mathrm{st}_0)$

th-41 (lem-32): $\neg\ \mathrm{st} \in_{sl} \mathrm{stl}_0 \rightarrow \neg\ \mathrm{st} \in_{sl} \mathrm{cdr}(\mathrm{stl}_0)$

th-42 (lem-34): $\mathrm{st} +_{sl} \mathrm{snil}\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} \leftrightarrow \mathrm{st} \in_s \mathrm{s}$

th-43 (lem-38): $\mathrm{st} +_{sl} \mathrm{stl}\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} +_s \mathrm{st} \leftrightarrow \mathrm{stl}\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} +_s \mathrm{st}$

th-44 (lem-41): $\mathrm{stl} \neq \mathrm{snil} \rightarrow \mathrm{cdr}(\mathrm{stl} \odot_{sl} \mathrm{stl}_0) = \mathrm{scdr}(\mathrm{stl}) \odot_{sl} \mathrm{stl}_0$

th-45 (lem-42): $\mathrm{st} \in_{sl} \mathrm{stl\ from\ st} \leftrightarrow \mathrm{st} \in_{sl} \mathrm{stl}$

th-46 (lem-44): $\mathrm{stl} \ll_s \mathrm{stl}_0 \rightarrow \mathrm{stl\ from\ st} \ll_s \mathrm{stl}_0\ \mathrm{from\ st} \vee \mathrm{stl\ from\ st} = \mathrm{stl}_0\ \mathrm{from\ st}$

th-47 (lem-46): $\mathrm{st}_0 +_{sl} \mathrm{stl} \ll_s \mathrm{stl}_0 \wedge \neg\ \mathrm{st} \in_{sl} \mathrm{stl} \wedge \mathrm{st} \neq \mathrm{st}_0 \wedge \mathrm{st} \in_{sl} \mathrm{stl}_0 \rightarrow \mathrm{st}_0 +_{sl} \mathrm{stl} \ll_s \mathrm{stl}_0\ \mathrm{from\ st}$

th-48 (lem-47): $\mathrm{st} \in_{sl} \mathrm{stl} \wedge \mathrm{st}_0 +_{sl} \mathrm{stl} = \mathrm{stack\ from\ st}_0 \rightarrow \mathrm{st} \in_{sl} \mathrm{stack}$

th-49 (lem-48): $\mathrm{st} \in_{sl} \mathrm{stl} \wedge \mathrm{stl} \ll_s \mathrm{stl}_0 \wedge \mathrm{stl}_0\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} \rightarrow \mathrm{st} \in_s \mathrm{s}$

th-5 (lem-08): $\mathrm{st} +_{sl} \mathrm{stl}\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} \leftrightarrow \mathrm{st} \in_s \mathrm{s} \wedge \mathrm{stl}\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s}$

th-50 (lem-49): $\mathrm{stl} \ll_s \mathrm{stl}_0 \wedge \mathrm{stl}_0\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} \rightarrow \neg\ \mathrm{snew(s)} \in_{sl} \mathrm{stl}$

th-51 (lem-50): $\mathrm{stl}\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} \rightarrow (\mathrm{stl} \odot_{sl} \mathrm{stl}_0\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} +_s \mathrm{st} \leftrightarrow \mathrm{stl}_0\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} +_s \mathrm{st})$

th-52 (lem-51): $\mathrm{sdgl\ cutptsin\ cdr(stack\ from\ st)\ from\ st}_0 \rightarrow \mathrm{sdgl\ cutptsin\ stack}$

th-6 (lem-12): $\mathrm{sdg} +_{sdl} \mathrm{sdgl\ cutptsin\ stl} \leftrightarrow \neg\ \neg\ (\ \ (\mathrm{sdg.s2} = \mathrm{bottom} \vee \mathrm{sdg.s2} \in_{sl} \mathrm{stl}) \wedge \mathrm{sdgl\ cutptsin\ stl\ from\ sdg.s2})$

th-7 (lem-15): $\mathrm{snil}\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s}$

th-8 (lem-20): $\mathrm{stl\ from\ st} = \mathrm{stl} \leftrightarrow \mathrm{stl} = \mathrm{snil} \vee \mathrm{scar(stl)} = \mathrm{st}$

th-9 (lem-04): $\mathrm{stl}\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s} \rightarrow \mathrm{stl\ from\ st}\ \mathrm{sl}{\subseteq}\mathrm{s}\ \mathrm{s}$

4.6 Axioms and Theorems from PrologTree

ax-01 : $\mathrm{mapclause(cnil,\ db)} = \mathrm{clnil}$

ax-02 : $\mathrm{mapclause(co} +_{col} \mathrm{col,\ db)} = \mathrm{clause(co,\ db)} +_{cli} \mathrm{mapclause(col,\ db)}$

ax-03 : $\mathrm{every(father,\ nnil,\ no)}$

ax-04 : $\mathrm{every(father,\ no}_1 +_{nl} \mathrm{nol,\ no)} \leftrightarrow \mathrm{father}\ \hat{}_{fa}\ \mathrm{no}_1 = \mathrm{no} \wedge \mathrm{every(father,\ nol,\ no)}$

ax-05 : $\mathrm{map(cll,\ nnil)} = \mathrm{cnil}$

ax-06 : $\mathrm{map(cll,\ no} +_{nl} \mathrm{nol)} = (\mathrm{cll}\ \hat{}_{cll}\ \mathrm{no}) +_{col} \mathrm{map(cll,\ nol)}$

ax-07 : $\mathrm{nodups(nnil)}$

ax-08 : $\mathrm{nodups(no} +_{nl} \mathrm{nol)} \leftrightarrow \neg\ \mathrm{no} \in_{nl} \mathrm{nol} \wedge \mathrm{nodups(nol)}$

ax-09 : $\mathrm{nol}\ \mathrm{nl}{\subseteq}\mathrm{s}\ \mathrm{ns} \leftrightarrow (\forall\ \mathrm{no.}\ \mathrm{no} \in_{nl} \mathrm{nol} \rightarrow \mathrm{no} \in_n \mathrm{ns})$

ax-10 : $\mathrm{disjoint(nol,\ nol}_0) \leftrightarrow (\forall\ \mathrm{no.}\ \mathrm{no} \in_{nl} \mathrm{nol} \rightarrow \neg\ \mathrm{no} \in_{nl} \mathrm{nol}_0)$

ax-11 : $\mathrm{disjointls(nol,\ ns)} \leftrightarrow (\forall\ \mathrm{no.}\ \mathrm{no} \in_{nl} \mathrm{nol} \rightarrow \neg\ \mathrm{no} \in_n \mathrm{ns})$

th-1 (lem-05): $\mathrm{every(father,\ nol,\ no)} \leftrightarrow (\forall\ \mathrm{no}_1.\ \mathrm{no}_1 \in_{nl} \mathrm{nol} \rightarrow \mathrm{father}\ \hat{}_{fa}\ \mathrm{no}_1 = \mathrm{no})$

th-10 (lem-12): $\neg\ \mathrm{disjoint(nol,\ nol}_0) \leftrightarrow (\exists\ \mathrm{no.}\ \mathrm{no} \in_{nl} \mathrm{nol} \wedge \mathrm{no} \in_{nl} \mathrm{nol}_0)$

th-11 (lem-13): $\neg\ \mathrm{no}_1 \in_{nl} \mathrm{nol} \rightarrow (\ \ \mathrm{every(father} +_{fa} \mathrm{no}_1\ /_{fa}\ \mathrm{no}_2, \mathrm{nol, no)} \leftrightarrow \mathrm{every(father, nol, no))}$

th-12 (lem-14): $\mathrm{every(father, nol, no)} \wedge \mathrm{no}_1 \in_{nl} \mathrm{nol} \rightarrow (\mathrm{father}\ \hat{}_{fa}\ \mathrm{no}_1 = \mathrm{no} \leftrightarrow \mathrm{true})$

th-13 (lem-15): $\mathrm{every(father,\ no} +_{nl} \mathrm{nol,\ no}_1) \rightarrow \mathrm{every(father,\ nol,\ no}_1)$

th-14 (lem-16): no $+_{nl}$ nol nl$\subseteq$s ns $\rightarrow$ nol nl$\subseteq$s ns

th-15 (lem-17): nodups(no $+_{nl}$ nol) $\rightarrow$ nodups(nol)

th-16 (lem-18): nodups(no $+_{nl}$ nol) $\rightarrow \neg$ no $\in_{nl}$ nol

th-17 (map-lemma): ($\forall$ no. no $\in_{nl}$ nol $\rightarrow$ cll $\hat{}_{cll}$ no = cll_1 $\hat{}_{cll}$ no)
$\rightarrow$ map(cll, nol) = map(cll_1, nol)

th-2 (lem-01): disjoint(nnil, nol_0)

th-3 (lem-02): disjoint(no $+_{nl}$ nol, nol_0)
$\leftrightarrow \neg$ no $\in_{nl}$ nol_0 $\wedge$ disjoint(nol, nol_0)

th-4 (lem-03): disjoint(nol_0, nnil)

th-5 (lem-04): disjoint(nol_0, no $+_{nl}$ nol)
$\leftrightarrow \neg$ no $\in_{nl}$ nol_0 $\wedge$ disjoint(nol_0, nol)

th-6 (lem-06): no $\in_{nl}$ nol $\wedge$ nol nl$\subseteq$s ns $\rightarrow$ no $\in_n$ ns

th-7 (lem-08): nol nl$\subseteq$s ns $\wedge$ ns $\subseteq_{ns}$ ns_1 $\rightarrow$ nol nl$\subseteq$s ns_1

th-8 (lem-09): nnil nl$\subseteq$s ns

th-9 (lem-10): disjoint(nol, nol_0) $\rightarrow$ disjoint(nol_0, nol)

4.7 Axioms and Theorems from Switching

Remark: (A $\supset$ B; C) abbreviates (A $\rightarrow$ B) $\wedge$ ($\neg$ A $\rightarrow$ C)

ax-03 : eqh(h, h_0, sdnil, sdnil)

ax-04 : $\neg$ eqh(h, h_0, mksdecgoal(go, st) $+_{sdl}$ sdgl, sdnil)

ax-05 : $\neg$ eqh(h, h_0, sdnil, mksdecgoal(go_0, st_0) $+_{sdl}$ sdgl_0)

ax-06 : eqh(h, h_0, mksdecgoal(go, st) $+_{sdl}$ sdgl,
mksdecgoal(go_0, st_0) $+_{sdl}$ sdgl_0)
$\leftrightarrow$ go = go_0
$\wedge$ (st = bottom
$\supset$ $\text{st}_0 \in_{sl} \text{h}_0$ $\hat{}_h$ bottom $\vee$ st_0 = bottom ;
$\text{st}_0 \in_{sl} \text{h}_0$ $\hat{}_h$ st
$\wedge \neg \text{st}_0 \in_{sl}$ scdr(h $\hat{}_h$ st))
$\wedge$ eqh(h, h_0, sdgl, sdgl_0)

th-1 (lem-09): $\neg$ eqh(h, h_0, sdnil, sdg_0 $+_{sdl}$ sdgl_0)

th-10 (lem-06): eqh(h, h_0, sdgl, sdgl_0) $\wedge$ sdgl cutptsin stl
$\wedge \neg$ st $\in_{sl}$ stl $\wedge$ st $\neq$ bottom
$\rightarrow$ eqh(h $+_h$ st $/_h$ stl_0, h_0 $+_h$ st $/_h$ stl_0, sdgl, sdgl_0)

th-11 (lem-04): eqh(h, h_0, sdgl, sdgl_0) $\wedge$ sdgl cutptsin snil
$\rightarrow$ eqh(h, h_0 $+_h$ bottom $/_h$ (stl_0 $\odot_{sl}$ (h_0 $\hat{}_h$ bottom)),
sdgl, sdgl_0)

th-12 (lem-03): sdgl cutptsin stl $\wedge \neg$ st $\in_{sl}$ stl
$\wedge \neg$ bottom $\in_{sl}$ stl $\wedge$ st $\neq$ bottom
$\rightarrow$ (eqh(h, h_0 $+_h$ st $/_h$ stl_0, sdgl, sdgl_0)
$\leftrightarrow$ eqh(h, h_0, sdgl, sdgl_0))

th-13 (lem-02): $\neg$ eqh(h, h_0, ssubres(sdgl, su), ssubres(sdgl_0, su))
$\rightarrow \neg$ eqh(h, h_0, sdgl, sdgl_0)

th-14 (elim-carcdr): stl $\neq$ snil
$\rightarrow$ (st = car(stl) $\wedge$ stl_0 = cdr(stl)
$\leftrightarrow$ stl = st $+_{sl}$ stl_0)

th-2 (lem-12): eqh(h, h_0, sdnil, sdgl_0) $\leftrightarrow$ sdgl_0 = sdnil

th-3 (lem-10): $\neg$ eqh(h, h_0, sdg $+_{sdl}$ sdgl, sdnil)

th-4 (lem-01): eqh(h, h_0, mksdecgoal(go, st) $+_{sdl}$ sdgl,
mksdecgoal(go_0, st_0) $+_{sdl}$ sdgl_0)
$\leftrightarrow \neg \neg$ (go = go_0
$\wedge$ (st = bottom

$\supset$ $st_0 \in_{sl} h_0 \,\hat{}_h$ bottom $\vee$ st_0 = bottom ;
$st_0 \in_{sl} h_0 \,\hat{}_h$ st $\wedge \neg\, st_0 \in_{sl}$ scdr(h $\hat{}_h$ st))
$\wedge$ eqh(h, h_0, sdgl, $sdgl_0$))

th-5 (lem-05): eqh(h, h_0, sdgl, $sdgl_0$)
$\rightarrow$ eqh(h, $h_0 +_h$ st $/_h$ ($stl_0 \odot_{sl}$ ($h_0 \,\hat{}_h$ st)),
sdgl, $sdgl_0$)

th-6 (lem-13): eqh(h, h_0, sdgl, sdnil) $\leftrightarrow$ sdgl = sdnil

th-7 (lem-11): eqh(h, h_0, sdgl, $sdgl_0$) $\wedge$ sdgl cutptsin st $+_{sl}$ stl
$\wedge$ st $\neq$ bottom $\wedge \neg$ bottom $\in_{sl}$ stl
$\wedge$ $sdgl_0$ cutptsin $st_0 +_{sl} stl_0 \odot_{sl} stl_1$
$\wedge$ ($\forall$ st'. st' $\in_{sl}$ $h_0 \,\hat{}_h$ st $\rightarrow \neg$ st' $\in_{sl} stl_1$)
$\wedge$ ($h_0 \,\hat{}_h$ st) from st_0 = $st_0 +_{sl} stl_0$
$\wedge \neg$ bottom $\in_{sl}$ stl_1 $\wedge \neg$ bottom $\in_{sl}$ $h_0 \,\hat{}_h$ st
$\wedge$ (h $\hat{}_h$ st) $\ll_s$ $st_0 +_{sl} stl_0$ $\vee$ h $\hat{}_h$ st = $st_0 +_{sl} stl_0$)
$\rightarrow$ eqh(h, $h_0 +_h$ st $/_h$ ($st_0 +_{sl} stl_0$), sdgl, $sdgl_0$)

th-8 (lem-08): eqh(h, h_0, sdgl, $sdgl_0$) $\wedge$ sdgl cutptsin snil
$\wedge$ $sdgl_0$ cutptsin ($h_0 \,\hat{}_h$ bottom) from st_1
$\rightarrow$ eqh(h, $h_0 +_h$ bottom $/_h$ (($h_0 \,\hat{}_h$ bottom) from st_1),
sdgl, $sdgl_0$)

th-9 (lem-07): eqh(h, h_0, sdgl, $sdgl_0$) $\wedge$ sdgl cutptsin snil
$\wedge$ $sdgl_0$ cutptsin snil
$\rightarrow$ eqh(h, $h_0 +_h$ bottom $/_h$ snil, sdgl, $sdgl_0$)

4.8 Axioms and Theorems from Tree+Stack+F

ax-01 : fnd(fn, sdnil) = dnil

ax-02 : fnd(fn, mksdecgoal(go, st) $+_{sdl}$ sdgl)
= mkdecgoal(go, fn $\hat{}_{fn}$ st) $+_{dl}$ fnd(fn, sdgl)

ax-03 : fns(fn, $@_s$) = $@_{ns}$

ax-04 : fns(fn, s $+_s$ st) = fns(fn, s) $+_{ns}$ (fn $\hat{}_{fn}$ st)

ax-05 : candsdisjoint(fn, cands, stl)
$\leftrightarrow$ ($\forall$ st, st_1. st $\in_{sl}$ stl $\wedge$ $st_1 \in_{sl}$ stl $\wedge$ st $\neq$ st_1
$\rightarrow$ disjoint(cands $\hat{}_n$ fn $\hat{}_{fn}$ st_1,
cands $\hat{}_n$ fn $\hat{}_{fn}$ st))

ax-06 : fn injonn stl
$\leftrightarrow$ ($\forall$ st, st_1. st $\in_{sl}$ stl $\wedge$ $st_1 \in_{sl}$ bottom $+_{sl}$ stl
$\wedge$ st $\neq$ st_1
$\rightarrow$ fn $\hat{}_{fn}$ st $\neq$ fn $\hat{}_{fn}$ st_1)

ax-07 : nocands(fn, cands, stl)
$\leftrightarrow$ ($\forall$ st, st_1. st $\in_{sl}$ stl $\wedge$ $st_1 \in_{sl}$ bottom $+_{sl}$ stl
$\rightarrow \neg$ (fn $\hat{}_{fn}$ st_1) $\in_{nl}$ cands $\hat{}_n$ fn $\hat{}_{fn}$ st)

th-1 (lem-01): candsdisjoint(fn, cands, snil)

th-10 (lem-11): fnd(fn, sdgl) = dnil $\leftrightarrow$ sdgl = sdnil

th-11 (lem-12): fn injonn stl $\rightarrow$ fn injonn stl from st

th-12 (lem-13): fn injonn snil

th-13 (lem-14): fn injonn st $+_{sl}$ stl $\rightarrow$ fn injonn stl

th-14 (lem-19): candsdisjoint(fn, cands, st $+_{sl}$ stl)
$\wedge$ $st_1 \in_{sl}$ stl $\wedge$ st $\neq$ st_1 $\wedge$ fn $\hat{}_{fn}$ st = no
$\rightarrow$ disjoint(cands $\hat{}_n$ fn $\hat{}_{fn}$ st_1, cands $\hat{}_n$ no)

th-15 (lem-20): fn $\hat{}_{fn}$ st_1 = no_1 $\wedge$ fn $\hat{}_{fn}$ st_2 = no_2
$\wedge$ $st_1 \neq st_2$ $\wedge$ $st_1 \in_{sl}$ stl $\wedge$ $st_2 \in_{sl}$ stl $\wedge$ fn injonn stl
$\rightarrow$ $no_1 \neq no_2$

th-16 (lem-22): $\mathrm{st}_1 \in_{sl} \mathrm{stl} \wedge \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_2 = \mathrm{no}$
$\wedge\ \mathrm{no} \in_{nl} \mathrm{cands}\ \hat{}_n\ \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_1$
$\rightarrow \neg\ \mathrm{nocands}(\mathrm{fn}, \mathrm{cands}, \mathrm{st}_2 +_{sl} \mathrm{stl})$

th-17 (lem-23): $\mathrm{st}_1 \in_{sl} \mathrm{stl} \wedge \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_2 = \mathrm{no}$
$\wedge\ (\mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_1) \in_{nl} \mathrm{cands}\ \hat{}_n\ \mathrm{no}$
$\rightarrow \neg\ \mathrm{nocands}(\mathrm{fn}, \mathrm{cands}, \mathrm{st}_2 +_{sl} \mathrm{stl})$

th-18 (lem-24): $\mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_1 = \mathrm{no} \wedge \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_2 = \mathrm{no}$
$\wedge\ \mathrm{st}_1 \in_{sl} \mathrm{stl} \wedge \mathrm{fn\ injonn}\ \mathrm{st}_2 +_{sl} \mathrm{stl}$
$\rightarrow \mathrm{st}_1 = \mathrm{st}_2$

th-19 (lem-25): $\mathrm{fn\ injonn\ stl} \wedge \mathrm{fn}\ \hat{}_{fn}\ \mathrm{bottom} = \mathrm{no}$
$\wedge\ \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_1 = \mathrm{no} \wedge (\mathrm{st}_1 \in_{sl} \mathrm{stl} \vee \mathrm{st}_1 = \mathrm{bottom})$
$\rightarrow \mathrm{st}_1 = \mathrm{bottom}$

th-2 (lem-02): $\mathrm{candsdisjoint}(\mathrm{fn}, \mathrm{cands}, \mathrm{st} +_{sl} \mathrm{stl})$
$\leftrightarrow\ (\forall\ \mathrm{st}_1.\ \ \mathrm{st}_1 \in_{sl} \mathrm{stl} \wedge \mathrm{st}_1 \neq \mathrm{st}$
$\rightarrow \mathrm{disjoint}(\mathrm{cands}\ \hat{}_n\ \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st},$
$\mathrm{cands}\ \hat{}_n\ \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_1))$
$\wedge\ \mathrm{candsdisjoint}(\mathrm{fn}, \mathrm{cands}, \mathrm{stl})$

th-20 (lem-26): $\mathrm{fn}\ \hat{}_{fn}\ \mathrm{st} = \mathrm{no} \wedge \mathrm{nocands}(\mathrm{fn}, \mathrm{cands}, \mathrm{stl}) \wedge \mathrm{st} \in_{sl} \mathrm{stl}$
$\rightarrow \neg\ \mathrm{no} \in_{nl} \mathrm{cands}\ \hat{}_n\ \mathrm{no}$

th-21 (lem-27): $\mathrm{fn}\ \hat{}_{fn}\ \mathrm{bottom} = \mathrm{no}_1 \wedge \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st} = \mathrm{no}_2$
$\wedge\ \mathrm{st} \in_{sl} \mathrm{stl} \wedge \mathrm{nocands}(\mathrm{fn}, \mathrm{cands}, \mathrm{stl})$
$\rightarrow \neg\ \mathrm{no}_1 \in_{nl} \mathrm{cands}\ \hat{}_n\ \mathrm{no}_2$

th-22 (lem-29): $\mathrm{sdgl\ cutptsin\ stl} \wedge \mathrm{bottom} \in_s \mathrm{s}$
$\wedge\ \mathrm{sdgl} \neq \mathrm{sdnil} \wedge \mathrm{stl\ sl}\subseteq\mathrm{s\ s}$
$\rightarrow\ \ \mathrm{sdcar}(\mathrm{sdgl}).\mathrm{s2} \neq \mathrm{snew}(\mathrm{s})$
$\wedge\ \ \ \mathrm{fnd}(\mathrm{fn} +_{fn} \mathrm{snew}(\mathrm{s}) /_{fn} \mathrm{no}, \mathrm{sdcdr}(\mathrm{sdgl}))$
$= \mathrm{fnd}(\mathrm{fn}, \mathrm{sdcdr}(\mathrm{sdgl}))$

th-23 (lem-30): $\mathrm{fnd}(\mathrm{fn}, \mathrm{ssubres}(\mathrm{sdgl}, \mathrm{su})) = \mathrm{subres}(\mathrm{fnd}(\mathrm{fn}, \mathrm{sdgl}), \mathrm{su})$

th-24 (lem-31): $\mathrm{candsdisjoint}(\mathrm{fn}, \mathrm{cands}, \mathrm{stl})$
$\wedge\ (\forall\ \mathrm{st}_1.\ \ \mathrm{st}_1 \in_{sl} \mathrm{stl}$
$\rightarrow\ \ \ \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_1 \neq \mathrm{no}$
$\wedge\ \mathrm{disjoint}(\mathrm{cands}\ \hat{}_n\ \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_1, \mathrm{nol}))$
$\rightarrow \mathrm{candsdisjoint}(\mathrm{fn} +_{fn} \mathrm{st} /_{fn} \mathrm{no},$
$\mathrm{cands} +_n \mathrm{no} /_n \mathrm{nol}, \mathrm{st} +_{sl} \mathrm{stl})$

th-25 (lem-32): $\mathrm{fn\ injonn\ stl}$
$\wedge\ (\forall\ \mathrm{st}_1.\ \mathrm{st}_1 \in_{sl} \mathrm{bottom} +_{sl} \mathrm{stl} \rightarrow \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_1 \neq \mathrm{no})$
$\rightarrow \mathrm{fn} +_{fn} \mathrm{st} /_{fn} \mathrm{no\ injonn\ st} +_{sl} \mathrm{stl}$

th-26 (lem-33): $\mathrm{nocands}(\mathrm{fn}, \mathrm{cands}, \mathrm{stl}) \wedge \neg\ \mathrm{no} \in_{nl} \mathrm{nol}$
$\wedge\ \neg\ (\mathrm{fn}\ \hat{}_{fn}\ \mathrm{bottom}) \in_{nl} \mathrm{nol}$
$\wedge\ (\forall\ \mathrm{st}_1.\ \ \mathrm{st}_1 \in_{sl} \mathrm{stl}$
$\rightarrow\ \ \ \neg\ (\mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_1) \in_{nl} \mathrm{nol}$
$\wedge\ \neg\ \mathrm{no} \in_{nl} \mathrm{cands}\ \hat{}_n\ \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st}_1)$
$\rightarrow \mathrm{nocands}(\mathrm{fn} +_{fn} \mathrm{st} /_{fn} \mathrm{no},$
$\mathrm{cands} +_n \mathrm{no} /_n \mathrm{nol}, \mathrm{st} +_{sl} \mathrm{stl})$

th-27 (lem-34): $\mathrm{sdgl\ cutptsin\ stl} \wedge \mathrm{stl\ sl}\subseteq\mathrm{s\ s}$
$\wedge\ \mathrm{bottom} \in_s \mathrm{s}$
$\rightarrow \mathrm{fnd}(\mathrm{fn} +_{fn} \mathrm{snew}(\mathrm{s}) /_{fn} \mathrm{no}, \mathrm{sdgl}) = \mathrm{fnd}(\mathrm{fn}, \mathrm{sdgl})$

th-28 (lem-35): $\mathrm{candsdisjoint}(\mathrm{fn}, \mathrm{cands}, \mathrm{stl})$
$\wedge\ (\forall\ \mathrm{st.}\ \mathrm{st} \in_{sl} \mathrm{stl} \rightarrow \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st} \neq \mathrm{no})$
$\rightarrow \mathrm{candsdisjoint}(\mathrm{fn}, \mathrm{cands} +_n \mathrm{no} /_n \mathrm{nol}, \mathrm{stl})$

th-29 (lem-36): $\mathrm{nocands}(\mathrm{fn}, \mathrm{cands}, \mathrm{stl})$
$\wedge\ (\forall\ \mathrm{st.}\ \mathrm{st} \in_{sl} \mathrm{stl} \rightarrow \mathrm{fn}\ \hat{}_{fn}\ \mathrm{st} \neq \mathrm{no})$
$\rightarrow \mathrm{nocands}(\mathrm{fn}, \mathrm{cands} +_n \mathrm{no} /_n \mathrm{nol}, \mathrm{stl})$

th-3 (lem-08): $\mathrm{nocands}(\mathrm{fn}, \mathrm{cands}, \mathrm{snil})$

th-30 (lem-37): candsdisjoint(fn, cands, stl)
→ candsdisjoint(fn, cands, stl from st)

th-31 (lem-38): nocands(fn, cands, stl) → nocands(fn, cands, stl from st)

th-32 (lem-21): b $\hat{}_b$ st_1 ≠ b $\hat{}_b$ st_2 → st_1 ≠ st_2

th-4 (lem-15): fn $\hat{}_{fn}$ st = no ∧ nocands(fn, cands, st $+_{sl}$ stl)
→ ¬ no $\in_{nl}$ cands $\hat{}_n$ no

th-5 (lem-16): fn $\hat{}_{fn}$ bottom = no_1
∧ fn $\hat{}_{fn}$ st = no_2 ∧ nocands(fn, cands, st $+_{sl}$ stl)
→ ¬ no_1 $\in_{nl}$ cands $\hat{}_n$ no_2

th-6 (lem-17): nocands(fn, cands, st $+_{sl}$ stl) → nocands(fn, cands, stl)

th-7 (lem-18): candsdisjoint(fn, cands, st $+_{sl}$ stl)
→ candsdisjoint(fn, cands, stl)

th-8 (lem-03): candsdisjoint(fn, cands, stl)
∧ (∀ st. st $\in_{sl}$ stl
→ cands $\hat{}_n$ fn $\hat{}_{fn}$ st = $cands_1$ $\hat{}_n$ fn $\hat{}_{fn}$ st)
→ candsdisjoint(fn, $cands_1$, stl)

th-9 (lem-04): nocands(fn, cands, stl)
∧ (∀ st. st $\in_{sl}$ bottom $+_{sl}$ stl
→ cands $\hat{}_n$ fn $\hat{}_{fn}$ st = $cands_1$ $\hat{}_n$ fn $\hat{}_{fn}$ st)
→ nocands(fn, $cands_1$, stl)

4.9 Axioms and Theorems from clause'fun

ax-01 : clause'(failcode, db) = null

4.10 Axioms and Theorems from clauseornull

ax-01 : the_clau(mkclau(cl)) = cl

ax-02 : mkclau(cl) = mkclau(cl_0) ↔ cl = cl_0

ax-03 : mkclau(cl) ≠ null

ax-04 : cln = mkclau(the_clau(cln)) ∨ cln = null

4.11 Axioms and Theorems from codefun

ax-01 : coa = start ↔ code(coa, db") = code_of_start

ax-02 : code(failcode, db") = nil'

th-1 (lem-02): code(start, db") = code_of_start

th-10 (lem-17): code(preg, db") = nil' → preg ≠ start

th-11 (lem-18): is_sw_const(code(preg, db")) → preg ≠ start

th-12 (lem-21): preg = start → code(preg, db") ≠ nil'

th-13 (lem-22): preg = start → ¬ is_sw_const(code(preg, db"))

th-14 (lem-23): preg = start → ¬ is_sw_struct(code(preg, db"))

th-15 (lem-24): preg = start → ¬ is_sw_term(code(preg, db"))

th-16 (lem-03): is_try(code(preg, db")) → preg ≠ start

th-17 (lem-04): is_try_me(code(preg, db")) → preg ≠ start

th-18 (lem-05): is_retry(code(preg, db")) → preg ≠ start

th-19 (lem-06): is_retry_me(code(preg, db")) → preg ≠ start

th-2 (lem-01): code(coa, db") = code_of_start ↔ coa = start

th-20 (lem-07): is_trust(code(preg, db")) → preg ≠ start

th-21 (lem-08): is_trust_me(code(preg, db")) → preg ≠ start

th-22 (lem-09): is_clause(code(preg, db")) → preg ≠ start
th-23 (lem-19): is_sw_struct(code(preg, db")) → preg ≠ start
th-24 (lem-20): is_sw_term(code(preg, db")) → preg ≠ start
th-25 (lem-25): ¬ is_sw_struct(code(preg, db"))
∧ ¬ is_sw_term(code(preg, db"))
∧ ¬ is_try(code(preg, db"))
∧ ¬ is_try_me(code(preg, db"))
∧ ¬ is_clause(code(preg, db"))
∧ ¬ is_retry(code(preg, db"))
∧ ¬ is_retry_me(code(preg, db"))
∧ ¬ is_trust(code(preg, db"))
∧ ¬ is_trust_me(code(preg, db"))
∧ code(preg, db") ≠ nil'
→ (¬ is_sw_const(code(preg, db")) ↔ preg = start)
th-26 (lem-26): start ≠ failcode
th-3 (lem-10): preg = start → ¬ is_try(code(preg, db"))
th-4 (lem-11): preg = start → ¬ is_try_me(code(preg, db"))
th-5 (lem-12): preg = start → ¬ is_retry(code(preg, db"))
th-6 (lem-13): preg = start → ¬ is_retry_me(code(preg, db"))
th-7 (lem-14): preg = start → ¬ is_trust(code(preg, db"))
th-8 (lem-15): preg = start → ¬ is_trust_me(code(preg, db"))
th-9 (lem-16): preg = start → ¬ is_clause(code(preg, db"))

4.12 Axioms and Theorems from enrnodeset

ax-02 : new($@_{ns}$) = root

4.13 Axioms and Theorems from enrset

ax-01 : ¬ new(s) ∈ s
th-1 (lem-01): a ∈ s → (¬ a ≠ new(s) ↔ false)

4.14 Axioms and Theorems from enrstateset

ax-01 : snew($@_s$) = bottom

4.15 Axioms and Theorems from enrterm

ax-01 : is_user_defined(trm)
↔ trm ≠ true' ∧ trm ≠ fail' ∧ trm ≠ !
ax-02 : arity(trm) = tlen(args(trm))+1
ax-03 : args(trm) = the_one(trm_1) → arg(trm, 0 +1) = trm_1
ax-04 : args(trm) = tcons(trm_1, trmli) → arg(trm, 0 +1) = trm_1 ∧ (0 < n → arg(trm, n +1) = arg(struct(funct(trm), trmli), n))
th-1 (lem-1): ¬ is_user_defined(!)
th-2 (lem-2): ¬ is_user_defined(true')
th-3 (lem-3): ¬ is_user_defined(fail')
th-4 (lem-4): ¬ is_user_defined(trm)

$\leftrightarrow$ trm = ! $\vee$ trm = true' $\vee$ trm = fail'

4.16 Axioms and Theorems from idfun

ax-01 : is_struct(trm)
$\rightarrow$ id(trm) = mkident(funct(trm), arity(trm))
ax-02 : is_const(trm)
$\rightarrow$ id(trm) = mkident(constsym(trm), 0)

4.17 Axioms and Theorems from instr+clau

ax-01 : where(try_me_else(coa)) = coa
ax-02 : where(retry_me_else(coa)) = coa
ax-03 : what(try'(coa)) = coa
ax-04 : what(retry'(coa)) = coa
ax-05 : what(trust(coa)) = coa
ax-06 : argindex(switch_on_term(n, coa, coa_0, coa_1, coa_2)) = n
ax-07 : vlabel(switch_on_term(n, coa, coa_0, coa_1, coa_2)) = coa
ax-08 : clabel(switch_on_term(n, coa, coa_0, coa_1, coa_2)) = coa_0
ax-09 : llabel(switch_on_term(n, coa, coa_0, coa_1, coa_2)) = coa_1
ax-10 : slabel(switch_on_term(n, coa, coa_0, coa_1, coa_2)) = coa_2
ax-100 : $\neg$ is_sw_term(nil')
ax-101 : $\neg$ is_sw_term(code_of_start)
ax-102 : $\neg$ is_sw_const(try_me_else(coa))
ax-103 : $\neg$ is_sw_const(retry_me_else(coa))
ax-104 : $\neg$ is_sw_const(trust_me)
ax-105 : $\neg$ is_sw_const(try'(coa))
ax-106 : $\neg$ is_sw_const(retry'(coa))
ax-107 : $\neg$ is_sw_const(trust(coa))
ax-108 : $\neg$ is_sw_const(switch_on_term(n, coa, coa_0, coa_1, coa_2))
ax-109 : is_sw_const(switch_on_constant(n, n_0, coa))
ax-11 : argindex(switch_on_constant(n, n_0, coa)) = n
ax-110 : $\neg$ is_sw_const(switch_on_structure(n, n_0, coa))
ax-111 : $\neg$ is_sw_const(mkcl(cl))
ax-112 : $\neg$ is_sw_const(nil')
ax-113 : $\neg$ is_sw_const(code_of_start)
ax-114 : $\neg$ is_sw_struct(try_me_else(coa))
ax-115 : $\neg$ is_sw_struct(retry_me_else(coa))
ax-116 : $\neg$ is_sw_struct(trust_me)
ax-117 : $\neg$ is_sw_struct(try'(coa))
ax-118 : $\neg$ is_sw_struct(retry'(coa))
ax-119 : $\neg$ is_sw_struct(trust(coa))
ax-12 : tabsize(switch_on_constant(n, n_0, coa)) = n_0
ax-120 : $\neg$ is_sw_struct(switch_on_term(n, coa, coa_0, coa_1, coa_2))
ax-121 : $\neg$ is_sw_struct(switch_on_constant(n, n_0, coa))
ax-122 : is_sw_struct(switch_on_structure(n, n_0, coa))
ax-123 : $\neg$ is_sw_struct(mkcl(cl))
ax-124 : $\neg$ is_sw_struct(nil')
ax-125 : $\neg$ is_sw_struct(code_of_start)
ax-126 : $\neg$ is_clause(try_me_else(coa))
ax-127 : $\neg$ is_clause(retry_me_else(coa))

ax-128 : $\neg$ is_clause(trust_me)

ax-129 : $\neg$ is_clause(try'(coa))

ax-13 : table(switch_on_constant(n, n_0, coa)) = coa

ax-130 : $\neg$ is_clause(retry'(coa))

ax-131 : $\neg$ is_clause(trust(coa))

ax-132 : $\neg$ is_clause(switch_on_term(n, coa, coa_0, coa_1, coa_2))

ax-133 : $\neg$ is_clause(switch_on_constant(n, n_0, coa))

ax-134 : $\neg$ is_clause(switch_on_structure(n, n_0, coa))

ax-135 : is_clause(mkcl(cl))

ax-136 : $\neg$ is_clause(nil')

ax-137 : $\neg$ is_clause(code_of_start)

ax-138 : try_me_else(coa) = try_me_else(coa_0) $\leftrightarrow$ coa = coa_0

ax-139 : retry_me_else(coa) = retry_me_else(coa_0) $\leftrightarrow$ coa = coa_0

ax-14 : argindex(switch_on_structure(n, n_0, coa)) = n

ax-140 : try'(coa) = try'(coa_0) $\leftrightarrow$ coa = coa_0

ax-141 : retry'(coa) = retry'(coa_0) $\leftrightarrow$ coa = coa_0

ax-142 : trust(coa) = trust(coa_0) $\leftrightarrow$ coa = coa_0

ax-143 : switch_on_term(n, coa, coa_0, coa_1, coa_2)
= switch_on_term(n_0, coa_3, coa_4, coa_5, coa_6)
$\leftrightarrow$ n = n_0 $\wedge$ coa = coa_3 $\wedge$ coa_0 = coa_4
$\wedge$ coa_1 = coa_5 $\wedge$ coa_2 = coa_6

ax-144 : switch_on_constant(n, n_0, coa)
= switch_on_constant(n_1, n_2, coa_0)
$\leftrightarrow$ n = n_1 $\wedge$ n_0 = n_2 $\wedge$ coa = coa_0

ax-145 : switch_on_structure(n, n_0, coa)
= switch_on_structure(n_1, n_2, coa_0)
$\leftrightarrow$ n = n_1 $\wedge$ n_0 = n_2 $\wedge$ coa = coa_0

ax-146 : mkcl(cl) = mkcl(cl_0) $\leftrightarrow$ cl = cl_0

ax-147 : nil' $\neq$ code_of_start

ax-148 : mkcl(cl) $\neq$ code_of_start

ax-149 : mkcl(cl) $\neq$ nil'

ax-15 : tabsize(switch_on_structure(n, n_0, coa)) = n_0

ax-150 : switch_on_structure(n, n_0, coa) $\neq$ code_of_start

ax-151 : switch_on_structure(n, n_0, coa) $\neq$ nil'

ax-152 : switch_on_structure(n, n_0, coa) $\neq$ mkcl(cl)

ax-153 : switch_on_constant(n, n_0, coa) $\neq$ code_of_start

ax-154 : switch_on_constant(n, n_0, coa) $\neq$ nil'

ax-155 : switch_on_constant(n, n_0, coa) $\neq$ mkcl(cl)

ax-156 : switch_on_constant(n, n_0, coa)
$\neq$ switch_on_structure(n_1, n_2, coa_0)

ax-157 : switch_on_term(n, coa, coa_0, coa_1, coa_2)
$\neq$ code_of_start

ax-158 : switch_on_term(n, coa, coa_0, coa_1, coa_2) $\neq$ nil'

ax-159 : switch_on_term(n, coa, coa_0, coa_1, coa_2) $\neq$ mkcl(cl)

ax-16 : table(switch_on_structure(n, n_0, coa)) = coa

ax-160 : switch_on_term(n, coa, coa_0, coa_1, coa_2)
$\neq$ switch_on_structure(n_0, n_1, coa_3)

ax-161 : switch_on_term(n, coa, coa_0, coa_1, coa_2)
$\neq$ switch_on_constant(n_0, n_1, coa_3)

ax-162 : trust(coa) $\neq$ code_of_start

ax-163 : trust(coa) $\neq$ nil'

ax-164 : trust(coa) $\neq$ mkcl(cl)

ax-165 : trust(coa) $\neq$ switch_on_structure(n, n_0, coa_0)

ax-166 : trust(coa) $\neq$ switch_on_constant(n, n_0, coa_0)

ax-167 : trust(coa) ≠ switch_on_term(n, coa_0, coa_1, coa_2, coa_3)
ax-168 : retry'(coa) ≠ code_of_start
ax-169 : retry'(coa) ≠ nil'
ax-17 : the_cl(mkcl(cl)) = cl
ax-170 : retry'(coa) ≠ mkcl(cl)
ax-171 : retry'(coa) ≠ switch_on_structure(n, n_0, coa_0)
ax-172 : retry'(coa) ≠ switch_on_constant(n, n_0, coa_0)
ax-173 : retry'(coa) ≠ switch_on_term(n, coa_0, coa_1, coa_2, coa_3)
ax-174 : retry'(coa) ≠ trust(coa_0)
ax-175 : try'(coa) ≠ code_of_start
ax-176 : try'(coa) ≠ nil'
ax-177 : try'(coa) ≠ mkcl(cl)
ax-178 : try'(coa) ≠ switch_on_structure(n, n_0, coa_0)
ax-179 : try'(coa) ≠ switch_on_constant(n, n_0, coa_0)
ax-18 : is_try_me(try_me_else(coa))
ax-180 : try'(coa) ≠ switch_on_term(n, coa_0, coa_1, coa_2, coa_3)
ax-181 : try'(coa) ≠ trust(coa_0)
ax-182 : try'(coa) ≠ retry'(coa_0)
ax-183 : trust_me ≠ code_of_start
ax-184 : trust_me ≠ nil'
ax-185 : trust_me ≠ mkcl(cl)
ax-186 : trust_me ≠ switch_on_structure(n, n_0, coa)
ax-187 : trust_me ≠ switch_on_constant(n, n_0, coa)
ax-188 : trust_me ≠ switch_on_term(n, coa, coa_0, coa_1, coa_2)
ax-189 : trust_me ≠ trust(coa)
ax-19 : ¬ is_try_me(retry_me_else(coa))
ax-190 : trust_me ≠ retry'(coa)
ax-191 : trust_me ≠ try'(coa)
ax-192 : retry_me_else(coa) ≠ code_of_start
ax-193 : retry_me_else(coa) ≠ nil'
ax-194 : retry_me_else(coa) ≠ mkcl(cl)
ax-195 : retry_me_else(coa) ≠ switch_on_structure(n, n_0, coa_0)
ax-196 : retry_me_else(coa) ≠ switch_on_constant(n, n_0, coa_0)
ax-197 : retry_me_else(coa)
≠ switch_on_term(n, coa_0, coa_1, coa_2, coa_3)
ax-198 : retry_me_else(coa) ≠ trust(coa_0)
ax-199 : retry_me_else(coa) ≠ retry'(coa_0)
ax-20 : ¬ is_try_me(trust_me)
ax-200 : retry_me_else(coa) ≠ try'(coa_0)
ax-201 : retry_me_else(coa) ≠ trust_me
ax-202 : try_me_else(coa) ≠ code_of_start
ax-203 : try_me_else(coa) ≠ nil'
ax-204 : try_me_else(coa) ≠ mkcl(cl)
ax-205 : try_me_else(coa) ≠ switch_on_structure(n, n_0, coa_0)
ax-206 : try_me_else(coa) ≠ switch_on_constant(n, n_0, coa_0)
ax-207 : try_me_else(coa)
≠ switch_on_term(n, coa_0, coa_1, coa_2, coa_3)
ax-208 : try_me_else(coa) ≠ trust(coa_0)
ax-209 : try_me_else(coa) ≠ retry'(coa_0)
ax-21 : ¬ is_try_me(try'(coa))
ax-210 : try_me_else(coa) ≠ try'(coa_0)
ax-211 : try_me_else(coa) ≠ trust_me
ax-212 : try_me_else(coa) ≠ retry_me_else(coa_0)
ax-213 : ioc = try_me_else(where(ioc))

$\vee$ ioc = retry_me_else(where(ioc))
$\vee$ ioc = trust_me $\vee$ ioc = try'(what(ioc))
$\vee$ ioc = retry'(what(ioc)) $\vee$ ioc = trust(what(ioc))
$\vee$ ioc = switch_on_term(argindex(ioc), vlabel(ioc),
clabel(ioc), llabel(ioc),
slabel(ioc))
$\vee$ ioc = switch_on_constant(argindex(ioc),
tabsize(ioc), table(ioc))
$\vee$ ioc = switch_on_structure(argindex(ioc),
tabsize(ioc), table(ioc))
$\vee$ ioc = mkcl(the_cl(ioc))
$\vee$ ioc = nil' $\vee$ ioc = code_of_start

ax-22 : $\neg$ is_try_me(retry'(coa))
ax-23 : $\neg$ is_try_me(trust(coa))
ax-24 : $\neg$ is_try_me(switch_on_term(n, coa, coa_0, coa_1, coa_2))
ax-25 : $\neg$ is_try_me(switch_on_constant(n, n_0, coa))
ax-26 : $\neg$ is_try_me(switch_on_structure(n, n_0, coa))
ax-27 : $\neg$ is_try_me(mkcl(cl))
ax-28 : $\neg$ is_try_me(nil')
ax-29 : $\neg$ is_try_me(code_of_start)
ax-30 : $\neg$ is_retry_me(try_me_else(coa))
ax-31 : is_retry_me(retry_me_else(coa))
ax-32 : $\neg$ is_retry_me(trust_me)
ax-33 : $\neg$ is_retry_me(try'(coa))
ax-34 : $\neg$ is_retry_me(retry'(coa))
ax-35 : $\neg$ is_retry_me(trust(coa))
ax-36 : $\neg$ is_retry_me(switch_on_term(n, coa, coa_0, coa_1, coa_2))
ax-37 : $\neg$ is_retry_me(switch_on_constant(n, n_0, coa))
ax-38 : $\neg$ is_retry_me(switch_on_structure(n, n_0, coa))
ax-39 : $\neg$ is_retry_me(mkcl(cl))
ax-40 : $\neg$ is_retry_me(nil')
ax-41 : $\neg$ is_retry_me(code_of_start)
ax-42 : $\neg$ is_trust_me(try_me_else(coa))
ax-43 : $\neg$ is_trust_me(retry_me_else(coa))
ax-44 : is_trust_me(trust_me)
ax-45 : $\neg$ is_trust_me(try'(coa))
ax-46 : $\neg$ is_trust_me(retry'(coa))
ax-47 : $\neg$ is_trust_me(trust(coa))
ax-48 : $\neg$ is_trust_me(switch_on_term(n, coa, coa_0, coa_1, coa_2))
ax-49 : $\neg$ is_trust_me(switch_on_constant(n, n_0, coa))
ax-50 : $\neg$ is_trust_me(switch_on_structure(n, n_0, coa))
ax-51 : $\neg$ is_trust_me(mkcl(cl))
ax-52 : $\neg$ is_trust_me(nil')
ax-53 : $\neg$ is_trust_me(code_of_start)
ax-54 : $\neg$ is_try(try_me_else(coa))
ax-55 : $\neg$ is_try(retry_me_else(coa))
ax-56 : $\neg$ is_try(trust_me)
ax-57 : is_try(try'(coa))
ax-58 : $\neg$ is_try(retry'(coa))
ax-59 : $\neg$ is_try(trust(coa))
ax-60 : $\neg$ is_try(switch_on_term(n, coa, coa_0, coa_1, coa_2))
ax-61 : $\neg$ is_try(switch_on_constant(n, n_0, coa))
ax-62 : $\neg$ is_try(switch_on_structure(n, n_0, coa))
ax-63 : $\neg$ is_try(mkcl(cl))

ax-64 : ¬ is_try(nil')
ax-65 : ¬ is_try(code_of_start)
ax-66 : ¬ is_retry(try_me_else(coa))
ax-67 : ¬ is_retry(retry_me_else(coa))
ax-68 : ¬ is_retry(trust_me)
ax-69 : ¬ is_retry(try'(coa))
ax-70 : is_retry(retry'(coa))
ax-71 : ¬ is_retry(trust(coa))
ax-72 : ¬ is_retry(switch_on_term(n, coa, coa_0, coa_1, coa_2))
ax-73 : ¬ is_retry(switch_on_constant(n, n_0, coa))
ax-74 : ¬ is_retry(switch_on_structure(n, n_0, coa))
ax-75 : ¬ is_retry(mkcl(cl))
ax-76 : ¬ is_retry(nil')
ax-77 : ¬ is_retry(code_of_start)
ax-78 : ¬ is_trust(try_me_else(coa))
ax-79 : ¬ is_trust(retry_me_else(coa))
ax-80 : ¬ is_trust(trust_me)
ax-81 : ¬ is_trust(try'(coa))
ax-82 : ¬ is_trust(retry'(coa))
ax-83 : is_trust(trust(coa))
ax-84 : ¬ is_trust(switch_on_term(n, coa, coa_0, coa_1, coa_2))
ax-85 : ¬ is_trust(switch_on_constant(n, n_0, coa))
ax-86 : ¬ is_trust(switch_on_structure(n, n_0, coa))
ax-87 : ¬ is_trust(mkcl(cl))
ax-88 : ¬ is_trust(nil')
ax-89 : ¬ is_trust(code_of_start)
ax-90 : ¬ is_sw_term(try_me_else(coa))
ax-91 : ¬ is_sw_term(retry_me_else(coa))
ax-92 : ¬ is_sw_term(trust_me)
ax-93 : ¬ is_sw_term(try'(coa))
ax-94 : ¬ is_sw_term(retry'(coa))
ax-95 : ¬ is_sw_term(trust(coa))
ax-96 : is_sw_term(switch_on_term(n, coa, coa_0, coa_1, coa_2))
ax-97 : ¬ is_sw_term(switch_on_constant(n, n_0, coa))
ax-98 : ¬ is_sw_term(switch_on_structure(n, n_0, coa))
ax-99 : ¬ is_sw_term(mkcl(cl))
th-1 (lem-1): is_try_me(ioc) → ¬ is_retry_me(ioc)
th-10 (lem-18): is_trust_me(ioc) → ¬ is_try(ioc)
th-11 (lem-19): is_trust(ioc) → ¬ is_retry(ioc)
th-12 (lem-2): is_try_me(ioc) → ¬ is_trust_me(ioc)
th-13 (lem-20): is_trust(ioc) → ¬ is_try(ioc)
th-14 (lem-21): is_retry(ioc) → ¬ is_try(ioc)
th-15 (lem-3): is_clause(ioc) → ¬ is_retry_me(ioc)
th-16 (lem-4): is_clause(ioc) → ¬ is_trust_me(ioc)
th-17 (lem-5): is_retry_me(ioc) → ¬ is_trust_me(ioc)
th-18 (lem-6): is_try_me(ioc) → ¬ is_clause(ioc)
th-19 (lem-64): is_try_me(ioc) → ¬ is_sw_struct(ioc)
th-2 (lem-10): is_clause(ioc) → ¬ is_trust(ioc)
th-20 (lem-65): is_retry_me(ioc) → ¬ is_sw_struct(ioc)
th-21 (lem-66): is_trust_me(ioc) → ¬ is_sw_struct(ioc)
th-22 (lem-67): is_try(ioc) → ¬ is_sw_struct(ioc)
th-23 (lem-68): is_retry(ioc) → ¬ is_sw_struct(ioc)
th-24 (lem-69): is_trust(ioc) → ¬ is_sw_struct(ioc)
th-25 (lem-7): is_try_me(ioc) → ¬ is_trust(ioc)

th-26 (lem-70): $\text{is\_clause}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_struct}(\text{ioc})$

th-27 (lem-71): $\text{is\_try\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_const}(\text{ioc})$

th-28 (lem-72): $\text{is\_retry\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_const}(\text{ioc})$

th-29 (lem-73): $\text{is\_trust\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_const}(\text{ioc})$

th-3 (lem-11): $\text{is\_clause}(\text{ioc}) \rightarrow \neg\ \text{is\_retry}(\text{ioc})$

th-30 (lem-74): $\text{is\_try}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_const}(\text{ioc})$

th-31 (lem-75): $\text{is\_retry}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_const}(\text{ioc})$

th-32 (lem-76): $\text{is\_trust}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_const}(\text{ioc})$

th-33 (lem-77): $\text{is\_clause}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_const}(\text{ioc})$

th-34 (lem-78): $\text{is\_try\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_term}(\text{ioc})$

th-35 (lem-79): $\text{is\_retry\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_term}(\text{ioc})$

th-36 (lem-8): $\text{is\_try\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_retry}(\text{ioc})$

th-37 (lem-80): $\text{is\_trust\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_term}(\text{ioc})$

th-38 (lem-81): $\text{is\_try}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_term}(\text{ioc})$

th-39 (lem-82): $\text{is\_retry}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_term}(\text{ioc})$

th-4 (lem-12): $\text{is\_clause}(\text{ioc}) \rightarrow \neg\ \text{is\_try}(\text{ioc})$

th-40 (lem-83): $\text{is\_trust}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_term}(\text{ioc})$

th-41 (lem-84): $\text{is\_clause}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_term}(\text{ioc})$

th-42 (lem-88): $\text{is\_sw\_const}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_struct}(\text{ioc})$

th-43 (lem-89): $\text{is\_sw\_term}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_struct}(\text{ioc})$

th-44 (lem-9): $\text{is\_try\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_try}(\text{ioc})$

th-45 (lem-90): $\text{is\_sw\_term}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_const}(\text{ioc})$

th-46 (elim-clause): $\text{is\_clause}(\text{ioc}) \rightarrow (\text{cl} = \text{the\_cl}(\text{ioc}) \leftrightarrow \text{ioc} = \text{mkcl}(\text{cl}))$

th-47 (elim-retry): $\text{is\_retry}(\text{ioc}) \rightarrow (\text{coa} = \text{what}(\text{ioc}) \leftrightarrow \text{ioc} = \text{retry'}(\text{coa}))$

th-48 (elim-retry_me): $\text{is\_retry\_me}(\text{ioc}) \rightarrow (\text{coa} = \text{where}(\text{ioc}) \leftrightarrow \text{ioc} = \text{retry\_me\_else}(\text{coa}))$

th-49 (elim-swconst): $\text{is\_sw\_const}(\text{ioc}) \rightarrow (\ \text{n} = \text{argindex}(\text{ioc}) \wedge \text{n}_0 = \text{tabsize}(\text{ioc}) \wedge \text{coa}_0 = \text{table}(\text{ioc}) \leftrightarrow \text{ioc} = \text{switch\_on\_constant}(\text{n}, \text{n}_0, \text{coa}_0))$

th-5 (lem-13): $\text{is\_retry\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_trust}(\text{ioc})$

th-50 (elim-swstruct): $\text{is\_sw\_struct}(\text{ioc}) \rightarrow (\ \text{n} = \text{argindex}(\text{ioc}) \wedge \text{n}_0 = \text{tabsize}(\text{ioc}) \wedge \text{coa}_0 = \text{table}(\text{ioc}) \leftrightarrow \text{ioc} = \text{switch\_on\_structure}(\text{n}, \text{n}_0, \text{coa}_0))$

th-51 (elim-swterm): $\text{is\_sw\_term}(\text{ioc}) \rightarrow (\ \text{n} = \text{argindex}(\text{ioc}) \wedge \text{coa}_0 = \text{vlabel}(\text{ioc}) \wedge \text{coa}_1 = \text{clabel}(\text{ioc}) \wedge \text{coa}_2 = \text{llabel}(\text{ioc}) \wedge \text{coa}_3 = \text{slabel}(\text{ioc}) \leftrightarrow \text{ioc} = \text{switch\_on\_term}(\text{n}, \text{coa}_0, \text{coa}_1, \text{coa}_2, \text{coa}_3))$

th-52 (elim-trust): $\text{is\_trust}(\text{ioc}) \rightarrow (\text{coa} = \text{what}(\text{ioc}) \leftrightarrow \text{ioc} = \text{trust}(\text{coa}))$

th-53 (elim-try): $\text{is\_try}(\text{ioc}) \rightarrow (\text{coa} = \text{what}(\text{ioc}) \leftrightarrow \text{ioc} = \text{try'}(\text{coa}))$

th-54 (elim-try_me): $\text{is\_try\_me}(\text{ioc}) \rightarrow (\text{coa} = \text{where}(\text{ioc}) \leftrightarrow \text{ioc} = \text{try\_me\_else}(\text{coa}))$

th-55 (lem-22): $\text{is\_retry\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_try\_me}(\text{ioc})$

th-56 (lem-23): $\text{is\_trust\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_try\_me}(\text{ioc})$

th-57 (lem-24): $\text{is\_retry\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_clause}(\text{ioc})$

th-58 (lem-25): $\text{is\_trust\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_clause}(\text{ioc})$

th-59 (lem-26): $\text{is\_trust\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_retry\_me}(\text{ioc})$

th-6 (lem-14): $\text{is\_retry\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_retry}(\text{ioc})$

th-60 (lem-27): $\text{is\_clause}(\text{ioc}) \rightarrow \neg\ \text{is\_try\_me}(\text{ioc})$

th-61 (lem-28): $\text{is\_trust}(\text{ioc}) \rightarrow \neg\ \text{is\_try\_me}(\text{ioc})$

th-62 (lem-29): $\text{is\_retry}(\text{ioc}) \rightarrow \neg\ \text{is\_try\_me}(\text{ioc})$

th-63 (lem-30): $\text{is\_try}(\text{ioc}) \rightarrow \neg\ \text{is\_try\_me}(\text{ioc})$

th-64 (lem-31): $\text{is\_trust}(\text{ioc}) \rightarrow \neg\ \text{is\_clause}(\text{ioc})$
th-65 (lem-32): $\text{is\_retry}(\text{ioc}) \rightarrow \neg\ \text{is\_clause}(\text{ioc})$
th-66 (lem-33): $\text{is\_try}(\text{ioc}) \rightarrow \neg\ \text{is\_clause}(\text{ioc})$
th-67 (lem-34): $\text{is\_trust}(\text{ioc}) \rightarrow \neg\ \text{is\_retry\_me}(\text{ioc})$
th-68 (lem-35): $\text{is\_retry}(\text{ioc}) \rightarrow \neg\ \text{is\_retry\_me}(\text{ioc})$
th-69 (lem-36): $\text{is\_try}(\text{ioc}) \rightarrow \neg\ \text{is\_retry\_me}(\text{ioc})$
th-7 (lem-15): $\text{is\_retry\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_try}(\text{ioc})$
th-70 (lem-37): $\text{is\_trust}(\text{ioc}) \rightarrow \neg\ \text{is\_trust\_me}(\text{ioc})$
th-71 (lem-38): $\text{is\_retry}(\text{ioc}) \rightarrow \neg\ \text{is\_trust\_me}(\text{ioc})$
th-72 (lem-39): $\text{is\_try}(\text{ioc}) \rightarrow \neg\ \text{is\_trust\_me}(\text{ioc})$
th-73 (lem-40): $\text{is\_retry}(\text{ioc}) \rightarrow \neg\ \text{is\_trust}(\text{ioc})$
th-74 (lem-41): $\text{is\_try}(\text{ioc}) \rightarrow \neg\ \text{is\_trust}(\text{ioc})$
th-75 (lem-42): $\text{is\_try}(\text{ioc}) \rightarrow \neg\ \text{is\_retry}(\text{ioc})$
th-76 (lem-43): $\text{is\_sw\_struct}(\text{ioc}) \rightarrow \neg\ \text{is\_try\_me}(\text{ioc})$
th-77 (lem-44): $\text{is\_sw\_struct}(\text{ioc}) \rightarrow \neg\ \text{is\_retry\_me}(\text{ioc})$
th-78 (lem-45): $\text{is\_sw\_struct}(\text{ioc}) \rightarrow \neg\ \text{is\_trust\_me}(\text{ioc})$
th-79 (lem-46): $\text{is\_sw\_struct}(\text{ioc}) \rightarrow \neg\ \text{is\_try}(\text{ioc})$
th-8 (lem-16): $\text{is\_trust\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_trust}(\text{ioc})$
th-80 (lem-47): $\text{is\_sw\_struct}(\text{ioc}) \rightarrow \neg\ \text{is\_retry}(\text{ioc})$
th-81 (lem-48): $\text{is\_sw\_struct}(\text{ioc}) \rightarrow \neg\ \text{is\_trust}(\text{ioc})$
th-82 (lem-49): $\text{is\_sw\_struct}(\text{ioc}) \rightarrow \neg\ \text{is\_clause}(\text{ioc})$
th-83 (lem-50): $\text{is\_sw\_term}(\text{ioc}) \rightarrow \neg\ \text{is\_try\_me}(\text{ioc})$
th-84 (lem-51): $\text{is\_sw\_term}(\text{ioc}) \rightarrow \neg\ \text{is\_retry\_me}(\text{ioc})$
th-85 (lem-52): $\text{is\_sw\_term}(\text{ioc}) \rightarrow \neg\ \text{is\_trust\_me}(\text{ioc})$
th-86 (lem-53): $\text{is\_sw\_term}(\text{ioc}) \rightarrow \neg\ \text{is\_try}(\text{ioc})$
th-87 (lem-54): $\text{is\_sw\_term}(\text{ioc}) \rightarrow \neg\ \text{is\_retry}(\text{ioc})$
th-88 (lem-55): $\text{is\_sw\_term}(\text{ioc}) \rightarrow \neg\ \text{is\_trust}(\text{ioc})$
th-89 (lem-56): $\text{is\_sw\_term}(\text{ioc}) \rightarrow \neg\ \text{is\_clause}(\text{ioc})$
th-9 (lem-17): $\text{is\_trust\_me}(\text{ioc}) \rightarrow \neg\ \text{is\_retry}(\text{ioc})$
th-90 (lem-57): $\text{is\_sw\_const}(\text{ioc}) \rightarrow \neg\ \text{is\_try\_me}(\text{ioc})$
th-91 (lem-58): $\text{is\_sw\_const}(\text{ioc}) \rightarrow \neg\ \text{is\_retry\_me}(\text{ioc})$
th-92 (lem-59): $\text{is\_sw\_const}(\text{ioc}) \rightarrow \neg\ \text{is\_trust\_me}(\text{ioc})$
th-93 (lem-60): $\text{is\_sw\_const}(\text{ioc}) \rightarrow \neg\ \text{is\_try}(\text{ioc})$
th-94 (lem-61): $\text{is\_sw\_const}(\text{ioc}) \rightarrow \neg\ \text{is\_retry}(\text{ioc})$
th-95 (lem-62): $\text{is\_sw\_const}(\text{ioc}) \rightarrow \neg\ \text{is\_trust}(\text{ioc})$
th-96 (lem-63): $\text{is\_sw\_const}(\text{ioc}) \rightarrow \neg\ \text{is\_clause}(\text{ioc})$
th-97 (lem-85): $\text{is\_sw\_struct}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_term}(\text{ioc})$
th-98 (lem-86): $\text{is\_sw\_struct}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_const}(\text{ioc})$
th-99 (lem-87): $\text{is\_sw\_const}(\text{ioc}) \rightarrow \neg\ \text{is\_sw\_term}(\text{ioc})$

4.18 Axioms and Theorems from list-data

ax-01 : $(a \oplus x).\text{first} = a$
ax-02 : $(a \oplus x).\text{rest} = x$
ax-03 : $a \oplus x = a_0 \oplus x_0 \leftrightarrow a = a_0 \wedge x = x_0$
ax-04 : $@ \neq a \oplus x$
ax-05 : $x = @ \vee x = x.\text{first} \oplus x.\text{rest}$
ax-06 : $\#(@) = 0$
ax-07 : $\#(a \oplus x) = \#(x)+1$
ax-08 : $@_p(@)$
ax-09 : $\neg\ @_p(a \oplus x)$
ax-10 : $\neg\ x \ll x$

ax-11 : $x \ll x_0 \wedge x_0 \ll x_1 \rightarrow x \ll x_1$

ax-12 : $\neg\ x \ll @$

ax-13 : $x_0 \ll a \oplus x \leftrightarrow x_0 = x \vee x_0 \ll x$

th-1 (ax-03a): $a \oplus x = a \oplus x_0 \leftrightarrow x = x_0$

th-10 (lfr): $n < \#(x) \rightarrow (x = x.\mathrm{first} \oplus y \leftrightarrow x.\mathrm{rest} = y)$

th-11 (lr-02): $x \neq @ \rightarrow \neg\ \#(x) < \#(x.\mathrm{rest})$

th-12 (ax-03b): $a \oplus x = a_0 \oplus x \leftrightarrow a = a_0$

th-13 (elim-list): $x \neq @ \rightarrow (a = x.\mathrm{first} \wedge y = x.\mathrm{rest} \leftrightarrow x = a \oplus y)$

th-14 (fr-01): $x \neq @ \wedge x \neq x.\mathrm{first} \oplus @$
$\rightarrow x.\mathrm{first} \oplus x.\mathrm{rest}.\mathrm{first} \oplus x.\mathrm{rest}.\mathrm{rest} = x$

th-15 (fr-03): $x.\mathrm{rest} = @ \wedge x \neq @ \rightarrow x.\mathrm{first} \oplus @ = x$

th-16 (fr-04): $x.\mathrm{first} = y.\mathrm{first}$
$\rightarrow (x.\mathrm{rest} = y.\mathrm{rest} \rightarrow x = y \vee x = @ \vee y = @)$

th-17 (lf): $0 + 1 < \#(x) \rightarrow x \neq x.\mathrm{first} \oplus @$

th-18 (lr): $\mathrm{true} \rightarrow (\#(x)+1 < \#(x.\mathrm{rest}) \rightarrow x = @)$

th-19 (lr-01): $x \neq @ \wedge y \neq @ \rightarrow \#(x.\mathrm{rest}) + \#(y.\mathrm{rest}) < \#(x) + \#(y)$

th-2 (fr-07): $x \neq @ \rightarrow (x = x.\mathrm{first} \oplus y \leftrightarrow x.\mathrm{rest} = y)$

th-20 (lr-03): $x \neq @ \wedge y \neq @ \rightarrow (\#(x.\mathrm{rest}) = \#(y.\mathrm{rest}) \leftrightarrow \#(x) = \#(y))$

th-21 (lr-04): $\#(x.\mathrm{rest}) < \#(x) \leftrightarrow x \neq @$

th-22 (lr-05): $\neg\ \#(x.\mathrm{rest})+1 < \#(x)$

th-23 (lr-06): $\#(x) = \#(x.\mathrm{rest})+1 \leftrightarrow x \neq @$

th-24 (ls-01): $\neg\ a \oplus x \ll x$

th-25 (ls-02): $\neg\ @ \ll x \leftrightarrow x = @$

th-26 (ls-trans1): $x \ll x_0 \wedge \neg\ x \ll x_1 \rightarrow \neg\ x_0 \ll x_1$

th-27 (ls-trans2): $x \ll x_0 \wedge \neg\ x_1 \ll x_0 \rightarrow \neg\ x_1 \ll x$

th-28 (p-fr): $a \oplus x = y \rightarrow a = y.\mathrm{first} \wedge x = y.\mathrm{rest}$

th-3 (fr-05): $x \neq @ \rightarrow (x \neq x.\mathrm{first} \oplus y \leftrightarrow x.\mathrm{rest} \neq y)$

th-4 (fr-02): $x \neq @ \rightarrow x.\mathrm{first} \oplus x.\mathrm{rest} = x$

th-5 (elim-list-c): $x \neq @ \rightarrow (a = x.\mathrm{first} \wedge y = x.\mathrm{rest} \leftrightarrow x = a \oplus y)$

th-6 (l): $\#(x) = 0 \leftrightarrow x = @$

th-7 (fr): $x \neq @ \wedge x \neq x.\mathrm{first} \oplus @$
$\rightarrow x.\mathrm{rest}.\mathrm{first} \oplus x.\mathrm{rest}.\mathrm{rest} = x.\mathrm{rest}$

th-8 (fr-06): $x \neq @ \wedge x.\mathrm{rest} = @ \rightarrow x.\mathrm{first} \oplus @ = x$

th-9 (lf-01): $\#(x) = 0 + 1 \leftrightarrow x = x.\mathrm{first} \oplus @$

4.19 Axioms and Theorems from list

ax-01 : $@ \odot x = x$

ax-02 : $a \oplus x \odot y = a \oplus (x \odot y)$

ax-03 : $a \in x \leftrightarrow (\exists\ y, z.\ x = y \odot a \oplus z)$

th-1 (app-a): $(x \odot y) \odot z = x \odot y \odot z$

th-10 (inapp): $a \in x \odot y \leftrightarrow \neg\ \neg\ (a \in x \vee a \in y)$

th-11 (in-04): $\neg\ a \in b \oplus x \leftrightarrow a \neq b \wedge \neg\ a \in x$

th-12 (in-03): $a \in b \oplus x \leftrightarrow \neg\ \neg\ (a = b \vee a \in x)$

th-13 (in-01): $a \in a \oplus x$

th-14 (inapp-03): $\neg\ a \in y \wedge \neg\ b \in x$
$\rightarrow (x \odot a \oplus z = y \odot b \oplus y' \leftrightarrow a = b \wedge x = y \wedge z = y')$

th-15 (lem-02): $x \ll y \wedge \neg\ a \in y \rightarrow \neg\ a \in x$

th-16 (lem-05): $\neg\ x \ll y \odot x \leftrightarrow y = @$

th-17 (app-07): $x \odot a \oplus @ = y \odot b \oplus @ \leftrightarrow x = y \wedge a = b$

th-18 (app-08): $x \neq @ \rightarrow x.\mathrm{first} \oplus (x.\mathrm{rest} \odot y) = x \odot y$

th-19 (app-09): $x \neq @ \rightarrow (x \odot y).\mathrm{rest} = x.\mathrm{rest} \odot y$

th-2 (app-06): $x \odot y = x \leftrightarrow y = @$

th-20 (app-10): $x \neq @ \rightarrow (x \odot y).\mathrm{first} = x.\mathrm{first}$

th-21 (in-06): $x.\mathrm{first} \in x \leftrightarrow x \neq @$

th-22 (in-09): $a \in b \oplus @ \leftrightarrow a = b$

th-23 (in-10): $\neg\, a \in x \rightarrow (a \in x.\mathrm{rest} \rightarrow x = @)$

th-24 (inapp-01): $\neg\, a \in x \rightarrow (x \odot a \oplus @ = y \odot a \oplus z \leftrightarrow y = x \wedge z = @)$

th-25 (inapp-02): $\neg\, a \in x \wedge \neg\, a \in z$
$\rightarrow (x \odot a \oplus y = z \odot a \oplus y' \leftrightarrow y = y' \wedge x = z)$

th-26 (lapp): $\#(x \odot y) = \#(x) + \#(y)$

th-27 (lem-01): $x \ll y \wedge a \in x \rightarrow a \in y$

th-28 (lem-08): $x \ll a \oplus (y \odot x)$

th-29 (app-01): $x \odot y = x \odot z \leftrightarrow y = z$

th-3 (app-04): $x = x \odot y \leftrightarrow y = @$

th-30 (app-03): $x \odot y = z \odot y \leftrightarrow x = z$

th-31 (app-11): $x \neq @ \wedge y \neq @ \wedge x.\mathrm{first} = y.\mathrm{first}$
$\rightarrow (x.\mathrm{rest} = y.\mathrm{rest} \odot z \leftrightarrow x = y \odot z)$

th-32 (app-12): $x \neq @ \rightarrow (x \odot y = x.\mathrm{first} \oplus @ \leftrightarrow x.\mathrm{rest} = @ \wedge y = @)$

th-33 (app-13): $a \neq b \wedge \neg\, b \in x \wedge \neg\, b \in x'$
$\rightarrow (\quad a \oplus (x \odot b \oplus y) = x' \odot b \oplus y'$
$\leftrightarrow a \oplus x = x' \wedge y = y')$

th-34 (firstoc): $a \in x \rightarrow (\exists\, y, z.\ x = y \odot a \oplus z \wedge \neg\, a \in y)$

th-35 (fw-01): $x \odot a \oplus @ = y \odot z \wedge \neg\, b \in x \wedge a \neq b$
$\rightarrow \neg\, b \in y \wedge \neg\, b \in z$

th-36 (fw-02): $b \oplus x = y \odot z \wedge \neg\, a \in x \wedge a \neq b \rightarrow \neg\, a \in y \wedge \neg\, a \in z$

th-37 (in-02): $a \neq b \rightarrow (a \in b \oplus x \leftrightarrow a \in x)$

th-38 (in-05): $\neg\, a \in x \wedge x \neq @ \rightarrow \neg\, a \in x.\mathrm{rest}$

th-39 (in-07): $\neg\, a \in x.\mathrm{rest} \rightarrow (a \in x \leftrightarrow a = x.\mathrm{first} \wedge x \neq @)$

th-4 (app): $x \odot @ = x$

th-40 (in-11): $a \in x \rightarrow x.\mathrm{first} \oplus x.\mathrm{rest} = x$

th-41 (inapp-04): $a \oplus x = y \odot a \oplus z \wedge \neg\, a \in x \rightarrow y = @ \wedge z = x$

th-42 (inapp-06): $b \oplus x = y \odot a \oplus z \wedge y \neq @ \rightarrow a \in x$

th-43 (lapp-01): $\#(x) = \#(z) \rightarrow (x \odot y = z \odot y' \leftrightarrow x = z \wedge y = y')$

th-44 (lapp-02): $a \oplus x = y \odot b \oplus z \rightarrow \neg\, \#(x) < \#(z)$

th-45 (lastdiv): $x \neq @ \rightarrow (\exists\, y, a.\ x = y \odot a \oplus @)$

th-46 (lastoc): $a \in x \rightarrow (\exists\, y, z.\ x = y \odot a \oplus z \wedge \neg\, a \in z)$

th-47 (lem-03): $x \ll a \oplus y \wedge \neg\, a \in y \rightarrow \neg\, a \in x$

th-48 (lem-04): $x \ll z \rightarrow x \ll y \odot z$

th-49 (lem-06): $x \neq @ \rightarrow (x \odot y = a \oplus @ \leftrightarrow x = a \oplus @ \wedge y = @)$

th-5 (app-02): $x \odot y = @ \leftrightarrow x = @ \wedge y = @$

th-50 (lem-07): $y \neq @ \rightarrow (x \odot y = a \oplus @ \leftrightarrow y = a \oplus @ \wedge x = @)$

th-51 (lem-09): $a \oplus (y \odot x) \neq x$

th-52 (lem-10): $\neg\, a \oplus (y \odot x) \ll x$

th-53 (lem-11): $a \oplus x \ll b \oplus y \wedge c \in x \rightarrow c \in y$

th-54 (lem-12): $x \ll b \oplus y \wedge a \in x \wedge a \neq b \rightarrow a \in y$

th-55 (ls-app-01): $x = y \odot z \rightarrow \neg\, x \ll y$

th-56 (ls-app-02): $x = y \odot z \rightarrow \neg\, x \ll z$

th-57 (p-app): $x \odot a \oplus @ = y \odot b \oplus @ \rightarrow x = y \wedge a = b$

th-58 (p-app-01): $x \odot a \oplus @ = y \odot b \oplus @ \rightarrow x = y$

th-59 (p-app-02): $a \oplus x = y \odot a \oplus z \wedge \neg\, a \in y \rightarrow y = @$

th-6 (app-05): $x = y \odot x \leftrightarrow y = @$

th-60 (p-app-03): $x \odot a \oplus @ = y \odot a \oplus z \wedge \neg\, a \in x \rightarrow y = x$

th-61 (p-app-04): $x \odot a \oplus @ = y \odot a \oplus z \wedge \neg\, a \in x \rightarrow z = @$

th-62 (p-lapp): $\#(a \oplus x \odot b \oplus y) = \#(b \oplus x \odot a \oplus y)$

th-63 (someoc): $a \in x \rightarrow (\exists\, y, z.\ x = y \odot a \oplus z)$

th-7 (in): $\neg\ a \in @$

th-8 (inapp-05): $a \in x \odot y \leftrightarrow a \in x \vee a \in y$

th-9 (in-08): $\neg\ a \in x \rightarrow (a \in b \oplus x \leftrightarrow a = b)$

4.20 Axioms and Theorems from mode

ax-01 : $\text{select} \neq \text{call}$

ax-02 : $\text{mode} = \text{select} \vee \text{mode} = \text{call}$

th-1 (lem-01): $\text{mode} \neq \text{select} \leftrightarrow \text{mode} = \text{call}$

4.21 Axioms and Theorems from nat-basic1

ax-01 : $n +1 -1 = n$

ax-02 : $n +1 = n_0 +1 \leftrightarrow n = n_0$

ax-03 : $0 \neq n +1$

ax-04 : $n = 0 \vee n = n -1 +1$

ax-05 : $\neg\ n < n$

ax-06 : $n < n_0 \wedge n_0 < n_1 \rightarrow n < n_1$

ax-07 : $\neg\ n < 0$

ax-08 : $n_0 < n +1 \leftrightarrow n_0 = n \vee n_0 < n$

th-1 (lem-14): $m \neq n \rightarrow (\neg\ m < n \leftrightarrow n < m)$

th-10 (lem-09): $m \neq 0 \rightarrow (\neg\ m -1 < n \leftrightarrow n < m)$

th-11 (lem-16): $\neg\ m -1 +1 < m$

th-12 (lem-32): $m -1 < m \leftrightarrow m \neq 0$

th-13 (lem-24): $m_0 < m \rightarrow (\neg\ m -1 < n \leftrightarrow n < m \wedge m \neq 0)$

th-14 (elim-pred): $n \neq 0 \rightarrow (m = n -1 \leftrightarrow n = m +1)$

th-15 (lem-04): $n \neq 0 \wedge n \neq 0 +1$
$\rightarrow (n -1 -1 < m \leftrightarrow n < m +1 +1)$

th-16 (lem-13): $\text{true} \rightarrow (m < m -1 \rightarrow m = 0)$

th-17 (lem-15): $m \neq m +1$

th-18 (lem-22): $m \neq 0 \rightarrow (m -1 = n \leftrightarrow m = n +1)$

th-19 (lem-25): $n_0 < n \rightarrow (n -1 < m \leftrightarrow \neg\ m < n \wedge n \neq 0)$

th-2 (lem-03): $m \neq 0 \rightarrow m -1 +1 = m$

th-20 (lem-27): $m \neq 0 \rightarrow (\neg\ 0 +1 < m \leftrightarrow m = 0 +1)$

th-21 (lem-30): $\neg\ n < m -1 \rightarrow (n +1 < m \leftrightarrow \text{false})$

th-22 (lem-18): $\neg\ m +1 < m$

th-23 (elim-pred-c): $n \neq 0 \rightarrow (m = n -1 \leftrightarrow n = m +1)$

th-24 (lem-01): $m \neq 0 \wedge m \neq 0 +1 \rightarrow m -1 -1 +1 = m -1$

th-25 (lem-05): $n \neq 0 \wedge n \neq 0 +1 \rightarrow (m < n -1 -1 \leftrightarrow m +1 +1 < n)$

th-26 (lem-06): $m \neq 0 \rightarrow (m -1 < n \leftrightarrow \neg\ n < m)$

th-27 (lem-10): $\text{true} \rightarrow (\neg\ m -1 -1 < m \rightarrow m = 0 \vee m = 0 +1)$

th-28 (lem-11): $\text{true} \rightarrow (m +1 < m -1 \rightarrow m = 0)$

th-29 (lem-12): $\text{true} \rightarrow (m = m -1 \rightarrow m = 0)$

th-3 (lem-29): $0 < m \leftrightarrow m \neq 0$

th-30 (lem-21): $\neg\ n < m \wedge n \neq 0 \rightarrow (n -1 < m \leftrightarrow m = n)$

th-31 (lem-23): $m \neq m +1 +1$

th-32 (lem-26): $m < n \rightarrow (\neg\ m +1 < n \leftrightarrow n = m +1)$

th-33 (lem-28): $n_0 < m \rightarrow (m -1 = n \leftrightarrow m = n +1)$

th-34 (lem-33): $m < n_0 \rightarrow (n < n_0 -1 \leftrightarrow n +1 < n_0)$

th-35 (lem-34): $m < 0 +1 \leftrightarrow m = 0$

th-36 (lem-35): $0 +1 < m \leftrightarrow \neg\ (m = 0 \vee m = 0 +1)$

th-37 (lem-36): $m \neq n \rightarrow (m < n+1 \leftrightarrow m < n)$
th-38 (prop-03): $m-1 = n-1 \wedge m \neq 0 \wedge n \neq 0 \rightarrow m = n$
th-39 (prop-04): $m+1 < n+1 \leftrightarrow m < n$
th-4 (lem-20): $m < n+1 \leftrightarrow \neg\, n < m$
th-40 (prop-05): $n \neq 0 \wedge m < n-1 \rightarrow m < n$
th-41 (q-01): $n \neq 0 \wedge n \neq 0+1 \rightarrow (\exists\ m.\ m+1+1 = n)$
th-42 (lem-17): $m < m+1$
th-43 (prop-01): $m < n \wedge \neg\, n_0 < n \rightarrow m < n_0$
th-44 (prop-02): $m+1 < n \rightarrow m < n$
th-5 (lem-07): $n \neq 0 \rightarrow (\neg\, m < n-1 \leftrightarrow \neg\, m+1 < n)$
th-6 (lem-19): $n-1+1 \neq n \leftrightarrow n = 0$
th-7 (lem-31): $\neg\, m < 0$
th-8 (lem-02): $n < m \rightarrow m-1+1 = m$
th-9 (lem-08): $n \neq 0 \rightarrow (m < n-1 \leftrightarrow m+1 < n)$

4.22 Axioms and Theorems from nat-basic2

ax-01 : $n + 0 = n$
ax-02 : $m + n + 1 = (m + n)+1$
ax-03 : $n < n_0 \vee n = n_0 \vee n_0 < n$
th-1 (prop-01): $m + 1 + n = (m + n)+1$
th-10 (lem-01): $(m + n)+1 < m + n_0 \leftrightarrow n + 1 < n_0$
th-11 (lem-04): $n \neq 0 \rightarrow m + n - 1 = (m + n)-1$
th-12 (lem-06): $m < m_0 \wedge n < n_0 \rightarrow (\neg\, m + n < m_0 + n_0 \leftrightarrow \text{false})$
th-13 (lem-12): $m < m + n \leftrightarrow n \neq 0$
th-14 (lem-34): $(n + n)+1 \neq m + m$
th-15 (lem-05): $n \neq 0 \wedge n \neq 0+1 \rightarrow m + n - 1 - 1 = (m + n)-1-1$
th-16 (lem-07): $m = m + n \leftrightarrow n = 0$
th-17 (lem-08): $m \neq 0 \rightarrow (m + n < m - 1 \leftrightarrow \text{false})$
th-18 (lem-11): $m + n < (m + n_0)+1 \leftrightarrow n < n_0 + 1$
th-19 (lem-13): $m + 1 < m + n \leftrightarrow 0 + 1 < n$
th-2 (prop-02): $0 + m = m$
th-20 (lem-15): $m + m < n + n \leftrightarrow m < n$
th-21 (lem-17): $m + n = m + n_0 \leftrightarrow n = n_0$
th-22 (lem-20): $n \neq 0 \rightarrow (m + n = 0+1 \leftrightarrow n = 0+1 \wedge m = 0)$
th-23 (lem-28): $m_0 < m \rightarrow n_0 + (m + n)-1 = ((n_0 + m) + n)-1$
th-24 (lem-29): $\neg\, m + n < m$
th-25 (lem-35): $m + m \neq 0+1$
th-26 (less-01): $\neg\, n_0 < n \wedge \neg\, n + m < m_0 \rightarrow \neg\, n_0 + m < m_0$
th-27 (less-02): $\neg\, n_0 + n_1 + n_2 < m \wedge \neg\, n < n_2 \rightarrow \neg\, n_0 + n_1 + n < m$
th-28 (less-03): $\neg\, m_0 < n_1 \wedge n + n_0 < m \rightarrow n + n_0 + n_1 < m + m_0$
th-29 (less-04): $m + m_0 < n + n_0 \wedge \neg\, m < n \rightarrow m_0 < n_0$
th-3 (lem-21): $m + n = n + m$
th-30 (less-05): $m < m_0 \wedge \neg\, n_0 < n \rightarrow m + n < m_0 + n_0$
th-31 (less-06): $\neg\, (m + n)+1 + 1 < m$
th-32 (prop-03): $m + n = 0+1 \leftrightarrow m = 0 \wedge n = 0+1 \vee n = 0 \wedge m = 0+1$
th-33 (prop-04): $(m + n)+1 = m + n + 1$
th-34 (q-01): $\exists\ n.\ m = n + n \vee m = (n + n)+1$
th-35 (subadd-01): $\neg\, n < m \rightarrow (\exists\ k.\ m + k = n)$
th-36 (lem-27): $m + m_0 < n + n_0 \rightarrow (\neg\, m + m_0 < n + n_0 \leftrightarrow \text{false})$
th-4 (lem-10): $(m + n_0) + n_1 = m + n_0 + n_1$
th-5 (lem-09): $m + n = 0 \leftrightarrow m = 0 \wedge n = 0$

th-6 (lem-02): $(m + n)+1 = m + n_0 \leftrightarrow n +1 = n_0$
th-7 (lem-03): $m + n < m + n_0 \leftrightarrow n < n_0$
th-8 (lem-30): $\neg\ (m + n)+1 < m$
th-9 (lem-33): $n + n = m + m \leftrightarrow n = m$

4.23 Axioms and Theorems from nat-lec

ax-01 : $1 = 0 +1$
ax-02 : $0 \neq 1$
ax-03 : $2 = 0 +1 +1$
ax-04 : $2 \neq 0$
ax-05 : $2 \neq 1$
ax-06 : $m \leq n \leftrightarrow \neg\ n < m$
ax-07 : $m > n \leftrightarrow n < m$
ax-08 : $m \geq n \leftrightarrow \neg\ m < n$

4.24 Axioms and Theorems from nat-sub

ax-01 : $m - 0 = m$
ax-02 : $m - n +1 = (m - n)-1$
th-1 (add-01): $m + n - m = n$
th-10 (less-01): $\neg\ m < n \wedge \neg\ n < n_1 \rightarrow \neg\ m + n_0 < (m - n) + n_0 + n_1$
th-11 (less-02): $\neg\ m < n \wedge n_0 < n \rightarrow (m - n) + n_0 < m$
th-12 (less-03): $m - n_0 < m - n_1 \wedge \neg\ m < n_0 \wedge \neg\ m < n_1 \rightarrow n_1 < n_0$
th-13 (ls): $\neg\ m < n \wedge \neg\ m < n_0 \rightarrow (m - n < m - n_0 \leftrightarrow n_0 < n)$
th-14 (ls-02): $\neg\ m < n \rightarrow (m - n < m \leftrightarrow n \neq 0)$
th-15 (pred-01): $n < m \rightarrow (m - n)-1 +1 = m - n$
th-16 (prop): $n < m \rightarrow m -1 - n = m - n +1$
th-17 (prop-01): $\neg\ m < n \rightarrow (m - n) + n = m$
th-18 (prop-02): $\neg\ m < n \rightarrow (m - n)+1 = m +1 - n$
th-19 (prop-03): $\neg\ m < n \rightarrow m +1 - n = (m - n)+1$
th-2 (ls-01): $n < m \rightarrow 0 < m - n$
th-20 (prop-04): $m < m - n \rightarrow m < n$
th-21 (prop-05): $\neg\ n < m \rightarrow (n - m) + n_0 = n + n_0 - m$
th-22 (prop-06): $m \neq 0 \wedge \neg\ n +1 < m \rightarrow n +1 - m = n - m -1$
th-23 (sub): $m - m = 0$
th-24 (sub-01): $n < m \rightarrow m - n \neq 0$
th-25 (succ): $n +1 - n = 0 +1$
th-26 (succ-02): $m +1 = m_0 \rightarrow (m + n)+1 - m_0 = n$
th-27 (succ-03): $n +1 = n_0 \wedge \neg\ m_0 < n_0 \rightarrow ((m + m_0)+1 - n_0 \neq m_0 + m - n \leftrightarrow \text{false})$
th-28 (succ-04): $\neg\ m < n \rightarrow (m +1 - n)-1 = m - n$
th-29 (succ-05): $\neg\ m < n \rightarrow (m +1 - n)-1 = m - n$
th-3 (add-03): $(m + n)+1 - n = m +1$
th-4 (elim-sub): $\neg\ m < n \rightarrow (k = m - n \leftrightarrow m = k + n)$
th-5 (add): $\neg\ n < n_0 \rightarrow (m < n - n_0 \leftrightarrow m + n_0 < n)$
th-6 (add-02): $\neg\ m < n \rightarrow ((m - n) + n_0 = m \leftrightarrow n = n_0)$
th-7 (pred): $n < m \rightarrow m -1 - n = (m - n)-1$
th-8 (succ-01): $\neg\ m < n \rightarrow m +1 - n = (m - n)+1$
th-9 (less): $\neg\ m < n \wedge n_1 < n \rightarrow (m - n) + n_0 + n_1 < m + n_0$

4.25 Axioms and Theorems from pair

ax-01 : mkpair(a', a").p1 = a'
ax-02 : mkpair(a', a").p2 = a"
ax-03 : mkpair(a', a") = mkpair(a'$_0$, a"$_0$) $\leftrightarrow$ a' = a'$_0$ $\wedge$ a" = a"$_0$
ax-04 : mkpair(p.p1, p.p2) = p
th-1 (elim-pair): a' = p.p1 $\wedge$ a" = p.p2 $\leftrightarrow$ p = mkpair(a', a")
th-2 (lem-1): p = mkpair(p.p1, a") $\leftrightarrow$ p.p2 = a"
th-3 (lem-2): p = mkpair(a', p.p2) $\leftrightarrow$ p.p1 = a'
th-4 (lem-3): mkpair(a', a") = mkpair(a'$_0$, a") $\leftrightarrow$ a' = a'$_0$
th-5 (lem-4): mkpair(a', a") = mkpair(a', a"$_0$) $\leftrightarrow$ a" = a"$_0$

4.26 Axioms and Theorems from paramterm

ax-01 : ! $\neq$ true'
ax-02 : ! $\neq$ fail'
ax-03 : true' $\neq$ fail'
ax-04 : is_user_defined(trm) $\leftrightarrow$ trm $\neq$ true' $\wedge$ trm $\neq$ fail' $\wedge$ trm $\neq$!
th-1 (lem-1): $\neg$ is_user_defined(!)
th-2 (lem-2): $\neg$ is_user_defined(true')
th-3 (lem-3): $\neg$ is_user_defined(fail')
th-4 (lem-4): $\neg$ is_user_defined(trm) $\leftrightarrow$ trm = ! $\vee$ trm = true' $\vee$ trm = fail'
th-5 (lem-5): is_user_defined(trm) $\rightarrow$ trm $\neq$!
th-6 (lem-6): is_user_defined(trm) $\rightarrow$ trm $\neq$ true'
th-7 (lem-7): is_user_defined(trm) $\rightarrow$ trm $\neq$ fail'

4.27 Axioms and Theorems from rmode

ax-01 : enter $\neq$ call'
ax-02 : retry $\neq$ call'
ax-03 : retry $\neq$ enter
ax-04 : try $\neq$ call'
ax-05 : try $\neq$ enter
ax-06 : try $\neq$ retry
ax-07 : rmode = try $\vee$ rmode = retry $\vee$ rmode = enter $\vee$ rmode = call'
th-1 (lem-01): rmode $\neq$ call' $\wedge$ rmode $\neq$ retry $\rightarrow$ (rmode $\neq$ try $\leftrightarrow$ rmode = enter)
th-2 (lem-03): rmode $\neq$ call' $\wedge$ rmode $\neq$ enter $\rightarrow$ (rmode $\neq$ try $\leftrightarrow$ rmode = retry)
th-3 (lem-02): rmode $\neq$ call' $\wedge$ rmode $\neq$ retry $\rightarrow$ (rmode $\neq$ enter $\leftrightarrow$ rmode = try)
th-4 (lem-04): rmode $\neq$ enter $\wedge$ rmode $\neq$ retry $\rightarrow$ (rmode $\neq$ try $\leftrightarrow$ rmode = call')

4.28 Axioms and Theorems from set

ax-01 : $s = s' \leftrightarrow (\forall\, a.\ a \in s \leftrightarrow a \in s')$
ax-02 : $\neg\ a \in \emptyset$
ax-03 : $a \in s +\!\!+ b \leftrightarrow a = b \lor a \in s$
ax-04 : $a\ ' = \emptyset +\!\!+ a$
ax-05 : $\#(\emptyset) = 0$
ax-06 : $\neg\ a \in s \rightarrow \#(s +\!\!+ a) = \#(s)+1$
ax-07 : $a \in s - b \leftrightarrow a \neq b \land a \in s$
ax-08 : $s \subseteq s' \leftrightarrow (\forall\, a.\ a \in s \rightarrow a \in s')$
th-1 (lem-01): $a \in s +\!\!+ a$
th-10 (lem-14): $s - b - a = s - a - b \leftrightarrow \text{true}$
th-11 (lem-16): $\neg\ a \in s \rightarrow (a \in s +\!\!+ b \leftrightarrow a = b)$
th-12 (lem-23): $s +\!\!+ a - a = s - a$
th-13 (lem-25): $a \in \emptyset +\!\!+ b \leftrightarrow a = b$
th-14 (lem-27): $\neg\ a \in s - a$
th-15 (lem-29): $\emptyset \subseteq s$
th-16 (prop-01): $a \neq b \rightarrow s +\!\!+ b - a = s - a +\!\!+ b$
th-17 (lem-03): $a \in s \rightarrow s - a +\!\!+ a = s$
th-18 (lem-07): $\neg\ a \in s \rightarrow (a \in s - a' +\!\!+ b +\!\!+ c \leftrightarrow a = b \lor a = c)$
th-19 (lem-10): $s \neq \emptyset \rightarrow (\#(s)-1 = n \leftrightarrow \#(s) = n+1)$
th-2 (add-01): $a \in s +\!\!+ b \leftrightarrow \neg\ \neg\ (a = b \lor a \in s)$
th-20 (lem-11): $\#(s)+1 = \#(s +\!\!+ a) \leftrightarrow \neg\ a \in s$
th-21 (lem-13): $a \in s \land a \neq b \rightarrow s - b +\!\!+ a = s - b$
th-22 (lem-15): $s - a = \emptyset \leftrightarrow s = \emptyset \lor s = \emptyset +\!\!+ a$
th-23 (lem-18): $\emptyset - a = \emptyset$
th-24 (lem-19): $s - a - a = s - a$
th-25 (lem-20): $s - a - b - a = s - b - a$
th-26 (lem-21): $\neg\ a \in s \rightarrow s +\!\!+ a - a = s$
th-27 (lem-22): $\#(s) = 0 \leftrightarrow s = \emptyset$
th-28 (lem-24): $a \in s \rightarrow \#(s - a) = \#(s)-1$
th-29 (lem-26): $\emptyset \neq s +\!\!+ a$
th-3 (lem-06): $\neg\ a \in s +\!\!+ b \leftrightarrow a \neq b \land \neg\ a \in s$
th-30 (lem-28): $s +\!\!+ a +\!\!+ a = s +\!\!+ a$
th-31 (lem-30): $s \subseteq s$
th-32 (lem-31): $a \in s \land s \subseteq s' \rightarrow a \in s'$
th-33 (prop-02): $s +\!\!+ b +\!\!+ a = s +\!\!+ a +\!\!+ b$
th-34 (prop-03): $s - b - a = s - a - b$
th-35 (prop-04): $s +\!\!+ a +\!\!+ b +\!\!+ c = s +\!\!+ c +\!\!+ a +\!\!+ b$
th-36 (q-01): $(\forall\, a.\ a \in s \rightarrow a = b) \land s \neq \emptyset \rightarrow s = \emptyset +\!\!+ b$
th-37 (selax): $s = \emptyset \lor (\exists\, a.\ a \in s)$
th-38 (sub-03): $a \in s' \rightarrow (s +\!\!+ a \subseteq s' \leftrightarrow s \subseteq s')$
th-39 (sub-04): $s \subseteq s' \rightarrow s - a \subseteq s' - a$
th-4 (lem-04): $a \in s \rightarrow s +\!\!+ a = s$
th-40 (sub-05): $s \subseteq s' \rightarrow s - a +\!\!+ b \subseteq s' - a +\!\!+ b$
th-41 (sub-06): $s \subseteq s' \rightarrow s - a +\!\!+ b +\!\!+ c \subseteq s' - a +\!\!+ b +\!\!+ c$
th-42 (sub-07): $s \subseteq s' \land \neg\ a \in s \rightarrow s \subseteq s' - a$
th-43 (sub-08): $s \subseteq s' \land \neg\ a \in s \rightarrow s \subseteq s' - a +\!\!+ b$
th-44 (sub-09): $s \subseteq s' \land \neg\ a \in s \rightarrow s \subseteq s' - a +\!\!+ b +\!\!+ c$
th-45 (sub-10): $s \subseteq s +\!\!+ a$
th-46 (sub-11): $s - a \subseteq s$
th-5 (lem-12): $a \neq b \rightarrow (a \in s +\!\!+ b \leftrightarrow a \in s)$
th-6 (lem-17): $a \neq b \rightarrow (\neg\ a \in s - b \leftrightarrow \neg\ a \in s)$
th-7 (lem-02): $\neg\ a \in s \rightarrow s - a = s$

th-8 (lem-08): $\neg\ a \in s \rightarrow (a \in s - b +\!+ c \leftrightarrow a = c)$
th-9 (lem-09): $a \in s \rightarrow (\#(s) - 1 = n \leftrightarrow \#(s) = n + 1)$

4.29 Axioms and Theorems from ssubres

ax-01 : ssubres(sdnil, su) = sdnil
ax-02 : ssubres(mksdecgoal(go, st) $+_{sdl}$ sdgl, su)
= mksdecgoal(su $\hat{}_{sg}$ go, st) $+_{sdl}$ ssubres(sdgl, su)

4.30 Axioms and Theorems from stopmode

ax-01 : failure $\neq$ run
ax-02 : success $\neq$ run
ax-03 : success $\neq$ failure
ax-04 : stop = success $\vee$ stop = failure $\vee$ stop = run
th-1 (lem-03): stop $\neq$ run $\rightarrow$ (stop $\neq$ failure $\leftrightarrow$ stop = success)
th-2 (lem-01): stop $\neq$ success $\rightarrow$ (stop $\neq$ failure $\leftrightarrow$ stop = run)
th-3 (lem-02): stop $\neq$ success $\rightarrow$ (stop $\neq$ run $\leftrightarrow$ stop = failure)

4.31 Axioms and Theorems from sublist

ax-01 : @ subli x
ax-02 : $\neg$ a $\oplus$ x subli @
ax-03 : a $\oplus$ x subli b $\oplus$ y
$\leftrightarrow$ a = b $\wedge$ x subli y $\vee$ a $\neq$ b $\wedge$ a $\oplus$ x subli y

4.32 Axioms and Theorems from subres

ax-01 : subres(dnil, su) = dnil
ax-02 : subres(mkdecgoal(go, no) $+_{dl}$ dgl, su)
= mkdecgoal(su $\hat{}_{sg}$ go, no) $+_{dl}$ subres(dgl, su)

4.33 Axioms and Theorems from subst

ax-01 : (su o su_1) o su_2 = su o su_1 o su_2
ax-02 : su o $@_{su}$ = su
ax-03 : $@_{su}$ o su = su

4.34 Axioms and Theorems from substgoal

ax-01 : su $\hat{}_{sg}$ gnil = gnil
ax-02 : su $\hat{}_{sg}$ trm $+_g$ go = (su $\hat{}_t$ trm) $+_g$ su $\hat{}_{sg}$ go

4.35 Axioms and Theorems from substorfail

ax-01 : the_subst(oksubst(su)) = su
ax-02 : oksubst(su) = oksubst(su_0) $\leftrightarrow$ su = su_0
ax-03 : oksubst(su) $\neq$ fail
ax-04 : subst = oksubst(the_subst(subst)) $\vee$ subst = fail

4.36 Axioms and Theorems from term

ax-01 : funct(struct(at, trmli)) = at
ax-02 : args(struct(at, trmli)) = trmli
ax-03 : constsym(mkconst(at)) = at
ax-04 : varsym(mkvar(va)) = va
ax-05 : thelist(mklist(trmli)) = trmli
ax-06 : and_only(the_one(trm)) = trm
ax-07 : tcar(tcons(trm, trmli)) = trm
ax-08 : tcdr(tcons(trm, trmli)) = trmli
ax-09 : is_struct(struct(at, trmli))
ax-10 : $\neg$ is_struct(mkconst(at))
ax-11 : $\neg$ is_struct(mkvar(va))
ax-12 : $\neg$ is_struct(mklist(trmli))
ax-13 : $\neg$ is_struct(!)
ax-14 : $\neg$ is_struct(true')
ax-15 : $\neg$ is_struct(fail')
ax-16 : $\neg$ is_const(struct(at, trmli))
ax-17 : is_const(mkconst(at))
ax-18 : $\neg$ is_const(mkvar(va))
ax-19 : $\neg$ is_const(mklist(trmli))
ax-20 : $\neg$ is_const(!)
ax-21 : $\neg$ is_const(true')
ax-22 : $\neg$ is_const(fail')
ax-23 : $\neg$ is_var(struct(at, trmli))
ax-24 : $\neg$ is_var(mkconst(at))
ax-25 : is_var(mkvar(va))
ax-26 : $\neg$ is_var(mklist(trmli))
ax-27 : $\neg$ is_var(!)
ax-28 : $\neg$ is_var(true')
ax-29 : $\neg$ is_var(fail')
ax-30 : $\neg$ is_list(struct(at, trmli))
ax-31 : $\neg$ is_list(mkconst(at))
ax-32 : $\neg$ is_list(mkvar(va))
ax-33 : is_list(mklist(trmli))
ax-34 : $\neg$ is_list(!)
ax-35 : $\neg$ is_list(true')
ax-36 : $\neg$ is_list(fail')
ax-37 : struct(at, trmli) = struct(at_0, $trmli_0$)
$\leftrightarrow$ at = at_0 $\wedge$ trmli = $trmli_0$
ax-38 : mkconst(at) = mkconst(at_0) $\leftrightarrow$ at = at_0
ax-39 : mkvar(va) = mkvar(va_0) $\leftrightarrow$ va = va_0
ax-40 : mklist(trmli) = mklist($trmli_0$) $\leftrightarrow$ trmli = $trmli_0$
ax-41 : the_one(trm) = the_one(trm_0) $\leftrightarrow$ trm = trm_0
ax-42 : tcons(trm, trmli) = tcons(trm_0, $trmli_0$)

$\leftrightarrow$ trm = $\text{trm}_0 \wedge$ trmli = trmli_0

ax-43 : true' $\neq$ fail'

ax-44 : ! $\neq$ fail'

ax-45 : ! $\neq$ true'

ax-46 : mklist(trmli) $\neq$ fail'

ax-47 : mklist(trmli) $\neq$ true'

ax-48 : mklist(trmli) $\neq$!

ax-49 : mkvar(va) $\neq$ fail'

ax-50 : mkvar(va) $\neq$ true'

ax-51 : mkvar(va) $\neq$!

ax-52 : mkvar(va) $\neq$ mklist(trmli)

ax-53 : mkconst(at) $\neq$ fail'

ax-54 : mkconst(at) $\neq$ true'

ax-55 : mkconst(at) $\neq$!

ax-56 : mkconst(at) $\neq$ mklist(trmli)

ax-57 : mkconst(at) $\neq$ mkvar(va)

ax-58 : struct(at, trmli) $\neq$ fail'

ax-59 : struct(at, trmli) $\neq$ true'

ax-60 : struct(at, trmli) $\neq$!

ax-61 : struct(at, trmli) $\neq$ mklist(trmli_0)

ax-62 : struct(at, trmli) $\neq$ mkvar(va)

ax-63 : struct(at, trmli) $\neq$ mkconst(at_0)

ax-64 : the_one(trm) $\neq$ tcons(trm_0, trmli)

ax-65 : trm = struct(funct(trm), args(trm))
$\vee$ trm = mkconst(constsym(trm))
$\vee$ trm = mkvar(varsym(trm))
$\vee$ trm = mklist(thelist(trm))
$\vee$ trm = ! $\vee$ trm = true' $\vee$ trm = fail'

ax-66 : trmli = the_one(and_only(trmli))
$\vee$ trmli = tcons(tcar(trmli), tcdr(trmli))

ax-67 : tlen(the_one(trm)) = 0

ax-68 : tlen(tcons(trm, trmli)) = tlen(trmli)+1

th-1 (lem-01): is_const(trm) $\rightarrow \neg$ is_struct(trm)

th-10 (el-var): is_var(trm) $\rightarrow$ (va = varsym(trm) $\leftrightarrow$ trm = mkvar(va))

th-11 (lem-04): is_struct(trm) $\rightarrow \neg$ is_const(trm)

th-12 (lem-07): is_var(trm) $\rightarrow \neg$ is_const(trm)

th-13 (lem-08): is_var(trm) $\rightarrow \neg$ is_struct(trm)

th-14 (lem-10): is_list(trm) $\rightarrow \neg$ is_const(trm)

th-15 (lem-11): is_list(trm) $\rightarrow \neg$ is_struct(trm)

th-16 (lem-12): is_list(trm) $\rightarrow \neg$ is_var(trm)

th-2 (lem-02): is_const(trm) $\rightarrow \neg$ is_var(trm)

th-3 (lem-03): is_const(trm) $\rightarrow \neg$ is_list(trm)

th-4 (lem-05): is_struct(trm) $\rightarrow \neg$ is_var(trm)

th-5 (lem-06): is_struct(trm) $\rightarrow \neg$ is_list(trm)

th-6 (lem-09): is_var(trm) $\rightarrow \neg$ is_list(trm)

th-7 (el-const): is_const(trm)
$\rightarrow$ (at = constsym(trm) $\leftrightarrow$ trm = mkconst(at))

th-8 (el-list): is_list(trm)
$\rightarrow$ (trmli = thelist(trm) $\leftrightarrow$ trm = mklist(trmli))

th-9 (el-struct): is_struct(trm)
$\rightarrow$ (at = funct(trm) $\wedge$ trmli = args(trm)
$\leftrightarrow$ trm = struct(at, trmli))

5 The Test Scenario

5.1 Sequential Test Discipline

The proof of each of the theorems shown in Sect. 4 could be tried using a theory which contains just the axioms of all subspecifications of the current specification. A far better strategy is the following: to prove a theorem add all previously proved theorems as lemmas to the theory. Although this enlarges the theory, the effect is positive, since proof lengths become much shorter, and the number of proofs which require induction decreases drastically. (in the case study described in [SR97], the success rate of the provers was doubled with the sequential test discipline).

To find a meaningful order in which the theorems should be proved we use a total order compatible with the partial order induced by the hierarchy of proofs in KIV (i.e. if the KIV proof of a theorem T uses another theorem L as a lemma, then the number of L is lower than that of T).

The sequential test discipline results in one input file for each of the theorems to prove and for each of the 3 provers.

5.2 Axiom Reduction

The full number of axioms and lemmas that are available for the proof of one WAM theorem is quite large. Most of these axioms and lemmas are redundant for the proof of a particular theorem. Therefore we have developed a reduction algorithm to find out irrelevant axioms in large specifications. It exploits the specification hierarchy and other properties of axioms. It is described in [RS97]. Often the number of relevant axioms and lemmas can be reduced drastically.

To study the effects of axiom reduction, for each of the theorems (and for each prover) another input file was generated, which contains only the reduced set of axioms as computed by the algorithm. This set depends on the signature, which occurs in the theorem. The following tables give an overview of the numbers of axioms and lemmas which are available for each theorem in the unreduced and in the reduced version. Adding the numbers of axioms used in all 1059 files together, we get 452200 axioms for the unreduced version, and 86720 for the reduced version. This means, that axiom reduction removes on the average 4 of 5 axioms. The following listing details the effects of axiom reduction for each theorem. As can be seen from the table, the benefit of axiom reduction increases with the size of the specification.

All specifications from case-study WAM

- Number of theorems: 1059
- Total number of axioms: 452200
- Total number of reduced axioms: 86720
- Reduction factor: 19.1 %

Specification: CompAssum2 from case-study WAM

- Number of theorems: 6
- Total number of axioms: 10491
- Total number of reduced axioms: 1123
- Reduction factor: 10.7 %

		#ax	#redax			#ax	#redax
th-1	lem-01	1748	183	th-1-ib	lem-01-ib	1748	183
th-1-is	lem-01-is	1748	183	th-1-pr	lem-01-pr	1748	183
th-2	lem-02	1749	184	th-3	lem-03	1750	207

Specification: PrologStack+F from case-study WAM

- Number of theorems: 70
- Total number of axioms: 62290
- Total number of reduced axioms: 9043
- Reduction factor: 14.5 %

		#ax	#redax			#ax	#redax
th-1	lem-14	877	112	th-10	lem-09	886	55
th-11	lem-10	887	126	th-11-ib	lem-10-ib	887	126
th-11-is	lem-10-is	887	126	th-12	lem-17	888	127
th-12-ib	lem-17-ib	888	127	th-12-is	lem-17-is	888	127
th-12-pr	lem-17-pr	888	127	th-13	lem-25	889	186
th-13-ib	lem-25-ib	889	186	th-13-is	lem-25-is	889	186
th-13-pr	lem-25-pr	889	186	th-14	lem-02	890	56
th-14-ib	lem-02-ib	890	56	th-14-is	lem-02-is	890	56
th-14-is-ib	lem-02-is-ib	890	56	th-14-pr	lem-02-pr	890	56
th-15	lem-03	891	25	th-15-ib	lem-03-ib	891	25
th-15-is	lem-03-is	891	25	th-15-pr	lem-03-pr	891	25
th-16	lem-05	892	119	th-16-ib	lem-05-ib	892	119
th-16-is	lem-05-is	892	119	th-16-pr	lem-05-pr	892	119
th-17	lem-06	893	139	th-17-ib	lem-06-ib	893	139
th-17-is	lem-06-is	893	139	th-18	lem-11	894	205
th-18-ib	lem-11-ib	894	205	th-18-is	lem-11-is	894	205
th-18-is-ib	lem-11-is-ib	894	205	th-18-pr	lem-11-pr	894	205
th-19	lem-19	895	135	th-2	lem-15	878	113
th-20	lem-21	896	200	th-21	lem-22	897	148
th-22	lem-23	898	64	th-22-ib	lem-23-ib	898	64
th-22-is-ib	lem-23-is-ib	898	64	th-23	lem-24	899	77
th-23-ib	lem-24-ib	899	77	th-23-is	lem-24-is	899	77
th-23-pr	lem-24-pr	899	77	th-24	lem-26	900	188
th-25	rew-1	901	57	th-26	lem-12	902	193
th-26-ib	lem-12-ib	902	193	th-26-is	lem-12-is	902	193
th-26-is-ib	lem-12-is-ib	902	193	th-3	lem-04	879	116
th-3-ib	lem-04-ib	879	116	th-3-is	lem-04-is	879	116
th-4	lem-08	880	132	th-4-ib	lem-08-ib	880	132
th-4-is	lem-08-is	880	132	th-5	lem-13	881	129
th-6	lem-16	882	134	th-6-ib	lem-16-ib	882	134
th-6-is	lem-16-is	882	134	th-7	lem-20	883	183
th-7-ib	lem-20-ib	883	183	th-7-is	lem-20-is	883	183
th-8	lem-18	884	41	th-9	lem-01	885	194
th-9-ib	lem-01-ib	885	194	th-9-is	lem-01-is	885	194
th-9-is-ib	lem-01-is-ib	885	194	th-9-pr	lem-01-pr	885	194

Specification: PrologStack+H from case-study WAM

- Number of theorems: 40
- Total number of axioms: 35307
- Total number of reduced axioms: 4371
- Reduction factor: 12.3 %

		#ax	#redax			#ax	#redax
th-1	lem-06	876	125	th-1-ib	lem-06-ib	876	125
th-1-is	lem-06-is	876	125	th-1-pr	lem-06-pr	876	125
th-10	lem-08	885	120	th-10-ib	lem-08-ib	885	120
th-10-is	lem-08-is	885	120	th-11	lem-09	886	113
th-11-ib	lem-09-ib	886	113	th-11-is	lem-09-is	886	113
th-12	lem-11	887	99	th-12-ib	lem-11-ib	887	99
th-12-is	lem-11-is	887	99	th-13	lem-12	888	125
th-14	lem-13	889	53	th-14-ib	lem-13-ib	889	53
th-14-is	lem-13-is	889	53	th-15	lem-14	890	100
th-16	lem-16	891	155	th-2	lem-01	877	112
th-3	lem-04	878	76	th-3-ib	lem-04-ib	878	76
th-3-is	lem-04-is	878	76	th-3-pr	lem-04-pr	878	76
th-4	lem-10	879	77	th-4-ib	lem-10-ib	879	77
th-4-is	lem-10-is	879	77	th-5	lem-15	880	128
th-6	lem-02	881	78	th-6-ib	lem-02-ib	881	78
th-6-is	lem-02-is	881	78	th-6-pr	lem-02-pr	881	78
th-7	lem-03	882	186	th-7-ib	lem-03-ib	882	186
th-7-is	lem-03-is	882	186	th-7-pr	lem-03-pr	882	186
th-8	lem-05	883	79	th-9	lem-07	884	142
th-9-ib	lem-07-ib	884	142	th-9-is	lem-07-is	884	142

Specification: PrologStack from case-study WAM

- Number of theorems: 110
- Total number of axioms: 92075
- Total number of reduced axioms: 10414
- Reduction factor: 11.3 %

		#ax	#redax			#ax	#redax
th-1	lem-10	813	104	th-10	lem-06	822	109
th-11	lem-21	823	183	th-11-ib	lem-21-ib	823	183
th-11-is	lem-21-is	823	183	th-11-is-ib	lem-21-is-ib	823	31
th-11-is-is	lem-21-is-is	823	31	th-12	cdr-1	824	20
th-13	lem-07	825	116	th-14	lem-40	826	163
th-15	lem-03	827	74	th-15-ib	lem-03-ib	827	74
th-15-is	lem-03-is	827	74	th-15-pr	lem-03-pr	827	74
th-16	lem-16	828	22	th-16-ib	lem-16-ib	828	22
th-16-is	lem-16-is	828	22	th-17	lem-17	829	166
th-18	lem-24	830	76	th-18-ib	lem-24-ib	830	76
th-18-is	lem-24-is	830	76	th-19	lem-27	831	120
th-2	lem-19	814	29	th-2-ib	lem-19-ib	814	29
th-2-is	lem-19-is	814	29	th-20	lem-33	832	107
th-21	lem-35	833	77	th-21-ib	lem-35-ib	833	77
th-21-is	lem-35-is	833	77	th-21-pr	lem-35-pr	833	77
th-22	lem-36	834	78	th-22-ib	lem-36-ib	834	78
th-22-is	lem-36-is	834	78	th-23	lem-37	835	79
th-23-ib	lem-37-ib	835	79	th-23-is	lem-37-is	835	79
th-23-pr	lem-37-pr	835	79	th-24	lem-39	836	174
th-25	lem-43	837	23	th-25-ib	lem-43-ib	837	23
th-25-is	lem-43-is	837	23	th-26	lem-45	838	101
th-26-ib	lem-45-ib	838	101	th-26-is	lem-45-is	838	101
th-26-pr	lem-45-pr	838	101	th-27	lem-ax-09	839	84
th-28	lem-05	840	108	th-29	lem-09	841	112
th-3	lem-01	815	72	th-3-ib	lem-01-ib	815	72
th-3-is	lem-01-is	815	72	th-3-pr	lem-01-pr	815	72
th-30	lem-11	842	109	th-31	lem-13	843	179
th-31-ib	lem-13-ib	843	179	th-31-is	lem-13-is	843	179
th-31-pr	lem-13-pr	843	179	th-32	lem-14	844	172

th-32-ib	lem-14-ib	844	172	th-32-is	lem-14-is	844	172
th-32-pr	lem-14-pr	844	172	th-33	lem-18	845	33
th-33-ib	lem-18-ib	845	33	th-33-is	lem-18-is	845	33
th-34	lem-23	846	81	th-35	lem-25	847	85
th-35-ib	lem-25-ib	847	85	th-35-is	lem-25-is	847	85
th-35-is-ib	lem-25-is-ib	847	85	th-36	lem-26	848	93
th-36-ib	lem-26-ib	848	93	th-36-ib2	lem-26-ib2	848	93
th-36-is	lem-26-is	848	93	th-36-is-ib	lem-26-is-ib	848	93
th-36-is-ib2	lem-26-is-ib2	848	93	th-36-is2	lem-26-is2	848	93
th-36-pr	lem-26-pr	848	93	th-37	lem-28	849	133
th-38	lem-29	850	124	th-39	lem-30	851	110
th-4	lem-02	816	160	th-4-ib	lem-02-ib	816	160
th-4-is	lem-02-is	816	160	th-4-is-ib	lem-02-is-ib	816	160
th-40	lem-31	852	85	th-41	lem-32	853	72
th-41-ib	lem-32-ib	853	72	th-41-is	lem-32-is	853	72
th-41-pr	lem-32-pr	853	72	th-42	lem-34	854	111
th-43	lem-38	855	112	th-44	lem-41	856	43
th-45	lem-42	857	82	th-46	lem-44	858	34
th-46-ib	lem-44-ib	858	34	th-46-is	lem-44-is	858	34
th-46-pr	lem-44-pr	858	34	th-47	lem-46	859	106
th-48	lem-47	860	83	th-49	lem-48	861	132
th-5	lem-08	817	105	th-50	lem-49	862	137
th-51	lem-50	863	113	th-52	lem-51	864	183
th-6	lem-12	818	161	th-7	lem-15	819	106
th-8	lem-20	820	21	th-9	lem-04	821	112
th-9-ib	lem-04-ib	821	112	th-9-is	lem-04-is	821	112

Specification: PrologTree from case-study WAM

- Number of theorems: 23
- Total number of axioms: 18866
- Total number of reduced axioms: 1857
- Reduction factor: 9.8 %

		#ax	#redax			#ax	#redax
th-1	lem-05	812	76	th-1-ib	lem-05-ib	812	76
th-1-is	lem-05-is	812	76	th-10	lem-12	821	75
th-11	lem-13	822	77	th-12	lem-14	823	78
th-12-ib	lem-14-ib	823	78	th-12-is	lem-14-is	823	78
th-13	lem-15	824	25	th-14	lem-16	825	106
th-15	lem-17	826	71	th-16	lem-18	827	72
th-17	map-lemma	828	94	th-17-ib	map-lemma-ib	828	94
th-17-is	map-lemma-is	828	94	th-2	lem-01	813	70
th-3	lem-02	814	71	th-4	lem-03	815	72
th-5	lem-04	816	73	th-6	lem-06	817	104
th-7	lem-08	818	118	th-8	lem-09	819	105
th-9	lem-10	820	74				

Specification: Tree+Stack+F from case-study WAM

- Number of theorems: 55
- Total number of axioms: 73540
- Total number of reduced axioms: 8967
- Reduction factor: 12.1 %

		#ax	#redax			#ax	#redax
th-1	lem-01	1320	156	th-10	lem-11	1329	82
th-10-ib	lem-11-ib	1329	82	th-10-is	lem-11-is	1329	82
th-10-is-ib	lem-11-is-ib	1329	82	th-10-pr	lem-11-pr	1329	82
th-11	lem-12	1330	127	th-12	lem-13	1331	112
th-13	lem-14	1332	113	th-14	lem-19	1333	160
th-15	lem-20	1334	114	th-16	lem-22	1335	191
th-17	lem-23	1336	192	th-18	lem-24	1337	115
th-19	lem-25	1338	116	th-2	lem-02	1321	157
th-20	lem-26	1339	193	th-20-ib	lem-26-ib	1339	193
th-20-is	lem-26-is	1339	193	th-20-pr	lem-26-pr	1339	193
th-21	lem-27	1340	194	th-21-ib	lem-27-ib	1340	194
th-21-is	lem-27-is	1340	194	th-21-pr	lem-27-pr	1340	194
th-22	lem-29	1341	228	th-22-ib	lem-29-ib	1341	228
th-22-is	lem-29-is	1341	228	th-22-pr	lem-29-pr	1341	228
th-23	lem-30	1342	92	th-23-ib	lem-30-ib	1342	92
th-23-is	lem-30-is	1342	92	th-24	lem-31	1343	161
th-25	lem-32	1344	117	th-26	lem-33	1345	195
th-27	lem-34	1346	229	th-27-ib	lem-34-ib	1346	229
th-27-is	lem-34-is	1346	229	th-28	lem-35	1347	162
th-29	lem-36	1348	196	th-3	lem-08	1322	186
th-30	lem-37	1349	178	th-30-ib	lem-37-ib	1349	178
th-30-is	lem-37-is	1349	178	th-31	lem-38	1350	212
th-31-ib	lem-38-ib	1350	212	th-31-is	lem-38-is	1350	212
th-32	lem-21	1351	5	th-4	lem-15	1323	187
th-5	lem-16	1324	188	th-6	lem-17	1325	189
th-7	lem-18	1326	158	th-8	lem-03	1327	159
th-8-ib	lem-03-ib	1327	159	th-8-is	lem-03-is	1327	159
th-9	lem-04	1328	190				

Specification: codefun from case-study WAM

- Number of theorems: 39
- Total number of axioms: 22906
- Total number of reduced axioms: 9205
- Reduction factor: 40.1 %

		#ax	#redax			#ax	#redax
th-1	lem-02	571	140	th-10	lem-17	580	142
th-11	lem-18	581	156	th-12	lem-21	582	143
th-13	lem-22	583	158	th-14	lem-23	584	157
th-15	lem-24	585	157	th-16	lem-03	586	158
th-17	lem-04	587	158	th-18	lem-05	588	158
th-19	lem-06	589	158	th-2	lem-01	572	141
th-20	lem-07	590	158	th-21	lem-08	591	157
th-22	lem-09	592	158	th-23	lem-19	593	158
th-24	lem-20	594	158	th-25	lem-25	595	383
th-25-ib	lem-25-ib	595	383	th-25-is	lem-25-is	595	383
th-25-is10	lem-25-is10	595	383	th-25-is11	lem-25-is11	595	383
th-25-is2	lem-25-is2	595	383	th-25-is3	lem-25-is3	595	383
th-25-is4	lem-25-is4	595	383	th-25-is5	lem-25-is5	595	383
th-25-is6	lem-25-is6	595	383	th-25-is7	lem-25-is7	595	383
th-25-is8	lem-25-is8	595	383	th-25-is9	lem-25-is9	595	383
th-25-pr	lem-25-pr	595	383	th-26	lem-26	596	144
th-3	lem-10	573	155	th-4	lem-11	574	155
th-5	lem-12	575	155	th-6	lem-13	576	155
th-7	lem-14	577	155	th-8	lem-15	578	154
th-9	lem-16	579	155				

Specification: enrset from case-study WAM

- Number of theorems: 1

- Total number of axioms: 186
- Total number of reduced axioms: 35
- Reduction factor: 18.8 %

		#ax	#redax			#ax	#redax
th-1	lem-01	186	35				

Specification: enrterm from case-study WAM

- Number of theorems: 4
- Total number of axioms: 878
- Total number of reduced axioms: 162
- Reduction factor: 18.4 %

		#ax	#redax			#ax	#redax
th-1	lem-1	218	39	th-2	lem-2	219	40
th-3	lem-3	220	41	th-4	lem-4	221	42

Specification: instr+clau from case-study WAM

- Number of theorems: 99
- Total number of axioms: 51381
- Total number of reduced axioms: 16056
- Reduction factor: 31.2 %

		#ax	#redax			#ax	#redax
th-1	lem-1	470	162	th-10	lem-18	479	162
th-11	lem-19	480	162	th-12	lem-2	481	162
th-13	lem-20	482	162	th-14	lem-21	483	162
th-15	lem-3	484	162	th-16	lem-4	485	162
th-17	lem-5	486	162	th-18	lem-6	487	162
th-19	lem-64	488	162	th-2	lem-10	471	162
th-20	lem-65	489	162	th-21	lem-66	490	162
th-22	lem-67	491	162	th-23	lem-68	492	162
th-24	lem-69	493	162	th-25	lem-7	494	162
th-26	lem-70	495	162	th-27	lem-71	496	162
th-28	lem-72	497	162	th-29	lem-73	498	162
th-3	lem-11	472	162	th-30	lem-74	499	162
th-31	lem-75	500	162	th-32	lem-76	501	162
th-33	lem-77	502	162	th-34	lem-78	503	162
th-35	lem-79	504	162	th-36	lem-8	505	162
th-37	lem-80	506	162	th-38	lem-81	507	162
th-39	lem-82	508	162	th-4	lem-12	473	162
th-40	lem-83	509	162	th-41	lem-84	510	162
th-42	lem-88	511	162	th-43	lem-89	512	162
th-44	lem-9	513	162	th-45	lem-90	514	162
th-46	elim-clause	515	150	th-47	elim-retry	516	150
th-48	elim-retry_me	517	150	th-49	elim-swconst	518	150
th-5	lem-13	474	162	th-50	elim-swstruct	519	150
th-51	elim-swterm	520	150	th-52	elim-trust	521	150
th-53	elim-try	522	150	th-54	elim-try_me	523	150

th-55	lem-22	524	165	th-56	lem-23	525	164
th-57	lem-24	526	165	th-58	lem-25	527	164
th-59	lem-26	528	164	th-6	lem-14	475	162
th-60	lem-27	529	165	th-61	lem-28	530	165
th-62	lem-29	531	165	th-63	lem-30	532	165
th-64	lem-31	533	165	th-65	lem-32	534	165
th-66	lem-33	535	165	th-67	lem-34	536	165
th-68	lem-35	537	165	th-69	lem-36	538	165
th-7	lem-15	476	162	th-70	lem-37	539	164
th-71	lem-38	540	164	th-72	lem-39	541	164
th-73	lem-40	542	165	th-74	lem-41	543	165
th-75	lem-42	544	165	th-76	lem-43	545	165
th-77	lem-44	546	165	th-78	lem-45	547	164
th-79	lem-46	548	165	th-8	lem-16	477	162
th-80	lem-47	549	165	th-81	lem-48	550	165
th-82	lem-49	551	165	th-83	lem-50	552	165
th-84	lem-51	553	165	th-85	lem-52	554	164
th-86	lem-53	555	165	th-87	lem-54	556	165
th-88	lem-55	557	165	th-89	lem-56	558	165
th-9	lem-17	478	162	th-90	lem-57	559	165
th-91	lem-58	560	165	th-92	lem-59	561	164
th-93	lem-60	562	165	th-94	lem-61	563	165
th-95	lem-62	564	165	th-96	lem-63	565	165
th-97	lem-85	566	165	th-98	lem-86	567	165
th-99	lem-87	568	165				

Specification: list-data from case-study WAM

- Number of theorems: 37
- Total number of axioms: 5759
- Total number of reduced axioms: 1095
- Reduction factor: 19.0 %

		#ax	#redax			#ax	#redax
th-1	ax-03a	143	5	th-10	lfr	152	69
th-11	lr-02	153	70	th-12	ax-03b	154	12
th-13	elim-list	155	13	th-14	fr-01	156	14
th-15	fr-03	157	15	th-16	fr-04	158	16
th-17	lf	159	76	th-18	lr	160	77
th-19	lr-01	161	116	th-2	fr-07	144	6
th-2-ib	fr-07-ib	144	6	th-2-is	fr-07-is	144	6
th-2-pr	fr-07-pr	144	6	th-20	lr-03	162	36
th-21	lr-04	163	79	th-22	lr-05	164	80
th-23	lr-06	165	37	th-24	ls-01	166	21
th-25	ls-02	167	22	th-25-ib	ls-02-ib	167	22
th-25-is	ls-02-is	167	22	th-25-pr	ls-02-pr	167	22
th-26	ls-trans1	168	23	th-27	ls-trans2	169	24
th-28	p-fr	170	17	th-3	fr-05	145	7
th-4	fr-02	146	8	th-5	elim-list-c	147	9
th-6	l	148	27	th-6-ib	l-ib	148	27
th-6-is	l-is	148	27	th-6-pr	l-pr	148	27
th-7	fr	149	10	th-8	fr-06	150	11
th-9	lf-01	151	30				

Specification: list from case-study WAM

- Number of theorems: 165
- Total number of axioms: 33609
- Total number of reduced axioms: 6949
- Reduction factor: 20.6 %

		#ax	#redax			#ax	#redax
th-1	app-a	174	20	th-1-ib	app-a-ib	174	20
th-1-is	app-a-is	174	20	th-10	inapp	183	30
th-11	in-04	184	31	th-12	in-03	185	32
th-13	in-01	186	33	th-14	inapp-03	187	34
th-14-ib	inapp-03-ib	187	34	th-14-ib-ib	inapp-03-ib-ib	187	34
th-14-ib-is	inapp-03-ib-is	187	34	th-14-is	inapp-03-is	187	34
th-14-is-ib	inapp-03-is-ib	187	34	th-14-is-is	inapp-03-is-is	187	34
th-14-pr	inapp-03-pr	187	34	th-15	lem-02	188	43
th-15-ib	lem-02-ib	188	43	th-15-is	lem-02-is	188	43
th-15-pr	lem-02-pr	188	43	th-16	lem-05	189	34
th-16-ib	lem-05-ib	189	34	th-16-is	lem-05-is	189	34
th-16-pr	lem-05-pr	189	34	th-17	app-07	190	26
th-17-ib	app-07-ib	190	26	th-17-ib-ib	app-07-ib-ib	190	26
th-17-ib-is	app-07-ib-is	190	26	th-17-is	app-07-is	190	26
th-17-is-ib	app-07-is-ib	190	26	th-17-is-is	app-07-is-is	190	26
th-17-pr	app-07-pr	190	26	th-18	app-08	191	27
th-19	app-09	192	28	th-2	app-06	175	21
th-2-ib	app-06-ib	175	21	th-2-is	app-06-is	175	21
th-20	app-10	193	29	th-21	in-06	194	39
th-22	in-09	195	40	th-23	in-10	196	41
th-24	inapp-01	197	42	th-24-ib	inapp-01-ib	197	42
th-24-ib-ib	inapp-01-ib-ib	197	30	th-24-ib-is	inapp-01-ib-is	197	30
th-24-is	inapp-01-is	197	42	th-24-is-ib	inapp-01-is-ib	197	42
th-24-is-is	inapp-01-is-is	197	42	th-24-pr	inapp-01-pr	197	42
th-25	inapp-02	198	43	th-26	lapp	199	70
th-26-ib	lapp-ib	199	70	th-26-is	lapp-is	199	70
th-27	lem-01	200	54	th-28	lem-08	201	39
th-29	app-01	202	30	th-29-ib	app-01-ib	202	30
th-29-is	app-01-is	202	30	th-29-pr	app-01-pr	202	30
th-3	app-04	176	22	th-30	app-03	203	31
th-30-ib	app-03-ib	203	31	th-30-is	app-03-is	203	31
th-30-pr	app-03-pr	203	31	th-31	app-11	204	32
th-32	app-12	205	33	th-33	app-13	206	48
th-33-ib	app-13-ib	206	48	th-33-is	app-13-is	206	48
th-33-pr	app-13-pr	206	48	th-34	firstoc	207	49
th-34-ib	firstoc-ib	207	49	th-34-is	firstoc-is	207	49
th-35	fw-01	208	50	th-36	fw-02	209	51
th-37	in-02	210	52	th-38	in-05	211	53
th-39	in-07	212	54	th-4	app	177	23
th-40	in-11	213	55	th-41	inapp-04	214	56
th-41-ib	inapp-04-ib	214	56	th-41-is	inapp-04-is	214	56
th-41-pr	inapp-04-pr	214	56	th-42	inapp-06	215	57
th-42-ib	inapp-06-ib	215	57	th-42-is	inapp-06-is	215	57
th-42-pr	inapp-06-pr	215	57	th-43	lapp-01	216	55
th-43-ib	lapp-01-ib	216	55	th-43-is	lapp-01-is	216	55
th-43-is-ib	lapp-01-is-ib	216	55	th-43-is-is	lapp-01-is-is	216	55
th-43-pr	lapp-01-pr	216	55	th-44	lapp-02	217	100
th-44-ib	lapp-02-ib	217	100	th-44-is	lapp-02-is	217	100
th-44-pr	lapp-02-pr	217	100	th-45	lastdiv	218	34
th-45-ib	lastdiv-ib	218	34	th-45-is	lastdiv-is	218	34
th-45-pr	lastdiv-pr	218	34	th-46	lastoc	219	59
th-46-ib	lastoc-ib	219	59	th-46-is	lastoc-is	219	59
th-47	lem-03	220	72	th-48	lem-04	221	45
th-48-ib	lem-04-ib	221	45	th-48-is	lem-04-is	221	45
th-49	lem-06	222	35	th-49-ib	lem-06-ib	222	35
th-49-is	lem-06-is	222	35	th-49-pr	lem-06-pr	222	35
th-5	app-02	178	24	th-5-ib	app-02-ib	178	24
th-5-is	app-02-is	178	24	th-5-pr	app-02-pr	178	24
th-50	lem-07	223	36	th-50-ib	lem-07-ib	223	36
th-50-is	lem-07-is	223	36	th-50-pr	lem-07-pr	223	36

th-51	lem-09	224	37	th-52	lem-10	225	49
th-53	lem-11	226	78	th-54	lem-12	227	79
th-55	ls-app-01	228	50	th-55-ib	ls-app-01-ib	228	50
th-55-is	ls-app-01-is	228	50	th-55-is-ib	ls-app-01-is-ib	228	38
th-55-is-ib2	ls-app-01-is-ib2	228	50	th-55-is-is	ls-app-01-is-is	228	38
th-55-is-is-ib	ls-app-01-is-is-ib	228	38	th-55-is-is-is	ls-app-01-is-is-is	228	38
th-55-is-is2	ls-app-01-is-is2	228	50	th-55-pr	ls-app-01-pr	228	50
th-56	ls-app-02	229	51	th-56-ib	ls-app-02-ib	229	51
th-56-is	ls-app-02-is	229	51	th-56-pr	ls-app-02-pr	229	51
th-57	p-app	230	38	th-58	p-app-01	231	39
th-59	p-app-02	232	65	th-59-ib	p-app-02-ib	232	65
th-59-is	p-app-02-is	232	65	th-59-pr	p-app-02-pr	232	65
th-6	app-05	179	25	th-6-ib	app-05-ib	179	25
th-6-is	app-05-is	179	25	th-6-is-ib	app-05-is-ib	179	25
th-6-is-is	app-05-is-is	179	25	th-6-pr	app-05-pr	179	25
th-60	p-app-03	233	66	th-61	p-app-04	234	67
th-62	p-lapp	235	62	th-63	someoc	236	68
th-7	in	180	27	th-8	inapp-05	181	28
th-8-ib	inapp-05-ib	181	28	th-8-is	inapp-05-is	181	28
th-8-is-ib	inapp-05-is-ib	181	28	th-8-is-is	inapp-05-is-is	181	28
th-8-pr	inapp-05-pr	181	28	th-9	in-08	182	29
th-9-ib	in-08-ib	182	29	th-9-is	in-08-is	182	29
th-9-pr	in-08-pr	182	29				

Specification: mode from case-study WAM

- Number of theorems: 4
- Total number of axioms: 8
- Total number of reduced axioms: 8
- Reduction factor: 100.0 %

		#ax	#redax			#ax	#redax
th-1	lem-01	2	2	th-1-ib	lem-01-ib	2	2
th-1-is	lem-01-is	2	2	th-1-pr	lem-01-pr	2	2

Specification: nat-basic1 from case-study WAM

- Number of theorems: 103
- Total number of axioms: 2746
- Total number of reduced axioms: 2277
- Reduction factor: 82.9 %

		#ax	#redax			#ax	#redax
th-1	lem-14	8	8	th-10	lem-09	17	17
th-10-ib	lem-09-ib	17	17	th-10-is	lem-09-is	17	17
th-10-pr	lem-09-pr	17	17	th-11	lem-16	18	18
th-11-ib	lem-16-ib	18	18	th-11-is	lem-16-is	18	18
th-12	lem-32	19	19	th-12-ib	lem-32-ib	19	19
th-12-is	lem-32-is	19	19	th-12-pr	lem-32-pr	19	19
th-13	lem-24	20	20	th-13-ib	lem-24-ib	20	20
th-13-is	lem-24-is	20	20	th-13-pr	lem-24-pr	20	20
th-14	elim-pred	21	6	th-14-ib	elim-pred-ib	21	6
th-14-is	elim-pred-is	21	6	th-14-pr	elim-pred-pr	21	6
th-15	lem-04	22	22	th-15-ib	lem-04-ib	22	22
th-15-is	lem-04-is	22	22	th-15-pr	lem-04-pr	22	22

th-16	lem-13	23	23	th-16-ib	lem-13-ib	23	23
th-16-is	lem-13-is	23	23	th-16-pr	lem-13-pr	23	23
th-17	lem-15	24	7	th-17-ib	lem-15-ib	24	7
th-17-is	lem-15-is	24	7	th-18	lem-22	25	8
th-19	lem-25	26	26	th-2	lem-03	9	4
th-2-ib	lem-03-ib	9	4	th-2-is	lem-03-is	9	4
th-2-pr	lem-03-pr	9	4	th-20	lem-27	27	27
th-20-ib	lem-27-ib	27	27	th-20-is	lem-27-is	27	27
th-20-pr	lem-27-pr	27	27	th-21	lem-30	28	28
th-21-ib	lem-30-ib	28	28	th-21-is	lem-30-is	28	28
th-21-pr	lem-30-pr	28	28	th-22	lem-18	29	29
th-23	elim-pred-c	30	9	th-24	lem-01	31	10
th-25	lem-05	32	32	th-25-ib	lem-05-ib	32	32
th-25-is	lem-05-is	32	32	th-25-pr	lem-05-pr	32	32
th-26	lem-06	33	33	th-27	lem-10	34	34
th-28	lem-11	35	35	th-28-ib	lem-11-ib	35	35
th-28-is	lem-11-is	35	35	th-28-pr	lem-11-pr	35	35
th-29	lem-12	36	11	th-3	lem-29	10	10
th-3-ib	lem-29-ib	10	10	th-3-is	lem-29-is	10	10
th-3-pr	lem-29-pr	10	10	th-30	lem-21	37	37
th-31	lem-23	38	12	th-31-ib	lem-23-ib	38	12
th-31-is	lem-23-is	38	12	th-32	lem-26	39	39
th-33	lem-28	40	40	th-33-ib	lem-28-ib	40	40
th-33-is	lem-28-is	40	40	th-33-pr	lem-28-pr	40	40
th-34	lem-33	41	41	th-34-ib	lem-33-ib	41	41
th-34-is	lem-33-is	41	41	th-34-pr	lem-33-pr	41	41
th-35	lem-34	42	42	th-36	lem-35	43	43
th-37	lem-36	44	44	th-38	prop-03	45	13
th-39	prop-04	46	46	th-4	lem-20	11	11
th-40	prop-05	47	47	th-41	q-01	48	14
th-41-ib	q-01-ib	48	14	th-41-is	q-01-is	48	14
th-41-pr	q-01-pr	48	14	th-42	lem-17	49	49
th-43	prop-01	50	50	th-44	prop-02	51	51
th-5	lem-07	12	12	th-5-ib	lem-07-ib	12	12
th-5-is	lem-07-is	12	12	th-5-pr	lem-07-pr	12	12
th-6	lem-19	13	5	th-7	lem-31	14	14
th-7-ib	lem-31-ib	14	14	th-7-is	lem-31-is	14	14
th-8	lem-02	15	15	th-9	lem-08	16	16
th-9-ib	lem-08-ib	16	16	th-9-is	lem-08-is	16	16
th-9-pr	lem-08-pr	16	16				

Specification: nat-basic2 from case-study WAM

- Number of theorems: 112
- Total number of axioms: 8028
- Total number of reduced axioms: 5471
- Reduction factor: 68.1 %

		#ax	#redax			#ax	#redax
th-1	prop-01	55	17	th-1-ib	prop-01-ib	55	17
th-1-is	prop-01-is	55	17	th-10	lem-01	64	64
th-10-ib	lem-01-ib	64	64	th-10-is	lem-01-is	64	64
th-11	lem-04	65	24	th-11-ib	lem-04-ib	65	24
th-11-is	lem-04-is	65	24	th-11-pr	lem-04-pr	65	24
th-12	lem-06	66	66	th-13	lem-12	67	67
th-13-ib	lem-12-ib	67	67	th-13-is	lem-12-is	67	67
th-13-pr	lem-12-pr	67	67	th-14	lem-34	68	25
th-14-ib	lem-34-ib	68	25	th-14-is	lem-34-is	68	25
th-14-is-ib	lem-34-is-ib	68	25	th-14-is-is	lem-34-is-is	68	25
th-15	lem-05	69	26	th-16	lem-07	70	27
th-16-ib	lem-07-ib	70	27	th-16-is	lem-07-is	70	27

th-16-pr	lem-07-pr	70	27	th-17	lem-08	71	71
th-18	lem-11	72	72	th-19	lem-13	73	73
th-19-ib	lem-13-ib	73	73	th-19-is	lem-13-is	73	73
th-19-pr	lem-13-pr	73	73	th-2	prop-02	56	18
th-2-ib	prop-02-ib	56	18	th-2-is	prop-02-is	56	18
th-20	lem-15	74	74	th-20-ib	lem-15-ib	74	74
th-20-is	lem-15-is	74	74	th-20-is-ib	lem-15-is-ib	74	74
th-20-is-is	lem-15-is-is	74	74	th-21	lem-17	75	28
th-21-ib	lem-17-ib	75	28	th-21-is	lem-17-is	75	28
th-21-pr	lem-17-pr	75	28	th-22	lem-20	76	29
th-22-ib	lem-20-ib	76	29	th-22-is	lem-20-is	76	29
th-22-pr	lem-20-pr	76	29	th-23	lem-28	77	77
th-23-ib	lem-28-ib	77	77	th-23-is	lem-28-is	77	77
th-23-pr	lem-28-pr	77	77	th-24	lem-29	78	78
th-24-ib	lem-29-ib	78	78	th-24-is	lem-29-is	78	78
th-25	lem-35	79	30	th-25-ib	lem-35-ib	79	30
th-25-is	lem-35-is	79	30	th-26	less-01	80	80
th-26-ib	less-01-ib	80	80	th-26-is	less-01-is	80	80
th-26-pr	less-01-pr	80	80	th-27	less-02	81	81
th-28	less-03	82	82	th-29	less-04	83	83
th-29-ib	less-04-ib	83	83	th-29-is	less-04-is	83	83
th-3	lem-21	57	19	th-3-ib	lem-21-ib	57	19
th-3-is	lem-21-is	57	19	th-30	less-05	84	84
th-30-ib	less-05-ib	84	84	th-30-is	less-05-is	84	84
th-30-pr	less-05-pr	84	84	th-31	less-06	85	85
th-31-ib	less-06-ib	85	85	th-31-is	less-06-is	85	85
th-32	prop-03	86	31	th-32-ib	prop-03-ib	86	31
th-32-is	prop-03-is	86	31	th-32-pr	prop-03-pr	86	31
th-33	prop-04	87	32	th-34	q-01	88	33
th-34-ib	q-01-ib	88	33	th-34-is	q-01-is	88	33
th-35	subadd-01	89	89	th-35-ib	subadd-01-ib	89	89
th-35-is	subadd-01-is	89	89	th-35-pr	subadd-01-pr	89	89
th-36	lem-27	90	90	th-4	lem-10	58	20
th-4-ib	lem-10-ib	58	20	th-4-is	lem-10-is	58	20
th-5	lem-09	59	21	th-5-ib	lem-09-ib	59	21
th-5-is	lem-09-is	59	21	th-5-pr	lem-09-pr	59	21
th-6	lem-02	60	22	th-6-ib	lem-02-ib	60	22
th-6-is	lem-02-is	60	22	th-6-pr	lem-02-pr	60	22
th-7	lem-03	61	61	th-7-ib	lem-03-ib	61	61
th-7-is	lem-03-is	61	61	th-8	lem-30	62	62
th-8-ib	lem-30-ib	62	62	th-8-is	lem-30-is	62	62
th-9	lem-33	63	23	th-9-ib	lem-33-ib	63	23
th-9-is	lem-33-is	63	23	th-9-is-ib	lem-33-is-ib	63	23
th-9-is-is	lem-33-is-is	63	23	th-9-pr	lem-33-pr	63	23

Specification: nat-sub from case-study WAM

- Number of theorems: 40
- Total number of axioms: 4216
- Total number of reduced axioms: 2852
- Reduction factor: 67.6 %

		#ax	#redax			#ax	#redax
th-1	add-01	93	36	th-1-ib	add-01-ib	93	36
th-1-is	add-01-is	93	36	th-10	less-01	102	102
th-11	less-02	103	103	th-12	less-03	104	58
th-13	ls	105	59	th-14	ls-02	106	60
th-15	pred-01	107	61	th-16	prop	108	62
th-17	prop-01	109	109	th-18	prop-02	110	63
th-19	prop-03	111	64	th-2	ls-01	94	55
th-20	prop-04	112	65	th-21	prop-05	113	113

th-22	prop-06	114	66	th-22-ib	prop-06-ib	114	91
th-22-is	prop-06-is	114	91	th-22-pr	prop-06-pr	114	114
th-23	sub	115	17	th-24	sub-01	116	68
th-25	succ	117	18	th-26	succ-02	118	40
th-27	succ-03	119	119	th-28	succ-04	120	70
th-29	succ-05	121	71	th-3	add-03	95	37
th-4	elim-sub	96	96	th-4-ib	elim-sub-ib	96	96
th-4-is	elim-sub-is	96	96	th-4-pr	elim-sub-pr	96	96
th-5	add	97	97	th-6	add-02	98	98
th-7	pred	99	56	th-7-ib	pred-ib	99	38
th-7-is	pred-is	99	38	th-7-pr	pred-pr	99	99
th-8	succ-01	100	57	th-9	less	101	101

Specification: pair from case-study WAM

- Number of theorems: 6
- Total number of axioms: 34
- Total number of reduced axioms: 34
- Reduction factor: 100.0 %

		#ax	#redax			#ax	#redax
th-1	elim-pair	4	4	th-1-ib	elim-pair-ib	4	4
th-2	lem-1	5	5	th-3	lem-2	6	6
th-4	lem-3	7	7	th-5	lem-4	8	8

Specification: paramterm from case-study WAM

- Number of theorems: 7
- Total number of axioms: 49
- Total number of reduced axioms: 49
- Reduction factor: 100.0 %

		#ax	#redax			#ax	#redax
th-1	lem-1	4	4	th-2	lem-2	5	5
th-3	lem-3	6	6	th-4	lem-4	7	7
th-5	lem-5	8	8	th-6	lem-6	9	9
th-7	lem-7	10	10				

Specification: rmode from case-study WAM

- Number of theorems: 9
- Total number of axioms: 69
- Total number of reduced axioms: 69
- Reduction factor: 100.0 %

		#ax	#redax			#ax	#redax
th-1	lem-01	7	7	th-1-ib	lem-01-ib	7	7
th-1-is	lem-01-is	7	7	th-1-is2	lem-01-is2	7	7
th-1-is3	lem-01-is3	7	7	th-1-pr	lem-01-pr	7	7
th-2	lem-03	8	8	th-3	lem-02	9	9
th-4	lem-04	10	10				

Specification: set from case-study WAM

- Number of theorems: 63
- Total number of axioms: 10243
- Total number of reduced axioms: 1894
- Reduction factor: 18.4 %

		#ax	#redax			#ax	#redax
th-1	lem-01	139	5	th-10	lem-14	148	13
th-11	lem-16	149	14	th-12	lem-23	150	15
th-13	lem-25	151	16	th-14	lem-27	152	17
th-15	lem-29	153	19	th-16	prop-01	154	18
th-17	lem-03	155	19	th-18	lem-07	156	20
th-19	lem-10	157	39	th-2	add-01	140	6
th-20	lem-11	158	40	th-21	lem-13	159	21
th-22	lem-15	160	22	th-23	lem-18	161	23
th-24	lem-19	162	24	th-25	lem-20	163	25
th-26	lem-21	164	26	th-27	lem-22	165	47
th-27-ib	lem-22-ib	165	47	th-27-ib2	lem-22-ib2	165	47
th-27-is	lem-22-is	165	47	th-27-is2	lem-22-is2	165	47
th-27-pr	lem-22-pr	165	47	th-28	lem-24	166	48
th-28-ib	lem-24-ib	166	48	th-28-is	lem-24-is	166	48
th-28-pr	lem-24-pr	166	48	th-29	lem-26	167	27
th-3	lem-06	141	7	th-30	lem-28	168	28
th-31	lem-30	169	31	th-32	lem-31	170	32
th-33	prop-02	171	29	th-34	prop-03	172	30
th-35	prop-04	173	31	th-36	q-01	174	32
th-36-ib	q-01-ib	174	32	th-36-is	q-01-is	174	32
th-36-pr	q-01-pr	174	32	th-37	selax	175	33
th-38	sub-03	176	38	th-38-ib	sub-03-ib	176	38
th-38-is	sub-03-is	176	38	th-38-pr	sub-03-pr	176	38
th-39	sub-04	177	39	th-4	lem-04	142	8
th-40	sub-05	178	40	th-41	sub-06	179	41
th-42	sub-07	180	42	th-43	sub-08	181	43
th-44	sub-09	182	44	th-45	sub-10	183	45
th-46	sub-11	184	46	th-5	lem-12	143	9
th-6	lem-17	144	10	th-7	lem-02	145	11
th-8	lem-08	146	12	th-9	lem-09	147	30
th-9-ib	lem-09-ib	147	30	th-9-is	lem-09-is	147	30
th-9-pr	lem-09-pr	147	30				

Specification: stopmode from case-study WAM

- Number of theorems: 7
- Total number of axioms: 31
- Total number of reduced axioms: 31
- Reduction factor: 100.0 %

		#ax	#redax			#ax	#redax
th-1	lem-03	4	4	th-1-ib	lem-03-ib	4	4
th-1-is	lem-03-is	4	4	th-1-is2	lem-03-is2	4	4
th-1-pr	lem-03-pr	4	4	th-2	lem-01	5	5
th-3	lem-02	6	6				

Specification: term from case-study WAM

- Number of theorems: 96
- Total number of axioms: 19488
- Total number of reduced axioms: 4758
- Reduction factor: 24.4 %

		#ax	#redax			#ax	#redax
th-1	lem-01	198	52	th-1-ib	lem-01-ib	198	52
th-1-is	lem-01-is	198	52	th-1-is2	lem-01-is2	198	52
th-1-is3	lem-01-is3	198	52	th-1-is4	lem-01-is4	198	52
th-1-is5	lem-01-is5	198	52	th-1-is6	lem-01-is6	198	52
th-1-pr	lem-01-pr	198	52	th-10	el-var	207	45
th-10-ib	el-var-ib	207	45	th-10-is	el-var-is	207	45
th-10-is2	el-var-is2	207	45	th-10-is3	el-var-is3	207	45
th-10-is4	el-var-is4	207	45	th-10-is5	el-var-is5	207	45
th-10-is6	el-var-is6	207	45	th-10-pr	el-var-pr	207	45
th-11	lem-04	208	55	th-12	lem-07	209	55
th-13	lem-08	210	55	th-14	lem-10	211	55
th-15	lem-11	212	55	th-16	lem-12	213	55
th-2	lem-02	199	52	th-2-ib	lem-02-ib	199	52
th-2-is	lem-02-is	199	52	th-2-is2	lem-02-is2	199	52
th-2-is3	lem-02-is3	199	52	th-2-is4	lem-02-is4	199	52
th-2-is5	lem-02-is5	199	52	th-2-is6	lem-02-is6	199	52
th-2-pr	lem-02-pr	199	52	th-3	lem-03	200	52
th-3-ib	lem-03-ib	200	52	th-3-is	lem-03-is	200	52
th-3-is2	lem-03-is2	200	52	th-3-is3	lem-03-is3	200	52
th-3-is4	lem-03-is4	200	52	th-3-is5	lem-03-is5	200	52
th-3-is6	lem-03-is6	200	52	th-3-pr	lem-03-pr	200	52
th-4	lem-05	201	52	th-4-ib	lem-05-ib	201	52
th-4-is	lem-05-is	201	52	th-4-is2	lem-05-is2	201	52
th-4-is3	lem-05-is3	201	52	th-4-is4	lem-05-is4	201	52
th-4-is5	lem-05-is5	201	52	th-4-is6	lem-05-is6	201	52
th-4-pr	lem-05-pr	201	52	th-5	lem-06	202	52
th-5-ib	lem-06-ib	202	52	th-5-is	lem-06-is	202	52
th-5-is2	lem-06-is2	202	52	th-5-is3	lem-06-is3	202	52
th-5-is4	lem-06-is4	202	52	th-5-is5	lem-06-is5	202	52
th-5-is6	lem-06-is6	202	52	th-5-pr	lem-06-pr	202	52
th-6	lem-09	203	52	th-6-ib	lem-09-ib	203	52
th-6-is	lem-09-is	203	52	th-6-is2	lem-09-is2	203	52
th-6-is3	lem-09-is3	203	52	th-6-is4	lem-09-is4	203	52
th-6-is5	lem-09-is5	203	52	th-6-is6	lem-09-is6	203	52
th-6-pr	lem-09-pr	203	52	th-7	el-const	204	45
th-7-ib	el-const-ib	204	45	th-7-is	el-const-is	204	45
th-7-is2	el-const-is2	204	45	th-7-is3	el-const-is3	204	45
th-7-is4	el-const-is4	204	45	th-7-is5	el-const-is5	204	45
th-7-is6	el-const-is6	204	45	th-7-pr	el-const-pr	204	45
th-8	el-list	205	45	th-8-ib	el-list-ib	205	45
th-8-is	el-list-is	205	45	th-8-is2	el-list-is2	205	45
th-8-is3	el-list-is3	205	45	th-8-is4	el-list-is4	205	45
th-8-is5	el-list-is5	205	45	th-8-is6	el-list-is6	205	45
th-8-pr	el-list-pr	205	45	th-9	el-struct	206	45
th-9-ib	el-struct-ib	206	45	th-9-is	el-struct-is	206	45
th-9-is2	el-struct-is2	206	45	th-9-is3	el-struct-is3	206	45
th-9-is4	el-struct-is4	206	45	th-9-is5	el-struct-is5	206	45
th-9-is6	el-struct-is6	206	45	th-9-pr	el-struct-pr	206	45

5.3 Operator Precedence

The specification hierarchy and the direction of rewrite rules in KIV can be exploited to define an operator precedence (see [ABH+98]), which can be exploited in the calculus of Otter and Spass by using an LPO. To see, if using such an LPO has positive effects, we generated additional files ordth-n and ordredth-n for every theorem which contain the ordering.

5.4 Input Syntax

Although each of the provers we tested has a different input syntax, a common translation for symbols was used in the generation of the 90 input files. Since most automated theorem provers cannot handle infix symbols or graphic symbols, as they are used in KIV, the symbols of the previous sections had to be translated to ASCII symbols (also some of the symbols are named differently in the KIV case study than in this paper). The translation has two cases: If the symbol name occurs in the left column of table 1, the result is shown in the right column. Otherwise, the symbol name is translated character by character according to table 2. Alphanumeric characters not mentioned in table 2 are left unchanged. As an exception, numbers are only translated, if they are the leading character of the symbol name. E.g. the symbol '33$\odot$' is translated to 'jthr3jodot'.

here	ASCII	here	ASCII
+1	jsuc	unknown	junknown
−1	jpre	name	jname
<=	jle	author	jauthor
clause	jclause	date	jdate
predicate	jpredicate	cnf	jcnf
status	jstatus	dnf	jdnf
logic	jlogic	formula	jformula
version	jversion	equal	jequal

here	ASCII	here	ASCII	here	ASCII	here	ASCII
+	jadd	$\downarrow$	jdoa	>	jgr	$\lfloor$	jlfl
*	jmul	$\equiv$	jev	=	jeq	$\rfloor$	jrfl
^	jexp	$\ni$	jni	!	jfac	$\ll$	jmls
–	jsub	$\in$	jin	?	jque	$\gg$	jmgr
~	jneg	$\forall$	jal	.	jdot	$\sqcup$	jlub
0	jzer	$\exists$	jex	%	jper	$\sqcap$	jglb
1	jone	$\neg$	jnot	’	jquo	$\triangle$	jtri
2	jtwo	$\cup$	jun	‘	jbqu	$\times$	jtim
3	jthr	$\cap$	jis	|	jdiv	$\sqsubseteq$	jlow
4	jfou	$\wedge$	jand	_	jusc	$\sqsupseteq$	jhig
5	jfiv	$\vee$	jor	$\circ$	jcir	$\Leftrightarrow$	jlra
6	jsix	$\top$	jtop	$\Box$	jbox	$\sqsubset$	jsql
7	jsev	$\bot$	jbot	$\Diamond$	jdia	$\sqsupset$	jsqg
8	jeig	$\Rightarrow$	jdra	$\models$	jfos	$\emptyset$	jemp
9	jnin	$\Leftarrow$	jdlr	$[\![$	jldb	$\bullet$	jbul
@	jemt	$\int$	jint	$]\!]$	jrdb	$\oplus$	jopl
#	jsiz	∞	jinf	$\subset$	jsub	$\ominus$	jomi
$	jdol	$\lceil$	jlce	$\supset$	jsup	$\otimes$	jomu
&	jcon	$\rceil$	jrce	$\leftarrow$	jire	$\oslash$	jodi
/	jsla	$\subseteq$	jsbe	$\rightarrow$	jimp	$\odot$	jodo
\	jbsl	$\supseteq$	jspe	$\uparrow$	jupa	j	jj
<	jls	$\equiv$	jcgr				

6 Experimental Results with 3 Provers

6.1 The Provers and their Settings

All experiments were done on a SPARC ULTRA-1 with Solaris 2.5.1. The provers were given a time limit of 2 minutes for the proof time (excluding preprocessing).

6.1.1 Otter

Otter (version 3.0.4,[WOLB92]) has a built-in equality predicate, but no sorts. These were encoded by mapping terms t of sort s to pairs *s(t)*, where s is a (unary) function. Otter was used with the settings of auto-mode, but with the (negated) theorem to prove as the set of support (sos, see p. 552 of [WOLB92]) and with binary resolution instead of hyper resolution (since the latter is not complete when combined with sos). Unit resulting resolution (ur_res) was added on advice of W. McCune, and improved the results slightly. The settings are contained in a file named 'settings'. The precedence order for every theorem is given in a file ordth-n.in for the unreduced version and ordredth-n.in for the reduced one. For th-n the output files were generated with the following commands:
cat settings th-n.in | otter > th-n.out
cat settings redth-n.in | otter > redth-n.out
cat settings ordth-n.in th-n.in | otter > ordth-n.out
cat settings ordredth-n.in redth-n.in | otter > ordredth-n.out. To avoid the generation of masses of useless output, a version of otter was used, in which the print-commands for derived clauses, derived demodulators etc. were removed. As can be seen from the results on the theorems over the *stopmode*-specification, the setting is incomplete. We tested various other settings on smaller examples (see [SR97]), but all resulted in fewer provable theorems.

6.1.2 Setheo

Setheo (V 3.3,[GLMS94]) can handle only clauses from an unsorted logic, and has no built-in equality predicate. Therefore we had to use a standard algorithm for encoding formulas as clauses. The resulting clauses are the same as the ones that Otter generates, except that clauses $\{x \neq t, L_1, \ldots L_1\}$ with $x \notin Vars(t)$ are optimized to $\{L_1[x \leftarrow t], \ldots L_1[x \leftarrow t]\}$. Sorts were encoded in the same way as for Otter. The equality predicate was explicitly axiomatized. Setheo was used with the '-wdr' option. For some theorems of *CompAssum2* and *Switching*, a new prerelease version of the components inwasm (V4.02b) and sam (V??) was used, which overcome some size limitations of the input file. The call to setheo is simply: setheo th-n.

6.1.3 Spass

Spass (v 0.81, [WGR96]) is a theorem prover for unsorted first-order logic with equality. Since unary predicates are treated by special "sort" inference rules we encoded sorts as unary predicates. For a few theorems, the experiments were done with v0.86 (both releases are beta-versions, which were kindly provided to us by C. Weidenbach). The output files were created by the following four calls to Spass:
(cat th-n ; echo "end_problem.") | spass -PGiven=0 -PProblem=0 -TimeLimit=120 -Stdin -DocProof=0 > res-th-n

(cat redth-n ; echo "end_problem.") | spass -PGiven=0 -PProblem=0 -TimeLimit=120 -Stdin -DocProof=0 > res-redth-n
cat th-n ; ordth-n | spass -PGiven=0 -PProblem=0 -TimeLimit=120 -Stdin -DocProof=0 -Auto=0 -Ordering=1 > res-ordth-n

cat redth-n ; ordth-n | spass -PGiven=0 -PProblem=0 -TimeLimit=120 -Stdin -DocProof=0 -Auto=0 -Ordering=1 > res-ordredth-n

6.2 The input and output files

The input files in DFG, Otter and Setheo-syntax are given as files WAM-DFG.tgz WAM-Otter.tgz and WAM-Setheo.tgz. Unzipping and untaring one of them (use either 'tar -xzf WAM-DFG.tar.gz' if you have the GNU-version of tar, or first 'gunzip WAM-DFG.tar.gz' then 'tar -xf WAM-DFG.tar') creates directories for every specification of the WAM which has theorems. Each directory contains a subdirectory 'DFG' resp. 'Otter' resp. 'Setheo-new'. The goals to prove for each specification are in that subdirectory. Additionally WAM-Otter.tgz contains the 'settings'-file.

6.3 Proof Times and Success Rates

The following tables give various summaries of the results we achieved with the provers. The first and second table give the number of proved theorems and the number of saved interactions for each specification and each prover setting. The third table gives a distribution of the automatically proved theorems in terms of complexity measured by interactions. The last table shows the influence of reducing the time given to the provers.

All results are given with (columns marked with 'red') and without axiom reduction. The settings in which the provers Otter and Spass were given a precedence order are marked with 'ord'. The provers Setheo, Otter and Spass are abbreviated as 'Se', 'Ot' and 'Sp'.

6.3.1 Summary 1: Proved Theorems

This table gives the number of proved theorems for each prover setting and each specification. The KIV column gives the number of theorems KIV can prove automatically. The 'ths' column gives the total number of theorems, the 'gos' column gives the number of noninductive goals generated from them. The 'nints' gives the number of interactions required to prove the theorems in KIV exclunding interactions for induction (this is the maximal number of interactions that can be saved using an automated prover), and the 'ints' columns gives the full number of interactions. The last line gives the percentage of automatically proved theorems.

	Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV	ths	gos	nints	ints
CompAssum2	0	1	0	2	1	2	0	2	0	0	3	3	3	0	0
PrologStack+F	23	26	26	28	31	33	16	23	18	23	28	26	47	46	61
PrologStack+H	19	20	20	20	24	25	12	18	11	17	19	16	29	15	20
PrologStack	53	53	53	56	59	59	38	57	22	59	51	52	80	66	77
PrologTree	14	14	14	15	14	15	10	12	3	15	13	17	19	12	14
Tree+Stack+F	9	13	16	19	15	16	7	13	3	9	7	32	38	113	118
Switching	6	30	29	31	27	30	3	29	3	28	38	14	48	41	64
codefun	37	25	38	38	38	38	35	35	37	38	34	26	38	12	13
enrset	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
enrterm	3	4	4	4	4	4	4	4	4	4	0	4	4	4	4
instr+clau	45	42	68	90	69	90	45	45	45	45	99	99	99	0	0
list	76	81	82	91	89	94	66	89	57	97	91	63	122	66	93
list-data	18	22	25	29	25	29	30	30	23	28	28	28	30	2	2
mode	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0
nat-basic1	42	40	41	39	41	39	43	41	43	41	43	44	44	1	1
nat-basic2	35	35	34	40	46	49	38	44	29	37	41	36	53	24	28
nat-sub	16	15	21	19	24	21	21	18	13	15	27	29	34	8	10
pair	4	4	5	5	5	5	5	5	5	5	5	5	5	0	0
paramterm	7	7	7	7	7	7	7	7	7	7	7	7	7	0	0
rmode	4	4	0	0	4	4	4	4	4	4	4	4	4	0	0
set	31	39	36	36	34	36	30	40	15	40	44	46	48	11	12
stopmode	3	3	0	0	3	3	3	3	3	3	3	3	3	0	0
term	6	6	16	16	15	16	6	15	6	16	16	16	16	0	0
Σ	453	486	536	586	577	617	425	536	353	533	603	572	773	421	517
%	58.6	62.8	69.3	75.8	74.6	79.8	54.9	69.3	45.6	68.9	78.0				

6.3.2 Summary 2: Saved Interactions

The following table gives each specification the number of interactions required to prove the theorems when using the various prover settings. The last column gives the full number of interactions required to prove the goals in KIV (excluding interactions for induction). The last lines gives the percentage of interactions still required when using the prover.

Spec	Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
Switching	39	38	39	38	39	38	41	38	41	38	41
PrologStack	47	48	46	47	45	47	57	25	66	37	66
PrologStack+F	37	34	35	32	31	30	39	38	46	36	46
PrologStack+H	6	6	6	6	4	4	13	6	15	9	15
Tree+Stack+F	102	98	93	79	98	96	110	99	112	110	113
nat-basic1	0	0	0	0	0	0	0	0	0	0	1
nat-basic2	12	12	12	11	2	2	5	4	18	17	24
list	39	38	36	34	41	35	52	38	64	33	66
list-data	0	0	0	0	0	0	0	0	1	0	2
set	10	1	0	0	5	0	1	0	10	0	11
PrologTree	9	9	9	7	9	7	10	10	12	7	12
enrset	0	0	0	0	0	0	0	0	0	0	0
nat-sub	5	7	5	7	4	6	2	3	5	6	8
CompAssum2	0	0	0	0	0	0	0	0	0	0	0
codefun	0	2	0	0	0	0	0	0	0	0	12
enrterm	1	0	0	0	0	0	0	0	0	0	4
instr+clau	0	0	0	0	0	0	0	0	0	0	0
mode	0	0	0	0	0	0	0	0	0	0	0
pair	0	0	0	0	0	0	0	0	0	0	0
paramterm	0	0	0	0	0	0	0	0	0	0	0
rmode	0	0	0	0	0	0	0	0	0	0	0
stopmode	0	0	0	0	0	0	0	0	0	0	0
term	0	0	0	0	0	0	0	0	0	0	0
Σ	307	293	281	261	278	265	330	261	390	293	421
%	73	70	67	62	66	63	78	62	93	70	100

6.3.3 Summary 3: Distribution of Interactions for Proved Theorems

The following table gives for each number of interactions and each prover setting the number of theorems that could be proved automatically. The last column gives the total number of theorems that required the given number of interactions.

ints	Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
0	388	415	457	500	490	526	374	455	332	459	603
1	42	45	52	54	62	61	34	50	17	48	85
2	12	13	11	14	12	15	8	13	3	14	34
3	4	4	6	7	5	5	3	5	0	3	18
4	2	3	5	5	2	4	2	5	0	5	12
5	4	5	4	4	4	4	2	4	0	3	9
6	0	0	0	0	1	1	1	1	0	0	1
7	0	0	0	0	0	0	0	1	0	0	3
8	1	1	1	1	1	1	1	2	1	1	2
9	0	0	0	1	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	0	3
11	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	1
15	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	1

6.3.4 Summary 4: Timing Characteristics

The following table gives for 1, 2, 5, 10, 30, 60 and 120 seconds and for each prover setting the numbers of proved theorems and the number of remaining interactions. The full number was 733 theorems and 421 interactions.

t/sec	Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord
1	358/343	391/337	409/358	451/340	434/346	474/336	71/419	201/397	66/419	204/410
2	367/341	410/333	445/330	480/316	463/331	503/315	114/418	324/390	104/415	305/391
5	388/335	435/324	472/313	501/292	496/317	527/297	195/396	423/360	170/411	419/373
10	413/332	458/313	488/294	513/287	517/307	539/291	284/392	453/343	221/402	446/357
30	430/318	468/302	503/291	530/271	537/298	553/283	325/372	497/297	280/395	500/327
60	444/309	477/294	509/281	580/264	550/279	611/268	388/354	513/279	327/391	515/314
120	453/307	486/293	536/281	586/261	577/278	615/266	425/330	536/261	353/390	533/293

6.4 Detailed Results

This section gives the results for each theorem and each prover setting in full details. The first column gives the the theorems named th-1, th-2, ... in the order, in which they were proved using the sequential test discipline. The second column gives the name of the theorem in KIV. Proof times are given in seconds. An entry '——' indicates, that the proof was not found. Entries '!!!' and '???' indicate, that the prover failed to produce an output file for various reasons (Setheowas not able to compile a few of the largest input files, Otter ran on a few occasions for half an hour, although a time limit of two minutes was given).

Specification: CompAssum2 from case-study WAM

- Number of theorems: 3
- Proof steps: 9
- Interactions: 0
- Number of noniductive goals: 3
- Noninductive Proof steps: 9
- Noninductive Interactions: 0

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-01	—	13.63	—	5.78	—	0.98	—	110.44	—	—	0/2
th-2	lem-02	!!!	—	—	—	—	—	—	—	—	—	0/6
th-3	lem-03	!!!	???	???	1.37	1.43	0.90	—	6.49	—	—	0/1
Σ		0	1	0	2	1	2	0	2	0	0	3
steps		9	8	9	8	9	8	9	8	9	9	9
ints		0	0	0	0	0	0	0	0	0	0	0

Specification: PrologStack+F from case-study WAM

- Number of theorems: 26
- Proof steps: 159
- Interactions: 61
- Number of noniductive goals: 47
- Noninductive Proof steps: 138
- Noninductive Interactions: 46

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-14	0.01	0.01	0.01	0.00	0.00	0.00	44.77	1.96	—	2.07	1/1
th-10	lem-09	—	—	—	—	—	—	—	—	—	—	0/4
th-11-is	lem-10	—	—	8.11	4.58	6.62	3.40	—	—	—	—	2/4
th-11-ib	lem-10	0.40	0.09	0.00	0.00	0.00	0.00	—	2.10	82.58	2.19	0/0
th-12-is	lem-17	—	—	—	—	46.84	24.40	—	—	—	—	0/2
th-12-ib	lem-17	0.01	0.01	0.00	0.00	0.00	0.00	46.41	1.96	45.52	2.05	0/0
th-12-pr	lem-17	0.04	0.01	0.00	0.00	0.89	0.51	17.02	0.89	17.87	1.00	0/1
th-13-is	lem-25	—	—	—	—	—	—	—	—	—	—	1/6
th-13-ib	lem-25	7.02	1.72	0.00	0.00	0.00	0.00	—	3.19	58.33	3.38	0/0
th-13-pr	lem-25	0.03	0.01	0.00	0.00	0.00	0.00	35.26	2.39	35.42	2.54	0/1
th-14	lem-02	—	4.23	—	7.36	—	—	—	—	—	—	2/4
th-15	lem-03	—	4.68	—	2.91	—	0.32	—	5.81	—	11.32	1/2

th-16-is	lem-05	—	—	—	—	???	—	—	—	—	105.44	2/7
th-16-ib	lem-05	0.01	0.01	0.00	0.00	0.00	0.00	46.61	2.01	46.36	2.12	0/0
th-16-pr	lem-05	—	—	—	—	—	—	—	—	—	—	0/1
th-17-is	lem-06	—	—	—	—	—	—	—	—	—	—	4/14
th-17-ib	lem-06	0.05	0.01	0.00	0.00	0.00	0.00	49.25	2.36	47.67	2.47	0/0
th-18-is-ib	lem-11	—	—	—	—	—	—	—	—	—	—	2/4
th-18-is	lem-11	—	—	—	—	—	—	—	—	—	—	0/1
th-18-ib	lem-11	0.60	0.03	0.00	0.00	0.00	0.00	—	3.74	85.00	3.82	0/0
th-18-pr	lem-11	0.04	0.01	1.14	0.50	0.60	0.26	30.12	2.34	37.51	2.95	0/1
th-19	lem-19	—	—	—	—	—	—	—	—	—	—	7/17
th-2	lem-15	—	—	—	—	—	—	—	—	—	—	3/6
th-20	lem-21	21.37	4.24	2.22	1.14	3.45	3.40	—	—	—	—	1/2
th-21	lem-22	0.18	0.04	0.96	0.61	0.96	0.59	—	—	—	—	1/2
th-22-is-ib	lem-23	—	—	—	—	0.00	0.00	—	—	—	—	0/2
th-22-ib	lem-23	—	3.85	0.00	0.00	0.00	0.00	47.79	0.75	60.14	0.78	0/0
th-23-is	lem-24	—	—	16.37	9.49	12.85	6.40	—	—	—	—	0/2
th-23-ib	lem-24	2.67	0.52	0.00	0.00	0.00	0.00	—	1.49	60.78	1.48	0/0
th-23-pr	lem-24	0.04	0.01	1.00	0.51	1.06	0.53	14.65	0.56	15.34	0.61	0/1
th-24	lem-26	—	—	—	—	5.54	4.17	—	—	—	—	1/6
th-25	rew-1	0.01	0.01	2.03	0.32	0.10	0.01	—	55.61	—	—	0/1
th-26	lem-12	0.01	0.01	0.55	0.35	0.53	0.32	107.51	12.65	—	16.37	5/8
th-3-is	lem-04	—	—	—	—	15.93	11.14	—	—	—	—	0/2
th-3-ib	lem-04	0.02	0.01	0.00	0.00	0.00	0.00	44.53	1.84	44.47	1.93	0/0
th-4-is	lem-08	—	—	—	—	—	—	—	—	—	—	3/8
th-4-ib	lem-08	0.03	0.01	0.00	0.00	0.00	0.00	45.30	2.02	44.38	2.13	0/0
th-5	lem-13	—	—	—	—	—	—	—	—	—	—	3/6
th-6-is	lem-16	—	—	—	—	37.17	24.74	—	—	—	—	3/8
th-6-ib	lem-16	0.25	0.10	0.00	0.00	0.00	0.00	46.24	2.12	45.54	2.26	0/0
th-7-is	lem-20	—	—	—	—	—	40.02	—	—	—	—	0/4
th-7-ib	lem-20	0.06	0.01	0.00	0.00	0.00	0.00	44.92	2.97	45.38	3.03	0/0
th-8	lem-18	17.83	0.25	0.24	0.04	0.44	0.07	55.15	4.24	—	7.82	1/2
th-9-is-ib	lem-01	—	—	—	—	—	—	—	—	—	—	3/6
th-9-is	lem-01	—	—	???	—	—	—	—	—	—	—	0/1
th-9-ib	lem-01	2.44	0.65	0.00	0.00	0.00	0.00	—	3.32	53.14	3.43	0/0
th-9-pr	lem-01	0.01	0.01	0.00	0.00	0.00	0.00	26.92	2.06	28.07	2.15	0/1
Σ		23	26	26	28	31	33	16	23	18	23	28
steps		128	124	124	120	109	105	130	129	138	123	138
ints		37	34	35	32	31	30	39	38	46	36	46

Specification: PrologStack+H from case-study WAM

- Number of theorems: 16
- Proof steps: 84
- Interactions: 20
- Number of noniductive goals: 29
- Noninductive Proof steps: 72
- Noninductive Interactions: 15

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1-is	lem-06	—	—	—	—	27.41	25.33	—	—	—	—	0/2
th-1-ib	lem-06	0.01	0.01	0.00	0.00	0.00	0.00	43.82	1.98	43.40	2.03	0/0
th-1-pr	lem-06	0.01	0.01	1.20	0.75	1.29	0.75	16.09	0.89	16.96	0.97	0/1
th-10-is	lem-08	—	—	—	—	—	—	—	—	—	—	0/4
th-10-ib	lem-08	—	6.26	0.00	0.00	0.00	0.00	44.55	1.30	56.89	1.40	0/0
th-11-is	lem-09	—	—	—	—	21.02	1.63	—	—	—	—	0/8
th-11-ib	lem-09	0.39	0.03	0.00	0.00	0.00	0.00	—	2.43	—	1.40	0/0
th-12-is	lem-11	—	—	—	—	—	—	—	—	—	—	3/18
th-12-ib	lem-11	0.03	0.01	0.00	0.00	0.00	0.00	45.20	1.74	44.80	1.81	0/0
th-13	lem-12	0.24	0.05	1.52	1.23	1.56	1.23	—	14.65	—	112.80	4/5

th-14-is	lem-13	—	—	—	—	8.92	1.71	—	—	—	14.52	1/4
th-14-ib	lem-13	1.43	0.12	0.00	0.00	0.00	0.00	46.99	0.61	45.59	0.71	0/0
th-15	lem-14	0.94	0.20	7.11	3.49	7.22	3.70	—	15.08	—	—	1/2
th-16	lem-16	0.44	0.13	2.12	1.39	2.50	1.68	—	25.20	—	—	1/1
th-2	lem-01	17.80	0.17	1.14	0.14	1.97	0.08	—	—	—	—	0/1
th-3-is	lem-04	—	—	—	—	—	23.87	—	—	—	90.27	0/2
th-3-ib	lem-04	0.03	0.01	0.00	0.00	0.00	0.00	45.01	1.36	43.75	1.34	0/0
th-3-pr	lem-04	0.03	0.01	—	—	—	—	47.35	1.64	—	1.58	0/1
th-4	lem-10	0.01	0.01	1.32	1.47	1.49	1.54	109.63	8.77	—	105.91	1/4
th-5	lem-15	0.01	0.01	1.26	1.52	1.44	1.49	86.52	8.50	—	—	1/1
th-6-is	lem-02	—	—	17.75	10.80	14.33	7.41	—	—	—	—	0/2
th-6-ib	lem-02	2.86	0.44	0.00	0.00	0.00	0.01	—	67.79	55.78	1.55	0/0
th-6-pr	lem-02	0.03	0.01	1.29	0.69	1.39	0.72	13.48	0.55	14.54	0.64	0/1
th-7-is	lem-03	—	—	—	—	—	—	—	—	—	—	1/6
th-7-ib	lem-03	9.52	1.79	0.00	0.00	0.00	0.00	—	—	56.02	3.25	0/0
th-7-pr	lem-03	0.04	0.01	0.00	0.00	0.00	0.00	35.44	2.36	36.05	2.57	0/1
th-8	lem-05	94.91	11.25	5.01	2.26	7.66	3.45	—	19.25	—	—	1/4
th-9-is	lem-07	—	—	—	—	4.28	2.42	—	—	—	—	1/4
th-9-ib	lem-07	5.01	1.31	0.00	0.00	0.00	0.00	45.07	2.34	44.77	2.43	0/0
Σ		19	20	20	20	24	25	12	18	11	17	19
steps		61	61	60	60	46	45	69	61	72	61	72
ints		6	6	6	6	4	4	13	6	15	9	15

Specification: PrologStack from case-study WAM

- Number of theorems: 52
- Proof steps: 275
- Interactions: 77
- Number of noniductive goals: 80
- Noninductive Proof steps: 249
- Noninductive Interactions: 66

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-10	0.01	0.01	0.17	0.10	0.16	0.10	65.00	5.95	—	9.60	1/4
th-10	lem-06	0.01	0.01	0.00	0.00	0.01	0.00	52.81	3.65	—	10.83	3/4
th-11-is-is	lem-21	—	—	—	—	—	—	—	16.11	—	—	7/14
th-11-is-ib	lem-21	0.03	0.01	0.00	0.00	0.00	0.00	40.86	0.24	38.99	0.22	0/0
th-11-is	lem-21	—	—	—	—	—	???	—	—	—	—	4/9
th-11-ib	lem-21	0.01	0.01	0.00	0.00	0.00	0.00	39.47	2.93	39.33	2.84	0/0
th-12	cdr-1	14.54	0.09	—	0.48	—	0.14	—	0.40	100.05	0.33	0/2
th-13	lem-07	0.09	0.04	0.49	0.36	0.49	0.33	71.89	6.61	—	—	1/2
th-14	lem-40	0.01	—	3.65	—	3.49	—	—	—	—	—	1/2
th-15	lem-03	—	—	19.91	—	40.50	—	—	—	—	—	1/6
th-16-is	lem-16	—	—	11.37	1.56	15.23	1.58	—	0.89	—	3.14	1/3
th-16-ib	lem-16	0.01	0.01	0.00	0.00	0.00	0.00	41.20	0.21	44.47	0.22	0/0
th-17	lem-17	—	—	1.88	—	2.53	—	—	—	—	—	2/5
th-18-is	lem-24	—	—	—	—	—	—	—	—	—	35.76	0/4
th-18-ib	lem-24	0.02	0.01	0.00	0.00	0.00	0.00	40.03	1.23	39.41	1.25	0/0
th-19	lem-27	46.98	18.05	—	—	—	—	—	23.58	—	5.03	2/3
th-2-is	lem-19	—	—	—	—	—	—	—	2.90	—	6.44	0/4
th-2-ib	lem-19	0.01	0.01	0.00	0.00	0.00	0.00	39.08	0.23	38.78	0.26	0/0
th-20	lem-33	—	—	—	—	—	—	—	—	—	—	0/18
th-21-is	lem-35	—	—	—	—	16.05	10.34	—	—	—	22.41	0/2
th-21-ib	lem-35	0.02	0.02	0.00	0.00	0.00	0.00	42.53	1.49	44.91	1.44	0/0
th-21-pr	lem-35	0.03	0.01	1.14	0.68	1.32	0.76	11.67	0.44	12.23	0.51	0/1
th-22-is	lem-36	—	—	—	—	—	—	—	—	—	—	0/4
th-22-ib	lem-36	0.03	0.01	0.00	0.00	0.00	0.00	41.71	1.34	39.66	1.32	0/0

th-23	lem-37	0.72	0.10	0.00	0.00	0.01	0.00	43.68	1.49	—	1.69	0/6
th-24	lem-39	—	—	—	—	—	—	—	57.60	—	—	4/11
th-25	lem-43	—	—	—	119.16	—	0.20	—	1.60	—	0.84	0/6
th-26-is	lem-45	—	—	—	—	—	—	—	—	—	—	2/6
th-26-ib	lem-45	0.03	0.02	0.00	0.00	0.00	0.00	41.74	1.63	40.03	1.63	0/0
th-26-pr	lem-45	0.01	0.01	0.09	0.07	0.09	0.07	14.51	0.58	14.91	0.68	0/1
th-27	lem-ax-09	0.31	0.06	0.78	0.21	0.83	0.21	—	37.24	—	112.62	0/1
th-28	lem-05	0.01	0.01	0.00	0.01	0.01	0.00	44.91	2.05	—	6.92	0/1
th-29	lem-09	0.01	0.01	0.00	0.00	0.01	0.00	43.72	1.98	—	2.18	0/1
th-3-is	lem-01	—	35.97	11.04	7.53	7.15	4.84	—	—	—	23.73	0/2
th-3-ib	lem-01	0.02	0.01	0.00	0.00	0.00	0.00	39.64	1.20	38.51	1.30	0/0
th-3-pr	lem-01	0.03	0.01	0.06	0.05	0.07	0.06	10.37	0.42	10.67	0.48	0/1
th-30	lem-11	—	—	—	—	—	—	—	—	—	42.49	2/4
th-31-is	lem-13	—	—	—	—	—	—	—	—	—	—	3/8
th-31-ib	lem-13	—	38.89	—	—	28.23	26.62	—	—	—	—	0/0
th-31-pr	lem-13	—	—	—	—	—	—	—	—	—	—	0/1
th-32	lem-14	0.01	0.01	0.63	0.47	0.59	0.45	48.64	3.34	—	19.02	2/6
th-33	lem-18	—	—	3.41	0.07	3.53	0.08	—	1.31	—	1.84	1/6
th-34	lem-23	0.01	0.01	0.01	0.00	0.01	0.00	46.59	1.77	—	3.10	0/1
th-35-is-ib	lem-25	33.68	1.25	1.82	0.35	1.91	0.37	—	5.54	—	—	0/2
th-35-is	lem-25	0.28	0.05	3.00	0.42	3.18	0.44	46.63	1.01	—	1.05	0/1
th-35-ib	lem-25	0.01	0.01	0.00	0.00	0.00	0.00	52.59	1.29	46.91	0.97	0/0
th-36-is-ib2	lem-26	34.87	0.54	—	0.47	1.96	0.36	—	—	—	—	0/2
th-36-is2	lem-26	0.23	0.04	3.31	0.42	3.43	0.43	44.13	1.02	—	1.13	0/1
th-36-ib2	lem-26	0.01	0.01	0.00	0.00	0.00	0.01	40.93	0.90	41.17	0.95	0/0
th-36-is-ib	lem-26	19.91	0.34	—	—	3.61	0.33	—	—	—	—	0/2
th-36-is	lem-26	0.23	0.03	5.91	0.47	4.68	0.48	43.77	1.02	—	1.15	0/1
th-36-ib	lem-26	0.01	0.01	0.00	0.00	0.00	0.00	40.38	0.80	40.10	0.89	0/0
th-36-pr	lem-26	0.08	0.02	9.97	0.16	8.96	0.17	11.16	0.43	11.77	0.50	0/3
th-37	lem-28	—	—	2.57	1.87	3.25	2.29	—	8.06	—	5.44	1/4
th-38	lem-29	20.54	2.33	—	—	—	—	—	—	—	8.53	2/5
th-39	lem-30	0.04	0.02	0.53	0.05	0.53	0.05	76.07	6.38	—	11.59	0/2
th-4-is-ib	lem-02	—	—	3.07	1.73	0.42	0.27	94.24	9.13	—	—	1/4
th-4-is	lem-02	0.28	0.05	2.86	6.17	3.12	6.13	41.58	2.77	49.91	2.77	0/1
th-4-ib	lem-02	0.01	0.01	0.00	0.00	0.00	0.00	38.42	2.35	38.74	2.61	0/0
th-40	lem-31	—	—	—	—	—	—	—	9.06	—	2.97	3/7
th-41	lem-32	24.80	—	1.08	0.68	1.46	1.15	—	1.54	—	3.22	0/2
th-42	lem-34	0.13	0.04	0.24	0.21	0.26	0.20	—	7.32	—	7.05	0/1
th-43	lem-38	0.20	0.04	1.95	0.86	1.95	0.87	—	—	—	98.70	0/2
th-44	lem-41	29.48	1.07	—	1.47	1.52	1.27	—	1.92	—	0.98	0/2
th-45	lem-42	0.09	0.05	0.75	1.40	0.81	1.34	66.94	4.34	—	1.71	0/1
th-46-is	lem-44	—	—	—	—	—	—	—	17.94	—	—	8/15
th-46-ib	lem-44	0.04	0.01	0.00	0.00	0.00	0.00	43.07	0.25	42.24	0.27	0/0
th-46-pr	lem-44	0.02	0.01	0.88	0.09	0.91	0.10	16.59	0.18	17.27	0.20	0/1
th-47	lem-46	0.32	0.10	—	118.29	1.81	1.24	—	27.96	—	—	1/2
th-48	lem-47	5.43	0.86	—	13.47	—	42.92	—	19.26	—	6.28	2/3
th-49	lem-48	0.21	0.05	0.67	0.42	0.68	0.42	106.48	16.32	—	30.13	0/2
th-5	lem-08	—	—	—	—	—	—	—	—	—	57.71	4/10
th-50	lem-49	0.01	0.01	0.27	0.16	0.27	0.16	80.48	10.26	—	28.42	0/2
th-51	lem-50	47.24	0.51	3.75	2.61	3.89	2.63	—	—	—	84.08	1/4
th-52	lem-51	0.01	0.01	1.22	0.77	1.27	0.76	—	105.99	—	18.50	2/4
th-6	lem-12	—	—	???	???	—	???	—	—	—	—	0/2
th-7	lem-15	0.01	0.01	0.00	0.00	0.00	0.00	47.63	2.19	—	1.88	1/1
th-8	lem-20	—	—	—	—	—	—	—	—	—	—	2/7
th-9-is	lem-04	—	—	1.72	1.13	1.62	1.07	—	—	—	47.07	1/4
th-9-ib	lem-04	0.01	0.01	0.00	0.00	0.00	0.00	40.18	1.81	42.75	1.99	0/0
Σ		53	53	53	56	59	59	38	57	22	59	51
steps		204	205	191	190	186	188	224	151	246	166	249
ints		47	48	46	47	45	47	57	25	66	37	66

Specification: PrologTree from case-study WAM

- Number of theorems: 17
- Proof steps: 77
- Interactions: 14

- Number of noniductive goals: 19
- Noninductive Proof steps: 75
- Noninductive Interactions: 12

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1-is	lem-05	—	—	???	—	—	—	—	—	—	—	2/13
th-1-ib	lem-05	0.04	0.01	0.00	0.00	0.00	0.00	57.75	3.09	44.64	1.52	0/0
th-10	lem-12	1.56	0.46	10.58	24.12	9.50	5.49	—	—	—	27.66	1/6
th-11	lem-13	—	—	—	—	—	—	—	—	—	—	3/9
th-12	lem-14	0.03	0.01	0.14	0.08	0.15	0.09	103.83	9.26	—	115.30	2/2
th-13	lem-15	0.01	0.01	0.01	0.00	0.00	0.00	50.18	0.39	—	4.63	0/2
th-14	lem-16	5.98	0.97	2.92	1.34	3.13	1.34	—	56.85	—	12.56	0/2
th-15	lem-17	0.01	0.01	0.00	0.00	0.01	0.00	42.48	1.27	99.19	1.28	0/1
th-16	lem-18	0.01	0.01	0.00	0.01	0.01	0.00	43.50	1.43	—	1.42	0/1
th-17-is	map-lemma	—	—	—	86.13	—	27.05	—	—	—	—	2/6
th-17-ib	map-lemma	0.39	0.02	0.00	0.00	0.00	0.00	43.18	1.55	42.32	1.69	0/0
th-2	lem-01	0.01	0.01	0.00	0.00	0.00	0.00	104.78	10.61	—	1.42	0/1
th-3	lem-02	—	—	—	—	—	—	—	—	—	115.35	2/10
th-4	lem-03	0.01	0.01	0.00	0.00	0.00	0.00	101.61	10.89	—	1.47	0/1
th-5	lem-04	—	—	—	—	—	—	—	—	—	—	0/14
th-6	lem-06	0.01	0.01	0.29	0.21	0.30	0.21	69.60	6.01	—	9.84	0/2
th-7	lem-08	12.00	0.81	1.65	0.24	1.69	0.25	—	35.61	—	22.25	0/2
th-8	lem-09	0.01	0.01	0.01	0.00	0.00	0.00	50.29	2.14	—	1.82	0/1
th-9	lem-10	0.39	0.13	2.18	0.21	2.48	0.23	—	—	—	13.58	0/2
Σ		14	14	14	15	14	15	10	12	3	15	13
steps		64	64	64	59	64	59	72	70	75	55	75
ints		9	9	9	7	9	7	10	10	12	7	12

Specification: Switching from case-study WAM

- Number of theorems: 14
- Proof steps: 175
- Interactions: 64
- Number of noniductive goals: 48
- Noninductive Proof steps: 149
- Noninductive Interactions: 41

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-09	0.02	0.01	1.33	0.03	1.33	0.03	—	35.26	—	46.72	1/2
th-10-is-is-ib	lem-06	!!!	—	—	—	—	—	—	—	—	—	2/8
th-10-is-is	lem-06	!!!	—	—	—	—	—	—	—	—	—	1/2
th-10-is-ib	lem-06	!!!	0.03	0.00	0.00	0.01	0.00	—	11.36	—	17.52	0/0
th-10-ib-is-ib	lem-06	!!!	0.03	0.00	0.00	0.00	0.00	—	13.01	—	19.14	0/0
th-10-ib-is	lem-06	!!!	0.03	0.00	0.00	0.00	0.00	—	13.34	—	19.79	0/1
th-10-ib-ib	lem-06	!!!	0.02	0.00	0.00	0.00	0.00	—	3.25	—	3.72	0/0
th-10-is	lem-06	!!!	???	—	—	—	—	—	—	—	—	0/3
th-10-ib	lem-06	!!!	0.02	0.12	0.01	0.13	0.01	113.31	2.47	114.98	2.86	0/1
th-10-pr	lem-06	!!!	0.01	3.64	0.63	3.16	0.71	—	2.78	—	3.23	0/1
th-11	lem-04	0.01	0.01	0.11	0.01	0.11	0.00	—	—	—	—	0/1
th-12-is-is-ib	lem-03	!!!	—	???	—	???	—	—	—	—	—	5/16
th-12-is-is	lem-03	!!!	—	—	—	—	—	—	4.04	—	4.75	1/2
th-12-is-ib	lem-03	!!!	0.05	0.00	0.00	0.00	0.00	—	—	—	—	0/0
th-12-ib-is-ib	lem-03	!!!	0.04	0.00	0.00	0.00	0.00	—	—	—	—	0/0
th-12-ib-ib	lem-03	!!!	0.04	0.00	0.00	0.00	0.00	—	3.30	—	3.81	0/0
th-12-is	lem-03	!!!	—	—	—	—	—	—	—	—	—	0/3

th-12-ib	lem-03	!!!	1.92	60.32	17.89	35.09	11.65	—	3.13	—	3.62	0/1
th-12-pr	lem-03	!!!	1.98	???	—	???	—	—	3.33	—	3.91	0/1
th-13-is-is-ib-ib	lem-02	!!!	—	—	—	—	—	—	—	—	—	0/2
th-13-is-is-ib	lem-02	!!!	—	—	—	—	—	—	—	—	—	0/1
th-13-is-is	lem-02	!!!	—	—	—	—	—	—	93.06	—	—	0/1
th-13-is-ib	lem-02	!!!	0.01	0.00	0.00	0.00	0.00	—	2.44	—	2.40	0/0
th-13-is	lem-02	!!!	0.04	20.36	0.47	17.40	0.50	—	75.89	—	114.41	0/1
th-13-ib	lem-02	!!!	5.21	0.64	0.01	0.20	0.00	—	2.24	—	2.86	0/0
th-13-pr	lem-02	!!!	0.01	1.56	0.11	1.59	0.13	64.57	0.54	65.80	0.58	0/1
th-14	elim-carcdr	???	—	???	0.86	—	—	—	0.66	—	0.73	0/2
th-2	lem-12	1.08	0.04	11.90	0.19	—	0.42	—	3.60	—	3.53	0/2
th-3	lem-10	0.01	0.01	1.31	0.03	1.32	0.03	—	31.89	—	59.90	1/1
th-4	lem-01	???	—	—	—	—	—	—	—	—	—	2/3
th-5-is-is-ib-ib	lem-05	!!!	—	—	—	—	—	—	—	—	—	0/8
th-5-is-is	lem-05	!!!	0.08	—	46.28	—	48.87	—	—	—	—	1/2
th-5-is-ib	lem-05	!!!	0.01	0.00	0.00	0.00	0.00	—	15.58	—	14.05	0/0
th-5-is	lem-05	!!!	0.05	30.00	1.31	26.45	1.29	—	—	—	—	0/1
th-5-ib	lem-05	!!!	0.08	0.34	0.01	0.01	0.00	—	5.08	—	6.53	0/0
th-6	lem-13	???	0.05	12.06	0.22	—	0.17	—	3.65	—	3.70	0/2
th-7-is	lem-11	—	—	???	???	???	???	—	—	—	—	25/62
th-7-ib	lem-11	???	—	0.35	0.02	0.02	0.01	—	13.56	—	19.88	0/0
th-8-is-is	lem-08	!!!	—	—	—	—	—	—	—	—	—	2/6
th-8-is-ib	lem-08	!!!	0.01	0.00	0.00	0.00	0.00	—	12.61	—	24.43	0/0
th-8-is	lem-08	!!!	—	—	—	—	—	—	—	—	—	0/3
th-8-ib	lem-08	!!!	2.91	0.54	0.03	0.20	0.00	—	11.13	—	25.02	0/0
th-8-pr	lem-08	!!!	0.01	0.00	0.00	0.01	0.01	—	3.32	—	3.71	0/1
th-9-is-is	lem-07	???	—	—	—	—	—	—	—	—	—	0/4
th-9-is-ib	lem-07	0.01	0.01	0.00	0.00	0.00	0.00	—	14.88	—	20.38	0/0
th-9-is	lem-07	—	—	—	—	—	—	—	—	—	—	0/3
th-9-ib	lem-07	???	0.18	0.53	0.01	0.20	0.00	—	13.65	—	19.34	0/0
th-9-pr	lem-07	0.01	0.01	0.00	0.00	0.01	0.00	103.11	2.28	105.38	2.48	0/1
Σ		6	30	29	31	27	30	3	29	3	28	38
steps		147	145	146	144	148	145	149	144	149	144	149
ints		39	38	39	38	39	38	41	38	41	38	41

Specification: Tree+Stack+F from case-study WAM

- Number of theorems: 32
- Proof steps: 229
- Interactions: 118
- Number of noniductive goals: 38
- Noninductive Proof steps: 224
- Noninductive Interactions: 113

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-01	0.01	0.01	0.01	0.00	0.01	0.01	108.93	3.27	—	23.82	1/1
th-10	lem-11	—	—	—	4.94	—	68.83	—	—	—	—	2/4
th-11	lem-12	—	—	—	—	—	—	—	—	—	—	1/2
th-12	lem-13	0.01	0.01	0.00	0.00	0.01	0.00	112.78	1.92	—	2.12	0/1
th-13	lem-14	—	—	—	—	—	—	—	—	—	—	1/2
th-14	lem-19	28.92	5.31	47.42	5.55	31.97	7.38	—	38.09	—	—	3/3
th-15	lem-20	—	33.17	—	—	—	—	—	110.34	—	—	2/2
th-16	lem-22	—	—	—	—	—	—	—	—	—	—	2/4
th-17	lem-23	—	—	—	—	—	—	—	—	—	—	1/4
th-18	lem-24	—	30.48	5.42	2.53	7.29	3.16	—	—	—	—	1/2
th-19	lem-25	—	19.43	—	—	13.43	2.74	—	63.88	—	—	1/2
th-2	lem-02	—	—	—	—	—	—	—	—	—	—	10/22
th-20	lem-26	—	—	—	11.44	—	—	—	—	—	—	3/7

th-21	lem-27	—	—	2.28	0.53	—	—	—	—	—	—	3/7
th-22-is	lem-29	—	—	—	—	—	—	—	—	—	—	2/8
th-22-ib	lem-29	—	—	0.56	0.66	8.53	2.15	—	109.55	—	5.66	0/0
th-22-pr	lem-29	—	—	—	—	—	—	—	—	—	—	0/5
th-23-is	lem-30	—	—	—	—	???	—	—	—	—	—	4/8
th-23-ib	lem-30	—	4.15	0.00	0.00	0.00	0.00	114.27	1.02	—	1.08	0/0
th-24	lem-31	—	—	—	—	—	—	—	—	—	—	10/20
th-25	lem-32	—	—	—	—	—	—	—	—	—	—	1/5
th-26	lem-33	—	—	—	—	—	—	—	—	—	—	10/28
th-27-is	lem-34	—	—	—	—	—	—	—	—	—	—	1/6
th-27-ib	lem-34	0.88	0.04	0.00	0.00	0.00	0.00	—	90.59	—	4.50	0/0
th-28	lem-35	—	—	—	—	—	—	—	—	—	—	5/10
th-29	lem-36	—	—	—	—	—	—	—	—	—	—	5/9
th-3	lem-08	0.01	0.01	0.01	0.00	0.00	0.00	113.43	3.86	—	28.86	1/1
th-30-is	lem-37	—	—	1.95	1.21	2.09	1.23	—	—	—	—	1/4
th-30-ib	lem-37	0.01	0.01	0.00	0.00	0.00	0.00	118.64	3.90	116.07	4.21	0/0
th-31-is	lem-38	—	—	2.06	1.29	2.18	1.31	—	—	—	—	1/4
th-31-ib	lem-38	0.01	0.01	0.00	0.00	0.00	0.00	116.32	4.58	115.88	5.19	0/0
th-32	lem-21	0.01	0.01	0.00	0.00	0.72	0.01	31.45	0.05	32.04	0.03	1/1
th-4	lem-15	—	—	—	—	—	—	—	—	—	—	4/4
th-5	lem-16	—	—	2.04	0.91	—	—	—	—	—	—	3/3
th-6	lem-17	—	—	—	—	—	—	—	—	—	—	5/5
th-7	lem-18	0.01	0.01	0.01	0.01	0.00	0.01	—	3.93	—	—	5/5
th-8	lem-03	—	—	—	13.39	—	—	—	—	—	—	9/21
th-9	lem-04	—	—	—	—	—	—	—	—	—	—	14/14
Σ		9	13	16	19	15	16	7	13	3	9	7
steps		218	215	203	174	210	207	224	216	224	224	224
ints		102	98	93	79	98	96	110	99	112	110	113

Specification: codefun from case-study WAM

- Number of theorems: 26
- Proof steps: 37
- Interactions: 13
- Number of noniductive goals: 38
- Noninductive Proof steps: 36
- Noninductive Interactions: 12

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-02	0.01	0.01	0.01	0.00	0.01	0.00	8.98	1.80	18.66	1.87	1/1
th-10	lem-17	—	33.82	0.00	0.00	0.00	0.00	9.29	1.72	9.53	1.79	0/1
th-11	lem-18	0.61	0.28	0.00	0.00	0.00	0.00	9.22	1.90	9.41	2.02	0/1
th-12	lem-21	9.88	4.20	0.00	0.00	0.00	0.00	9.55	1.76	9.55	1.78	0/1
th-13	lem-22	0.03	0.02	0.00	0.00	0.00	0.00	9.31	1.91	9.45	2.05	0/1
th-14	lem-23	0.62	0.28	0.00	0.00	0.00	0.00	9.24	1.92	9.52	1.99	0/1
th-15	lem-24	0.62	0.28	0.00	0.00	0.00	0.00	9.15	2.02	9.54	2.14	0/1
th-16	lem-03	0.03	0.01	0.00	0.00	0.00	0.00	9.13	1.87	9.50	1.96	0/1
th-17	lem-04	0.02	0.01	0.00	0.01	0.00	0.00	9.27	1.87	9.44	1.97	0/1
th-18	lem-05	0.03	0.02	0.00	0.00	0.00	0.00	9.25	1.87	9.57	1.98	0/1
th-19	lem-06	0.03	0.01	0.00	0.00	0.00	0.00	9.26	1.85	9.58	1.96	0/1
th-2	lem-01	9.40	0.22	0.07	0.07	0.02	0.02	17.19	4.63	20.65	1.91	1/1
th-20	lem-07	0.03	0.02	0.00	0.00	0.00	0.00	9.32	1.86	9.42	1.97	0/1
th-21	lem-08	0.01	0.01	0.00	0.00	0.00	0.00	9.20	1.84	9.59	1.95	0/1
th-22	lem-09	0.04	0.02	0.00	0.00	0.00	0.00	9.28	1.92	9.52	1.93	0/1
th-23	lem-19	0.04	0.02	0.00	0.00	0.00	0.00	9.19	1.92	9.46	1.99	0/1
th-24	lem-20	0.03	0.01	0.00	0.00	0.00	0.00	9.32	2.06	9.52	2.13	0/1
th-25-is11	lem-25	0.01	???	0.00	0.00	0.07	0.07	9.85	3.51	9.39	3.38	0/0
th-25-is10	lem-25	0.01	???	0.00	0.00	0.38	0.17	—	—	17.80	4.50	0/0

th-25-is9	lem-25	0.01	???	0.00	0.00	0.37	0.17	—	—	17.66	4.52	0/0
th-25-is8	lem-25	0.01	???	0.00	0.00	0.39	0.26	—	—	—	39.01	0/0
th-25-is7	lem-25	0.01	???	0.12	0.11	0.09	0.09	10.22	3.82	9.39	3.44	0/0
th-25-is6	lem-25	0.01	???	0.11	0.11	0.09	0.08	10.31	3.84	9.34	3.41	0/0
th-25-is5	lem-25	0.01	???	0.12	0.11	0.08	0.08	10.31	3.85	9.36	3.42	0/0
th-25-is4	lem-25	0.01	???	0.12	0.12	0.09	0.08	10.37	3.84	9.33	3.41	0/0
th-25-is3	lem-25	0.01	???	0.12	0.11	0.09	0.08	10.31	3.84	9.39	3.42	0/0
th-25-is2	lem-25	0.17	???	0.00	0.00	0.00	0.00	10.03	3.64	19.16	4.46	0/0
th-25-is	lem-25	0.01	???	0.00	0.00	0.00	0.00	2.54	1.16	2.85	1.26	0/0
th-25-ib	lem-25	0.01	???	0.00	0.00	0.00	0.00	5.39	2.12	5.60	2.24	0/0
th-25-pr	lem-25	3.39	???	2.10	2.06	0.88	0.85	50.09	35.20	11.56	4.39	2/4
th-26	lem-26	0.17	0.08	0.00	0.00	0.00	0.00	2.73	0.70	6.02	1.37	8/8
th-3	lem-10	0.61	0.26	0.00	0.00	0.00	0.00	9.12	1.88	9.35	1.95	0/1
th-4	lem-11	0.58	0.26	0.00	0.00	0.00	0.00	9.10	1.87	9.30	1.96	0/1
th-5	lem-12	0.62	0.26	0.00	0.00	0.00	0.00	9.13	1.84	9.21	1.94	0/1
th-6	lem-13	0.61	0.26	0.00	0.00	0.00	0.00	9.21	1.85	9.23	1.94	0/1
th-7	lem-14	0.59	0.26	0.00	0.00	0.00	0.00	9.08	1.85	9.37	1.95	0/1
th-8	lem-15	0.40	0.17	0.00	0.00	0.00	0.00	9.09	1.84	9.32	1.93	0/1
th-9	lem-16	0.63	0.28	0.00	0.01	0.00	0.00	9.17	1.86	9.46	1.90	0/1
Σ		37	25	38	38	38	38	35	35	37	38	34
steps		26	29	26	26	26	26	26	26	26	26	36
ints		0	2	0	0	0	0	0	0	0	0	12

Specification: enrset from case-study WAM

- Number of theorems: 1
- Proof steps: 1
- Interactions: 0
- Number of noniductive goals: 1
- Noninductive Proof steps: 1
- Noninductive Interactions: 0

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-01	0.01	0.01	0.11	0.07	0.08	0.05	2.04	0.32	2.08	0.33	0/1
Σ		1	1	1	1	1	1	1	1	1	1	1
steps		1	1	1	1	1	1	1	1	1	1	1
ints		0	0	0	0	0	0	0	0	0	0	0

Specification: enrterm from case-study WAM

- Number of theorems: 4
- Proof steps: 8
- Interactions: 4
- Number of noniductive goals: 4
- Noninductive Proof steps: 8
- Noninductive Interactions: 4

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-1	0.01	0.01	0.00	0.00	0.00	0.00	2.05	0.23	2.11	0.27	1/2
th-2	lem-2	0.01	0.01	0.01	0.00	0.00	0.00	2.12	0.22	2.12	0.23	1/2
th-3	lem-3	0.01	0.01	0.00	0.00	0.00	0.00	2.13	0.22	2.11	0.25	1/2
th-4	lem-4	—	75.67	0.06	0.05	0.07	0.05	2.19	0.28	2.94	0.45	1/2

Σ	3	4	4	4	4	4	4	4	4	4	0
steps	5	4	4	4	4	4	4	4	4	4	8
ints	1	0	0	0	0	0	0	0	0	0	4

Specification: instr+clau from case-study WAM

- Number of theorems: 99
- Proof steps: 153
- Interactions: 0
- Number of noniductive goals: 99
- Noninductive Proof steps: 153
- Noninductive Interactions: 0

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-1	—	—	—	37.53	—	38.69	—	—	—	—	0/2
th-10	lem-18	—	—	106.08	35.41	107.74	36.86	—	—	—	—	0/2
th-11	lem-19	—	—	107.27	35.54	107.52	36.59	—	—	—	—	0/2
th-12	lem-2	—	—	106.19	34.61	109.63	36.84	—	—	—	—	0/2
th-13	lem-20	—	—	105.89	44.99	107.20	37.03	—	—	—	—	0/2
th-14	lem-21	—	—	105.63	42.12	107.84	36.68	—	—	—	—	0/2
th-15	lem-3	—	—	111.99	34.40	111.19	37.17	—	—	—	—	0/2
th-16	lem-4	—	—	114.53	33.66	115.66	36.76	—	—	—	—	0/2
th-17	lem-5	—	—	112.33	34.53	111.03	36.53	—	—	—	—	0/2
th-18	lem-6	—	???	—	34.71	114.29	37.38	—	—	—	—	0/2
th-19	lem-64	—	???	117.05	35.15	112.63	37.44	—	—	—	—	0/2
th-2	lem-10	—	—	—	34.14	—	37.25	—	—	—	—	0/2
th-20	lem-65	—	—	—	34.01	—	37.22	—	—	—	—	0/2
th-21	lem-66	—	—	—	33.55	—	36.70	—	—	—	—	0/2
th-22	lem-67	—	—	—	34.29	—	37.23	—	—	—	—	0/2
th-23	lem-68	—	—	—	35.32	—	37.38	—	—	—	—	0/2
th-24	lem-69	—	—	—	35.25	—	37.17	—	—	—	—	0/2
th-25	lem-7	—	—	—	34.52	—	36.94	—	—	—	—	0/2
th-26	lem-70	—	—	—	40.16	—	41.78	—	—	—	—	0/2
th-27	lem-71	—	???	115.98	38.81	112.35	37.73	—	—	—	—	0/2
th-28	lem-72	—	—	—	38.55	—	37.28	—	—	—	—	0/2
th-29	lem-73	—	—	—	37.39	—	36.84	—	—	—	—	0/2
th-3	lem-11	—	—	—	38.42	—	37.10	—	—	—	—	0/2
th-30	lem-74	—	—	—	38.16	—	37.32	—	—	—	—	0/2
th-31	lem-75	—	—	—	39.26	—	37.19	—	—	—	—	0/2
th-32	lem-76	—	—	—	36.77	—	37.36	—	—	—	—	0/2
th-33	lem-77	—	—	—	47.04	—	41.81	—	—	—	—	0/2
th-34	lem-78	—	—	—	36.73	—	36.77	—	—	—	—	0/2
th-35	lem-79	—	—	105.10	36.23	103.51	36.76	—	—	—	—	0/2
th-36	lem-8	—	—	106.17	38.60	103.25	36.75	—	—	—	—	0/2
th-37	lem-80	—	—	104.50	37.27	103.92	36.76	—	—	—	—	0/2
th-38	lem-81	—	—	108.54	36.41	103.59	36.93	—	—	—	—	0/2
th-39	lem-82	—	—	112.12	36.37	104.63	36.94	—	—	—	—	0/2
th-4	lem-12	—	—	—	37.67	—	37.42	—	—	—	—	0/2
th-40	lem-83	—	—	107.56	37.63	104.46	36.77	—	—	—	—	0/2
th-41	lem-84	—	—	109.92	37.71	106.27	37.02	—	—	—	—	0/2
th-42	lem-88	—	—	—	43.54	—	41.88	—	—	—	—	0/2
th-43	lem-89	—	—	107.43	37.85	106.51	37.14	—	—	—	—	0/2
th-44	lem-9	—	—	105.85	37.08	104.14	36.86	—	—	—	—	0/2
th-45	lem-90	—	—	109.08	37.71	105.87	37.09	—	—	—	—	0/2
th-46	elim-clause	—	—	—	—	—	—	—	—	—	—	0/2

th-47	elim-retry	—	—	—	—	—	—	—	—	—	—	0/2
th-48	elim-retry_me	—	—	—	—	—	—	—	—	—	—	0/2
th-49	elim-swconst	—	—	—	—	—	—	—	—	—	—	0/2
th-5	lem-13	—	—	—	36.94	—	36.63	—	—	—	—	0/2
th-50	elim-swstruct	—	—	—	—	—	—	—	—	—	—	0/2
th-51	elim-swterm	—	—	—	—	—	—	—	—	—	—	0/2
th-52	elim-trust	—	—	—	—	—	—	—	—	—	—	0/2
th-53	elim-try	???	—	—	—	—	—	—	—	—	—	0/2
th-54	elim-try_me	???	—	—	—	—	—	—	—	—	—	0/2
th-55	lem-22	0.01	0.01	0.00	0.00	0.01	0.01	8.99	1.94	9.30	2.01	0/1
th-56	lem-23	0.01	0.01	0.00	0.00	0.00	0.00	9.00	1.90	9.48	2.02	0/1
th-57	lem-24	0.01	0.01	0.00	0.00	0.01	0.00	9.00	1.89	9.60	2.04	0/1
th-58	lem-25	0.01	0.01	0.01	0.01	0.00	0.00	8.90	1.89	9.66	2.00	0/1
th-59	lem-26	0.01	0.01	0.00	0.00	0.00	0.00	8.86	1.92	9.66	1.96	0/1
th-6	lem-14	—	—	111.70	35.85	108.00	36.77	—	—	—	—	0/2
th-60	lem-27	0.01	???	0.00	0.00	0.00	0.00	9.06	1.90	9.81	2.02	0/1
th-61	lem-28	0.01	0.01	0.00	0.00	0.00	0.00	9.01	1.96	10.12	1.98	0/1
th-62	lem-29	0.01	0.01	0.00	0.00	0.00	0.01	9.02	1.97	10.45	2.00	0/1
th-63	lem-30	0.01	0.01	0.00	0.00	0.00	0.00	8.97	1.96	10.61	2.02	0/1
th-64	lem-31	0.01	0.01	0.00	0.00	0.00	0.00	9.08	1.93	9.29	2.03	0/1
th-65	lem-32	0.01	0.01	0.00	0.00	0.00	0.00	9.09	1.94	9.22	1.99	0/1
th-66	lem-33	0.01	0.01	0.00	0.00	0.00	0.00	9.09	1.93	9.35	1.99	0/1
th-67	lem-34	0.01	0.01	0.00	0.00	0.00	0.00	9.05	1.96	9.50	1.97	0/1
th-68	lem-35	0.01	0.01	0.00	0.00	0.00	0.00	9.11	1.97	9.50	1.99	0/1
th-69	lem-36	0.01	0.01	0.00	0.00	0.00	0.00	9.13	1.96	9.56	2.00	0/1
th-7	lem-15	—	—	108.72	35.70	111.03	36.71	—	—	—	—	0/2
th-70	lem-37	0.01	0.01	0.00	0.00	0.00	0.00	9.48	1.89	9.98	1.98	0/1
th-71	lem-38	0.01	0.01	0.00	0.00	0.00	0.00	9.48	1.94	10.00	1.94	0/1
th-72	lem-39	0.01	0.01	0.00	0.01	0.00	0.00	9.47	1.89	9.88	1.98	0/1
th-73	lem-40	0.01	0.01	0.00	0.00	0.00	0.00	9.48	1.92	9.89	2.08	0/1
th-74	lem-41	0.01	0.01	0.00	0.00	0.00	0.00	9.37	1.92	10.10	1.99	0/1
th-75	lem-42	0.01	0.01	0.00	0.00	0.00	0.00	9.38	1.93	10.21	1.96	0/1
th-76	lem-43	0.01	???	0.00	0.00	0.01	0.00	9.32	1.86	10.32	2.01	0/1
th-77	lem-44	0.01	0.01	0.00	0.00	0.01	0.00	9.36	1.88	10.54	1.97	0/1
th-78	lem-45	0.01	0.01	0.00	0.00	0.00	0.00	9.45	1.87	10.67	1.96	0/1
th-79	lem-46	0.01	0.01	0.00	0.00	0.00	0.00	9.45	1.89	10.51	1.98	0/1
th-8	lem-16	—	—	—	34.88	—	36.85	—	—	—	—	0/2
th-80	lem-47	0.01	0.01	0.00	0.00	0.00	0.00	9.81	1.90	10.73	2.02	0/1
th-81	lem-48	0.01	0.01	0.00	0.00	0.00	0.00	9.75	1.87	10.64	2.02	0/1
th-82	lem-49	0.01	0.01	0.00	0.00	0.00	0.00	10.19	1.93	11.19	2.00	0/1
th-83	lem-50	0.01	0.01	0.00	0.00	0.01	0.00	9.49	2.02	11.25	2.10	0/1
th-84	lem-51	0.01	0.01	0.00	0.00	0.00	0.00	9.40	2.01	11.37	2.14	0/1
th-85	lem-52	0.01	0.01	0.01	0.01	0.00	0.00	9.62	2.08	11.42	2.16	0/1
th-86	lem-53	0.01	0.01	0.00	0.00	0.00	0.00	9.64	2.05	11.55	2.17	0/1
th-87	lem-54	0.01	0.01	0.00	0.00	0.00	0.00	9.64	2.08	11.61	2.13	0/1
th-88	lem-55	0.01	0.01	0.00	0.00	0.00	0.00	9.76	2.03	11.55	2.13	0/1
th-89	lem-56	0.01	0.01	0.00	0.00	0.00	0.00	9.65	2.05	11.47	2.16	0/1
th-9	lem-17	—	—	113.80	35.01	107.10	36.81	—	—	—	—	0/2
th-90	lem-57	0.01	???	0.01	0.00	0.00	0.00	9.45	1.99	10.91	2.04	0/1
th-91	lem-58	0.01	0.01	0.00	0.00	0.01	0.00	9.86	1.96	11.52	2.00	0/1
th-92	lem-59	0.01	0.01	0.00	0.00	0.01	0.00	9.93	1.90	11.05	1.97	0/1
th-93	lem-60	0.01	0.01	0.00	0.00	0.00	0.00	9.86	1.98	11.14	2.02	0/1
th-94	lem-61	0.01	0.01	0.00	0.00	0.01	0.00	9.89	1.97	11.63	1.97	0/1
th-95	lem-62	0.01	0.01	0.00	0.00	0.00	0.00	9.92	1.95	11.43	1.98	0/1
th-96	lem-63	0.01	0.01	0.00	0.00	0.00	0.00	9.93	1.90	11.43	2.03	0/1
th-97	lem-85	0.01	0.01	0.00	0.01	0.00	0.00	9.87	2.11	11.72	2.18	0/1
th-98	lem-86	0.01	0.01	0.00	0.00	0.01	0.00	9.98	1.95	11.41	2.02	0/1
th-99	lem-87	0.01	0.01	0.01	0.00	0.00	0.00	9.92	2.10	11.54	2.19	0/1
Σ		45	42	68	90	69	90	45	45	45	45	99
steps		153	153	130	108	129	108	153	153	153	153	153
ints		0	0	0	0	0	0	0	0	0	0	0

Specification: list-data from case-study WAM

- Number of theorems: 28
- Proof steps: 43
- Interactions: 2
- Number of noniductive goals: 30
- Noninductive Proof steps: 42
- Noninductive Interactions: 2

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	ax-03a	0.01	0.01	0.01	0.00	0.03	0.03	2.16	0.33	1.82	0.20	0/1
th-10	lfr	—	—	0.46	0.18	0.50	0.19	5.02	0.94	2.11	0.86	0/2
th-11	lr-02	—	—	—	17.61	5.37	3.09	2.84	0.98	—	78.56	0/2
th-12	ax-03b	0.02	0.01	0.00	0.00	0.18	0.10	2.65	0.68	1.94	0.29	0/1
th-13	elim-list	—	—	1.44	0.20	0.65	0.17	3.06	0.76	2.02	0.28	0/1
th-14	fr-01	0.16	0.05	0.55	0.07	0.01	0.00	1.93	0.31	2.02	0.39	0/1
th-15	fr-03	0.01	0.01	0.00	0.00	0.00	0.00	1.92	0.17	1.97	0.20	0/1
th-16	fr-04	—	8.35	0.08	0.05	0.07	0.05	1.95	0.20	1.98	0.20	0/2
th-17	lf	0.05	0.02	0.12	0.05	0.15	0.05	1.92	0.56	2.36	0.88	0/1
th-18	lr	0.05	0.01	41.45	1.25	0.30	0.11	2.10	0.77	2.88	1.08	0/2
th-19	lr-01	—	—	—	—	17.73	30.04	7.65	3.18	—	—	0/2
th-2	fr-07	0.34	0.09	0.10	0.04	0.09	0.07	1.99	0.24	1.86	0.23	1/2
th-20	lr-03	—	—	—	1.38	—	0.53	3.87	0.77	—	0.82	0/4
th-21	lr-04	—	—	0.73	1.81	0.72	9.27	2.89	1.18	3.87	—	0/2
th-22	lr-05	—	47.55	—	22.99	—	4.05	4.23	1.41	3.95	79.32	0/2
th-23	lr-06	—	7.71	8.32	0.82	4.72	0.29	3.33	0.50	4.07	0.47	0/2
th-24	ls-01	0.01	0.01	0.00	0.01	0.00	0.00	2.34	0.48	—	0.42	1/2
th-25-is	ls-02	0.26	0.03	0.18	0.08	0.13	0.07	2.07	0.31	96.45	0.44	0/0
th-25-ib	ls-02	0.01	0.01	0.00	0.00	0.00	0.00	0.53	0.10	0.63	0.10	0/0
th-25-pr	ls-02	0.02	0.02	0.18	0.07	0.12	0.06	1.87	0.21	1.93	0.27	0/1
th-26	ls-trans1	0.01	0.01	0.05	0.02	0.04	0.02	2.69	0.58	—	0.85	0/1
th-27	ls-trans2	0.01	0.01	0.05	0.02	0.05	0.02	2.60	0.59	—	0.85	0/1
th-28	p-fr	100.47	2.39	0.12	0.05	0.24	0.21	2.58	0.45	2.14	0.29	0/1
th-3	fr-05	0.26	0.08	0.04	0.03	0.08	0.09	1.95	0.26	1.87	0.24	0/1
th-4	fr-02	0.01	0.01	0.00	0.00	0.00	0.00	1.72	0.13	1.84	0.14	0/1
th-5	elim-list-c	—	—	0.13	0.06	—	—	2.43	0.44	1.84	0.25	0/1
th-6	l	—	2.29	—	0.78	—	0.15	2.54	0.51	3.51	0.39	0/2
th-7	fr	0.29	0.05	0.03	0.03	0.42	0.02	1.82	0.17	1.88	0.20	0/1
th-8	fr-06	0.13	0.03	0.01	0.00	0.00	0.00	1.82	0.15	1.88	0.17	0/1
th-9	lf-01	—	—	119.06	0.82	—	0.36	2.54	0.51	—	26.37	0/1
Σ		18	22	25	29	25	29	30	30	23	28	28
steps		39	35	35	29	33	28	28	28	34	30	42
ints		0	0	0	0	0	0	0	0	1	0	2

Specification: list from case-study WAM

- Number of theorems: 63
- Proof steps: 263
- Interactions: 93
- Number of noniductive goals: 122
- Noninductive Proof steps: 229
- Noninductive Interactions: 66

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1-is	app-a	—	7.10	—	—	0.00	0.00	—	—	—	0.97	0/2
th-1-ib	app-a	0.19	0.07	0.00	0.00	0.01	0.00	7.68	0.50	2.09	0.31	0/0
th-10	inapp	0.16	0.12	0.46	0.09	0.49	0.10	—	2.92	—	5.66	0/1
th-11	in-04	—	—	9.45	4.78	—	2.75	—	2.73	—	91.98	4/12
th-12	in-03	0.95	0.67	0.53	0.09	0.43	0.12	—	1.46	—	3.79	0/1
th-13	in-01	0.01	0.01	0.00	0.00	0.00	0.00	2.29	0.36	113.54	0.45	0/1
th-14-is-is	inapp-03	—	—	—	—	—	—	—	—	—	—	1/2
th-14-is-ib	inapp-03	5.72	2.96	0.00	0.00	0.00	0.00	—	39.15	2.50	0.52	0/0
th-14-ib-is	inapp-03	23.06	8.77	10.48	5.98	0.54	0.38	—	—	—	3.35	0/0
th-14-ib-ib	inapp-03	0.19	0.10	0.01	0.00	0.77	0.36	17.09	3.07	2.36	0.42	0/0
th-14-is	inapp-03	—	—	—	—	—	—	—	—	—	—	0/1
th-14-ib	inapp-03	7.86	2.52	0.00	0.00	0.32	0.19	34.44	7.36	2.44	0.52	0/1
th-14-pr	inapp-03	—	—	—	—	—	—	2.08	0.38	2.11	0.44	0/1
th-15-is	lem-02	6.04	3.52	3.42	1.41	—	—	2.82	0.82	—	11.39	0/2
th-15-ib	lem-02	0.01	0.01	0.00	0.00	0.00	0.00	2.23	0.48	2.20	0.46	0/0
th-15-pr	lem-02	0.01	0.01	0.15	0.11	0.14	0.10	0.72	0.22	0.86	0.23	0/1
th-16-is	lem-05	—	—	—	1.88	1.50	0.54	—	6.51	—	1.28	0/2
th-16-ib	lem-05	0.01	0.01	0.00	0.00	0.00	0.00	0.67	0.12	0.79	0.13	0/0
th-16-pr	lem-05	2.70	1.11	1.89	0.50	0.85	0.24	2.33	0.27	2.33	0.31	0/1
th-17-is-is	app-07	—	—	—	—	—	—	—	—	—	45.69	1/2
th-17-is-ib	app-07	3.17	—	0.06	0.02	7.20	0.03	—	2.32	6.07	0.49	0/0
th-17-ib-is	app-07	—	—	—	10.20	14.94	0.14	—	3.25	—	2.39	0/0
th-17-ib-ib	app-07	0.02	0.01	0.00	0.00	0.01	0.00	4.46	0.45	2.52	0.38	0/0
th-17-is	app-07	—	—	—	55.82	???	—	—	—	—	—	0/1
th-17-ib	app-07	0.23	0.03	0.00	0.00	0.04	0.01	19.13	1.17	2.60	0.42	0/1
th-17-pr	app-07	39.83	9.18	—	—	—	—	1.88	0.29	1.91	0.33	0/1
th-18	app-08	35.56	6.33	0.77	0.19	—	2.18	—	49.51	—	3.23	0/2
th-19	app-09	34.75	5.95	0.58	0.33	35.53	0.75	28.49	1.07	—	1.25	0/2
th-2-is	app-06	—	—	—	3.92	14.02	1.54	—	2.13	—	1.07	1/2
th-2-ib	app-06	8.68	0.13	0.00	0.01	0.00	0.00	2.10	0.28	2.36	0.40	0/0
th-20	app-10	43.70	8.54	—	17.16	42.70	1.57	20.57	1.07	—	1.77	0/2
th-21	in-06	1.16	0.50	0.47	10.79	0.77	0.71	5.62	2.12	3.45	0.78	0/2
th-22	in-09	0.45	0.23	0.01	0.01	0.49	0.43	4.40	1.61	—	11.01	0/1
th-23	in-10	2.03	0.79	—	11.03	—	49.24	5.58	2.48	—	1.23	0/2
th-24-is-is	inapp-01	—	—	—	—	—	—	—	—	—	—	1/2
th-24-is-ib	inapp-01	4.93	2.95	0.01	0.01	0.01	0.01	—	20.09	2.68	0.67	0/0
th-24-ib-is	inapp-01	—	—	8.27	0.57	6.10	1.01	—	5.68	101.94	0.78	0/0
th-24-ib-ib	inapp-01	0.52	0.12	0.00	0.00	0.01	0.01	6.18	0.76	2.59	0.42	0/0
th-24-is	inapp-01	—	—	—	—	—	—	—	—	—	—	0/1
th-24-ib	inapp-01	0.58	0.19	0.00	0.00	0.13	0.09	51.25	14.43	2.80	0.79	0/1
th-24-pr	inapp-01	—	—	99.53	70.25	80.87	54.24	2.02	0.54	2.12	0.61	0/1
th-25	inapp-02	—	—	—	—	4.46	8.05	—	—	—	—	0/1
th-26-is	lapp	—	—	—	—	0.00	0.00	—	—	—	25.25	0/2
th-26-ib	lapp	—	16.29	0.00	0.00	0.00	0.00	12.92	0.88	2.58	0.64	0/0
th-27	lem-01	0.01	0.01	0.09	0.06	0.09	0.06	6.67	3.51	—	22.63	0/1
th-28	lem-08	0.54	0.27	0.50	0.20	0.47	0.19	6.15	0.93	—	0.64	0/1
th-29-is	app-01	—	—	—	2.07	7.88	1.42	—	3.20	—	—	0/2
th-29-ib	app-01	0.05	0.01	0.00	0.00	0.00	0.00	2.75	0.39	2.56	0.37	0/0
th-29-pr	app-01	0.09	0.02	1.68	0.35	3.51	0.65	0.83	0.14	0.94	0.16	0/1
th-3	app-04	5.49	1.73	3.95	0.05	0.00	0.01	2.67	0.46	37.74	0.41	0/1
th-30-is	app-03	—	—	—	—	—	—	—	—	—	—	1/2
th-30-ib	app-03	0.05	0.03	0.00	0.00	0.00	0.00	2.78	0.41	2.61	0.41	0/0
th-30-pr	app-03	0.09	0.02	1.78	0.39	3.69	0.84	0.84	0.14	0.99	0.16	0/1
th-31	app-11	—	—	2.54	1.10	6.13	1.30	99.65	2.81	—	1.58	0/2
th-32	app-12	—	—	—	—	—	—	29.52	2.38	72.76	0.81	0/2
th-33-is	app-13	—	—	—	—	—	—	—	—	—	—	0/0
th-33-ib	app-13	0.34	0.28	0.02	0.02	0.02	0.03	—	110.85	3.01	0.89	0/0
th-33-pr	app-13	—	—	—	—	—	—	25.68	15.95	—	6.05	0/1
th-34-is	firstoc	—	—	—	—	—	—	—	—	—	—	3/10
th-34-ib	firstoc	0.01	0.01	0.01	0.00	0.00	0.00	2.78	0.75	2.75	0.83	0/0

th-35	fw-01	—	—	—	—	—	—	—	—	—	—	2/3
th-36	fw-02	—	—	—	—	—	15.51	—	—	—	63.48	2/3
th-37	in-02	0.35	0.26	3.35	3.70	3.06	3.15	—	62.55	—	17.72	0/1
th-38	in-05	0.01	0.01	0.03	0.03	0.03	0.03	3.32	1.13	80.23	1.28	0/1
th-39	in-07	—	—	—	—	—	—	—	69.11	—	42.86	0/2
th-4	app	0.01	0.01	0.00	0.00	0.00	0.00	2.09	0.21	25.95	0.25	0/1
th-40	in-11	0.06	0.03	0.10	0.09	0.10	0.09	3.20	1.09	4.68	1.58	0/2
th-41	inapp-04	—	—	—	—	8.63	10.68	—	—	—	13.11	0/2
th-42	inapp-06	—	—	1.13	0.95	2.27	1.79	—	—	—	21.06	1/2
th-43-is-is	lapp-01	—	—	—	—	—	—	—	—	—	—	1/2
th-43-is-ib	lapp-01	0.32	0.07	0.00	0.00	0.00	0.00	3.73	0.60	3.15	0.50	0/0
th-43-is	lapp-01	—	—	—	—	—	—	—	—	—	—	0/1
th-43-ib	lapp-01	—	68.93	0.63	0.26	2.45	0.56	3.64	0.76	4.57	0.97	0/0
th-43-pr	lapp-01	—	67.21	50.68	46.83	—	—	2.25	0.46	2.37	0.50	0/1
th-44-is	lapp-02	—	—	—	—	???	—	—	—	—	—	0/0
th-44-ib	lapp-02	—	—	0.03	0.01	8.77	1.40	—	8.40	3.53	1.23	0/0
th-44-pr	lapp-02	0.03	0.01	0.00	0.00	0.92	0.56	1.20	0.48	1.34	0.52	0/1
th-45-is	lastdiv	52.73	6.90	—	29.61	—	13.93	—	—	—	87.71	1/6
th-45-ib	lastdiv	0.02	0.01	0.00	0.00	0.00	0.00	0.89	0.14	1.05	0.17	0/0
th-45-pr	lastdiv	0.01	0.01	0.07	0.01	0.05	0.01	3.67	0.41	4.56	0.38	0/1
th-46-is	lastoc	—	—	—	—	—	—	—	—	—	—	4/11
th-46-ib	lastoc	0.01	0.01	0.00	0.00	0.00	0.00	3.17	1.00	3.23	1.04	0/0
th-47	lem-03	4.92	3.93	1.03	0.89	3.70	3.12	17.88	11.52	—	86.65	1/2
th-48	lem-04	0.19	0.12	34.48	4.69	0.95	0.22	20.21	2.03	—	3.56	1/4
th-49	lem-06	—	—	—	—	—	144.81	—	4.06	83.23	0.98	0/4
th-5	app-02	—	—	—	8.96	—	20.83	—	22.31	—	2.09	0/2
th-50	lem-07	—	—	—	—	17.32	3.73	21.35	4.41	87.85	1.16	1/2
th-51	lem-09	0.01	6.16	—	—	0.70	—	—	91.33	11.33	77.02	1/1
th-52	lem-10	0.01	0.01	0.00	0.00	0.00	0.01	—	79.93	—	3.05	2/3
th-53	lem-11	46.35	30.42	46.70	35.28	41.32	37.18	52.80	39.70	—	—	4/7
th-54	lem-12	0.13	0.14	2.59	2.18	0.98	0.81	30.93	22.26	—	41.05	1/2
th-55-is-is-is	ls-app-01	4.93	1.36	6.33	2.34	0.03	0.00	—	—	38.63	2.08	0/2
th-55-is-is-ib	ls-app-01	0.01	0.01	0.00	0.00	0.00	0.00	12.76	1.05	3.82	0.58	0/0
th-55-is-is2	ls-app-01	19.76	7.77	—	—	—	—	—	—	—	—	5/9
th-55-is-ib2	ls-app-01	0.01	0.01	0.00	0.00	0.00	0.00	8.19	1.05	3.69	0.63	0/0
th-55-is-is	ls-app-01	5.51	1.49	5.89	0.80	0.02	0.00	—	8.91	49.58	0.78	0/2
th-55-is-ib	ls-app-01	0.01	0.01	0.00	0.00	0.00	0.00	3.77	0.43	3.69	0.43	0/0
th-55-is	ls-app-01	0.17	0.11	8.67	1.48	0.02	0.00	—	11.56	—	16.15	5/9
th-55-ib	ls-app-01	0.01	0.01	0.00	0.00	0.00	0.00	3.65	0.59	3.59	0.59	0/0
th-55-pr	ls-app-01	0.01	0.01	0.00	0.00	1.28	0.51	0.98	0.19	1.14	0.23	0/1
th-56	ls-app-02	0.01	0.01	1.06	0.43	1.45	0.32	23.25	2.59	—	1.51	3/6
th-57	p-app	0.29	0.04	0.04	0.01	6.01	0.64	5.50	0.70	—	—	0/1
th-58	p-app-01	0.01	0.01	0.03	0.00	0.04	0.01	5.31	0.66	—	—	0/1
th-59	p-app-02	6.72	3.14	1.22	0.90	3.03	2.17	—	—	55.00	1.54	0/2
th-6-is-is	app-05	—	—	—	—	—	—	—	—	—	—	1/2
th-6-is-ib	app-05	0.01	0.01	0.00	0.00	0.00	0.00	0.59	0.11	0.70	0.11	0/0
th-6-is	app-05	0.54	0.13	0.52	0.21	0.58	0.22	2.30	0.32	2.18	0.27	0/1
th-6-ib	app-05	0.05	0.01	0.00	0.00	0.00	0.00	2.05	0.21	1.98	0.25	0/0
th-6-pr	app-05	—	3.80	0.80	0.17	0.40	0.15	2.09	0.23	2.07	0.25	0/1
th-60	p-app-03	0.01	0.01	0.02	0.02	0.04	0.02	47.20	26.52	—	—	0/1
th-61	p-app-04	0.01	0.01	0.02	0.02	0.04	0.02	—	—	—	—	0/1
th-62	p-lapp	—	—	—	—	0.00	—	—	—	—	—	0/1
th-63	someoc	0.01	0.01	0.00	0.00	0.00	0.00	9.46	4.39	—	27.65	1/1
th-7	in	1.77	0.37	0.54	0.31	0.32	0.19	5.03	0.43	—	4.63	1/1
th-8-is-is	inapp-05	—	—	—	—	—	—	—	—	—	—	3/8
th-8-is-ib	inapp-05	—	50.73	0.20	0.11	2.81	1.05	—	37.82	—	7.82	1/4
th-8-is	inapp-05	32.95	14.23	0.77	0.42	2.44	1.20	—	—	—	50.34	0/1
th-8-ib	inapp-05	0.17	0.05	0.25	0.12	0.00	0.00	3.69	0.95	—	4.12	1/4
th-8-pr	inapp-05	—	—	—	—	—	—	—	—	—	—	7/13
th-9-is	in-08	0.04	0.03	1.03	0.38	0.23	0.17	—	—	—	1.81	0/2
th-9-ib	in-08	7.28	3.63	0.01	0.00	0.00	0.00	15.15	2.74	2.28	0.38	0/0

th-9-pr	in-08	—	—	7.49	4.30	—	—	—	—	—	9.62	4/10
Σ		76	81	82	91	89	94	66	89	57	97	91
steps		176	172	166	155	180	156	201	167	219	150	229
ints		39	38	36	34	41	35	52	38	64	33	66

Specification: mode from case-study WAM

- Number of theorems: 1
- Proof steps: 2
- Interactions: 0
- Number of noniductive goals: 1
- Noninductive Proof steps: 2
- Noninductive Interactions: 0

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-01	0.01	0.01	???	???	0.00	0.00	0.02	0.02	0.01	0.05	0/2
Σ		1	1	0	0	1	1	1	1	1	1	1
steps		1	1	2	2	1	1	1	1	1	1	2
ints		0	0	0	0	0	0	0	0	0	0	0

Specification: nat-basic1 from case-study WAM

- Number of theorems: 44
- Proof steps: 69
- Interactions: 1
- Number of noniductive goals: 44
- Noninductive Proof steps: 69
- Noninductive Interactions: 1

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-14	—	—	—	—	—	—	—	—	—	—	0/1
th-10	lem-09	4.39	4.36	0.08	0.07	0.10	0.11	3.66	3.23	5.56	4.99	0/2
th-11	lem-16	0.01	0.01	0.00	0.01	0.01	0.00	0.31	0.34	0.31	0.28	0/2
th-12	lem-32	0.03	0.03	0.06	0.05	0.05	0.05	0.33	0.28	0.31	0.27	0/2
th-13	lem-24	0.81	0.80	0.09	0.08	0.09	0.09	0.43	0.37	1.72	1.56	0/2
th-14	elim-pred	37.56	0.47	0.11	0.02	0.41	0.03	0.62	0.11	0.24	0.09	0/2
th-15	lem-04	—	—	—	—	—	—	3.92	3.68	11.40	11.24	0/2
th-16	lem-13	0.02	0.02	0.04	0.04	0.04	0.04	0.20	0.19	0.87	0.81	0/2
th-17	lem-15	0.01	—	0.00	—	0.00	—	0.18	—	0.18	—	0/2
th-18	lem-22	22.10	0.48	0.23	0.06	0.27	0.07	0.46	0.14	0.26	0.13	0/1
th-19	lem-25	1.69	1.73	0.17	0.17	0.20	0.20	0.71	0.66	2.49	2.42	0/1
th-2	lem-03	0.01	0.01	0.01	0.01	0.01	0.00	0.10	0.03	0.10	0.06	0/2
th-20	lem-27	0.95	0.98	0.13	0.13	0.17	0.16	2.33	2.24	1.51	1.38	0/2
th-21	lem-30	0.03	0.02	0.01	0.01	0.01	0.00	0.32	0.30	11.05	11.45	0/2
th-22	lem-18	0.01	0.01	0.00	0.00	0.00	0.00	0.31	0.29	0.30	0.30	0/1
th-23	elim-pred-c	83.97	0.74	0.24	0.04	0.31	0.12	0.53	0.13	0.30	0.15	1/1
th-24	lem-01	0.11	0.05	0.11	0.05	0.11	0.02	0.24	0.09	0.25	0.08	0/1
th-25	lem-05	69.17	70.08	—	—	—	—	8.49	8.47	3.30	3.46	0/2
th-26	lem-06	0.12	0.12	0.26	0.27	0.29	0.30	0.56	0.54	1.32	1.34	0/1
th-27	lem-10	0.04	0.04	0.13	0.13	0.11	0.10	0.35	0.36	0.36	0.34	0/1

th-28	lem-11	0.02	0.01	0.08	0.08	0.07	0.07	0.31	0.30	0.99	0.95	0/2
th-29	lem-12	0.01	0.02	0.00	0.00	0.00	0.00	0.24	0.09	0.28	0.09	0/1
th-3	lem-29	0.01	0.01	0.03	0.03	0.04	0.03	0.70	0.62	0.57	0.55	0/2
th-30	lem-21	3.02	3.00	0.78	0.79	0.88	0.90	1.19	1.05	2.10	2.06	0/1
th-31	lem-23	0.01	—	0.13	—	0.11	—	0.68	—	0.35	—	0/2
th-32	lem-26	2.06	1.90	0.13	0.13	0.41	0.41	3.96	4.02	2.28	2.30	0/1
th-33	lem-28	95.72	95.75	1.91	1.99	0.42	0.44	1.00	1.02	0.47	0.45	0/2
th-34	lem-33	2.36	2.38	0.47	0.46	0.46	0.45	1.14	1.19	4.94	5.24	0/2
th-35	lem-34	0.31	0.30	0.00	0.01	0.01	0.01	1.08	1.08	1.95	1.92	0/1
th-36	lem-35	0.97	0.97	0.51	0.52	0.55	0.54	0.96	0.90	0.71	0.69	0/1
th-37	lem-36	0.11	0.11	0.82	0.82	0.58	0.59	0.82	0.77	2.32	2.27	0/1
th-38	prop-03	9.92	0.31	0.00	0.00	0.33	0.05	0.37	0.10	0.37	0.10	0/1
th-39	prop-04	0.36	0.35	0.15	0.15	0.17	0.18	5.40	6.07	1.58	1.56	0/1
th-4	lem-20	0.23	0.23	0.17	0.16	0.17	0.17	0.33	0.28	0.54	0.52	0/1
th-40	prop-05	0.05	0.04	0.15	0.16	0.19	0.20	0.49	0.48	16.52	18.51	0/1
th-41	q-01	0.33	0.04	0.20	0.04	0.23	0.05	0.63	0.17	1.83	0.19	0/6
th-42	lem-17	0.01	0.01	0.00	0.01	0.00	0.00	0.37	0.37	0.39	0.40	0/1
th-43	prop-01	0.57	0.58	0.14	0.14	0.57	0.56	1.52	1.64	15.95	18.60	0/1
th-44	prop-02	0.01	0.01	0.01	0.00	0.01	0.01	0.56	0.53	3.85	4.10	0/1
th-5	lem-07	102.19	105.42	15.56	16.64	31.76	34.39	4.71	4.63	17.31	19.69	0/2
th-6	lem-19	0.12	0.03	0.01	0.00	0.00	0.00	0.20	0.08	0.19	0.09	0/1
th-7	lem-31	0.01	0.01	0.00	0.00	0.00	0.00	0.10	0.11	0.12	0.10	0/2
th-8	lem-02	0.01	0.01	0.00	0.01	0.01	0.01	0.24	0.20	0.25	0.24	0/1
th-9	lem-08	0.03	0.03	0.09	0.09	0.12	0.13	0.28	0.24	1.24	1.19	0/2
Σ		42	40	41	39	41	39	43	41	43	41	43
steps		45	47	46	48	46	48	44	46	44	46	69
ints		0	0	0	0	0	0	0	0	0	0	1

Specification: nat-basic2 from case-study WAM

- Number of theorems: 36
- Proof steps: 120
- Interactions: 28
- Number of noniductive goals: 53
- Noninductive Proof steps: 107
- Noninductive Interactions: 24

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1-is	prop-01	2.62	0.32	1.57	0.58	0.00	0.00	1.11	0.24	0.88	0.21	0/2
th-1-ib	prop-01	0.47	0.03	0.00	0.00	0.00	0.00	0.84	0.16	0.47	0.18	0/0
th-10	lem-01	7.24	7.24	—	—	0.51	0.48	—	—	—	—	0/1
th-11	lem-04	81.61	34.94	1.34	2.69	2.57	0.15	—	20.86	—	—	0/2
th-12	lem-06	—	—	—	—	—	—	—	—	—	—	1/2
th-13	lem-12	—	—	0.51	0.51	0.55	0.53	74.34	73.54	—	—	0/4
th-14-is-is	lem-34	35.94	8.08	—	1.38	1.50	0.29	89.77	4.11	—	0.87	0/2
th-14-is-ib	lem-34	0.22	0.04	0.00	0.00	0.00	0.00	1.46	0.38	0.62	0.25	0/0
th-14-is	lem-34	21.02	1.45	1.97	0.57	0.01	0.01	52.56	3.96	—	49.55	0/1
th-14-ib	lem-34	0.05	0.04	0.00	0.01	0.00	0.00	1.45	0.35	0.60	0.26	0/0
th-15	lem-05	—	—	—	2.36	0.43	0.04	98.46	4.55	—	3.10	0/2
th-16	lem-07	5.28	—	0.58	0.39	0.62	69.85	2.11	—	1.57	12.77	0/4
th-17	lem-08	0.01	0.01	0.03	0.03	0.03	0.02	2.07	2.04	32.08	31.91	0/1
th-18	lem-11	27.36	27.37	—	—	1.09	1.05	—	—	27.98	28.10	0/1
th-19	lem-13	—	—	0.40	0.41	0.40	0.42	116.41	116.64	—	—	0/2
th-2-is	prop-02	0.03	0.01	0.08	0.02	0.00	0.00	2.53	0.36	0.90	0.22	0/0
th-2-ib	prop-02	0.01	0.01	0.00	0.00	0.01	0.00	0.37	0.11	0.40	0.12	0/0
th-20	lem-15	—	—	45.27	45.42	41.03	40.92	—	—	—	—	2/5
th-21	lem-17	—	—	—	—	—	7.36	—	—	—	—	0/4
th-22	lem-20	—	—	—	—	7.41	3.39	3.61	1.37	2.18	14.46	0/2

th-23	lem-28	114.39	115.00	18.78	18.72	0.27	0.26	—	—	—	—	0/4
th-24	lem-29	0.23	0.23	0.06	0.06	0.06	0.06	3.60	3.53	—	—	2/3
th-25	lem-35	1.03	0.36	0.01	0.00	0.00	0.00	0.93	0.41	2.35	0.75	0/2
th-26	less-01	6.34	6.34	24.98	25.85	31.64	31.63	24.85	24.60	—	—	0/4
th-27	less-02	6.27	6.28	—	—	—	—	—	—	—	—	0/1
th-28	less-03	—	—	—	—	—	—	86.86	86.28	—	—	1/2
th-29	less-04	0.05	0.05	0.41	0.41	0.41	0.40	24.23	24.26	—	—	3/5
th-3-is	lem-21	4.15	0.75	3.53	0.47	0.00	0.00	1.10	0.25	5.81	0.38	0/2
th-3-ib	lem-21	0.02	0.01	0.00	0.00	0.00	0.00	0.44	0.13	0.47	0.13	0/0
th-30	less-05	22.46	22.50	—	—	5.70	5.79	13.20	13.06	—	—	2/4
th-31	less-06	0.01	0.01	0.00	0.01	0.01	0.01	14.32	14.23	1.30	1.30	2/4
th-32	prop-03	—	—	—	—	—	—	34.74	2.03	3.46	5.50	0/2
th-33	prop-04	0.01	0.01	0.00	0.01	0.00	0.00	1.02	0.33	1.77	0.34	0/1
th-34-is	q-01	—	—	—	2.91	0.33	0.17	—	3.03	—	7.63	1/4
th-34-ib	q-01	0.01	0.01	0.00	0.00	0.00	0.00	0.83	0.27	0.87	0.28	1/2
th-35-is	subadd-01	—	—	—	—	38.17	39.46	70.10	70.45	—	—	6/11
th-35-ib	subadd-01	0.01	0.01	0.00	0.00	0.00	0.00	0.87	0.86	0.87	0.87	0/2
th-35-pr	subadd-01	0.01	0.01	0.14	0.13	0.14	0.15	1.08	1.07	0.90	0.89	0/1
th-36	lem-27	0.01	0.01	0.00	0.00	0.01	0.00	0.32	0.32	0.35	0.38	0/1
th-4-is	lem-10	—	—	—	—	0.00	0.00	—	18.23	—	0.57	0/2
th-4-ib	lem-10	0.49	0.03	0.00	0.00	0.00	0.00	1.06	0.33	0.56	0.23	0/0
th-5	lem-09	—	—	—	7.55	10.75	0.13	—	6.42	—	11.12	0/2
th-6-is	lem-02	—	—	—	8.53	2.48	0.59	—	1.60	—	—	0/2
th-6-ib	lem-02	0.60	0.08	0.00	0.00	0.00	0.00	0.50	0.16	0.51	0.19	0/0
th-6-pr	lem-02	—	3.25	6.38	1.07	7.74	1.18	1.76	0.35	2.57	0.24	0/1
th-7-is	lem-03	—	—	—	—	113.84	120.63	—	—	7.50	7.54	1/2
th-7-ib	lem-03	—	—	0.00	0.00	0.00	0.00	0.57	0.57	0.51	0.52	0/0
th-8	lem-30	2.49	2.50	0.24	0.23	0.25	0.28	36.90	36.71	51.88	51.45	2/4
th-9-is-is	lem-33	—	—	—	5.98	—	6.24	—	68.70	—	0.76	0/2
th-9-is-ib	lem-33	0.25	0.07	0.00	0.00	0.00	0.00	1.41	0.38	0.55	0.25	0/0
th-9-is	lem-33	89.23	6.30	—	—	13.15	2.19	—	15.64	—	12.10	0/1
th-9-ib	lem-33	6.81	0.97	0.00	0.00	0.00	0.00	1.03	0.39	1.16	0.35	0/0
th-9-pr	lem-33	4.91	0.09	0.78	0.16	—	33.20	0.22	0.08	0.25	0.09	0/1
Σ		35	35	34	40	46	49	38	44	29	37	41
steps		76	79	72	64	49	45	62	57	90	82	107
ints		12	12	12	11	2	2	5	4	18	17	24

Specification: nat-sub from case-study WAM

- Number of theorems: 29
- Proof steps: 69
- Interactions: 10
- Number of noniductive goals: 34
- Noninductive Proof steps: 66
- Noninductive Interactions: 8

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1-is	add-01	—	—	—	—	0.43	0.21	—	1.59	10.01	0.63	1/2
th-1-ib	add-01	0.01	0.01	0.01	0.00	0.00	0.00	1.06	0.32	1.10	0.34	0/0
th-10	less-01	—	—	—	—	32.94	33.25	—	—	—	—	0/2
th-11	less-02	12.73	12.42	—	—	0.77	0.78	—	—	—	—	0/2
th-12	less-03	—	—	1.04	—	1.02	—	50.21	—	—	—	0/2
th-13	ls	—	—	2.36	18.25	2.33	12.67	—	—	—	—	0/2
th-14	ls-02	—	—	2.61	1.02	26.84	7.39	85.83	—	—	—	0/2
th-15	pred-01	0.03	0.02	0.53	0.21	0.56	0.24	18.74	1.68	10.79	4.07	0/2
th-16	prop	6.79	0.85	0.02	0.01	0.00	0.00	1.56	0.60	—	11.51	0/1
th-17	prop-01	0.01	0.01	0.04	0.05	0.04	0.05	2.35	2.37	—	—	0/1
th-18	prop-02	0.04	0.02	0.00	0.00	0.00	0.00	1.55	0.57	63.32	16.05	0/2

th-19	prop-03	0.01	0.01	0.00	0.00	0.01	0.00	1.58	0.58	63.58	16.06	0/2
th-2	ls-01	—	—	—	—	—	—	23.08	—	—	—	1/2
th-20	prop-04	0.01	6.32	0.00	0.20	0.01	0.20	8.82	1.77	—	85.33	0/1
th-21	prop-05	—	—	—	—	—	—	—	—	—	—	0/2
th-22	prop-06	—	—	4.64	4.76	0.94	—	26.76	—	—	—	0/6
th-23	sub	0.30	—	0.04	—	0.05	—	5.63	—	1.94	—	1/1
th-24	sub-01	0.15	0.06	0.00	0.00	0.00	0.00	7.90	1.44	10.23	4.07	0/2
th-25	succ	4.66	—	0.24	—	0.23	—	3.49	—	14.96	—	1/1
th-26	succ-02	—	107.42	0.56	0.14	0.63	0.10	7.09	0.87	—	2.44	0/1
th-27	succ-03	—	—	—	—	—	—	—	—	—	—	0/3
th-28	succ-04	7.07	1.03	0.03	0.01	0.03	0.01	—	11.13	63.83	16.29	0/1
th-29	succ-05	0.01	0.01	0.01	0.00	0.00	0.01	1.75	0.63	67.21	16.34	0/1
th-3	add-03	42.95	6.98	0.69	0.29	0.97	0.40	—	1.49	—	15.09	1/1
th-4-is	elim-sub	—	—	—	—	—	—	61.08	60.27	—	—	1/6
th-4-ib	elim-sub	0.06	0.06	0.00	0.00	0.00	0.00	1.10	1.11	1.05	1.08	0/0
th-4-pr	elim-sub	—	—	—	—	—	—	75.18	75.35	—	—	2/6
th-5	add	—	—	—	—	—	—	—	—	—	—	0/2
th-6	add-02	—	—	—	—	—	—	—	—	—	—	0/2
th-7-is	pred	—	—	—	82.10	—	2.51	14.96	1.54	9.31	0.97	0/0
th-7-ib	pred	0.01	0.01	0.00	0.00	0.00	0.00	0.35	0.14	0.41	0.14	0/0
th-7-pr	pred	—	—	—	—	—	—	—	—	—	—	0/4
th-8	succ-01	—	—	0.99	0.42	0.87	0.40	—	—	—	—	0/2
th-9	less	—	—	—	—	—	—	—	—	—	—	0/2
Σ		16	15	21	19	24	21	21	18	13	15	27
steps		61	61	53	54	50	56	44	51	61	61	66
ints		5	7	5	7	4	6	2	3	5	6	8

Specification: pair from case-study WAM

- Number of theorems: 5
- Proof steps: 5
- Interactions: 0
- Number of noniductive goals: 5
- Noninductive Proof steps: 5
- Noninductive Interactions: 0

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	elim-pair	—	—	0.06	0.06	0.39	0.39	0.19	0.21	0.19	0.20	0/1
th-2	lem-1	0.04	0.03	0.02	0.03	0.03	0.03	0.24	0.24	0.20	0.21	0/1
th-3	lem-2	0.04	0.04	0.03	0.03	0.03	0.02	0.26	0.25	0.23	0.23	0/1
th-4	lem-3	0.01	0.01	0.00	0.00	0.05	0.05	0.45	0.45	0.22	0.25	0/1
th-5	lem-4	0.01	0.01	0.00	0.01	0.04	0.04	0.47	0.46	0.26	0.24	0/1
Σ		4	4	5	5	5	5	5	5	5	5	5
steps		5	5	5	5	5	5	5	5	5	5	5
ints		0	0	0	0	0	0	0	0	0	0	0

Specification: paramterm from case-study WAM

- Number of theorems: 7
- Proof steps: 7
- Interactions: 0
- Number of noniductive goals: 7
- Noninductive Proof steps: 7
- Noninductive Interactions: 0

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-1	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.04	0.01	0.01	0/1
th-2	lem-2	0.01	0.01	0.00	0.01	0.00	0.00	0.01	0.03	0.02	0.03	0/1
th-3	lem-3	0.01	0.01	0.00	0.00	0.00	0.00	0.03	0.02	0.02	0.02	0/1
th-4	lem-4	0.11	0.11	0.01	0.00	0.01	0.01	0.03	0.03	0.04	0.06	0/1
th-5	lem-5	0.01	0.01	0.00	0.00	0.00	0.00	0.03	0.01	0.02	0.02	0/1
th-6	lem-6	0.01	0.01	0.00	0.00	0.00	0.00	0.03	0.02	0.03	0.02	0/1
th-7	lem-7	0.01	0.01	0.00	0.00	0.00	0.00	0.03	0.02	0.04	0.03	0/1
Σ		7	7	7	7	7	7	7	7	7	7	7
steps		7	7	7	7	7	7	7	7	7	7	7
ints		0	0	0	0	0	0	0	0	0	0	0

Specification: rmode from case-study WAM

- Number of theorems: 4
- Proof steps: 5
- Interactions: 0
- Number of noniductive goals: 4
- Noninductive Proof steps: 5
- Noninductive Interactions: 0

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-01	0.03	0.03	???	???	0.00	0.00	0.02	0.04	0.28	0.27	0/2
th-2	lem-03	0.03	0.03	???	???	0.00	0.01	0.04	0.03	0.06	0.08	0/1
th-3	lem-02	0.05	0.06	???	???	0.01	0.01	0.04	0.05	0.09	0.07	0/1
th-4	lem-04	0.05	0.04	???	???	0.00	0.00	0.04	0.03	0.07	0.09	0/1
Σ		4	4	0	0	4	4	4	4	4	4	4
steps		4	4	5	5	4	4	4	4	4	4	5
ints		0	0	0	0	0	0	0	0	0	0	0

Specification: set from case-study WAM

- Number of theorems: 46
- Proof steps: 121
- Interactions: 12
- Number of noniductive goals: 48
- Noninductive Proof steps: 120
- Noninductive Interactions: 11

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-01	0.01	0.01	0.01	0.00	0.00	0.01	1.71	0.12	72.34	0.15	0/1
th-10	lem-14	—	—	—	—	—	—	—	75.96	—	—	0/1
th-11	lem-16	1.58	0.17	0.09	0.09	0.12	0.10	2.12	0.45	—	0.96	0/1
th-12	lem-23	—	—	—	—	—	—	—	15.69	—	—	0/1
th-13	lem-25	0.66	0.07	0.01	0.00	0.09	0.07	—	0.69	—	0.88	0/1
th-14	lem-27	0.01	0.01	0.01	0.01	0.00	0.00	1.85	0.23	—	0.45	0/1
th-15	lem-29	0.01	0.01	0.00	0.00	0.00	0.00	2.07	0.33	2.98	0.33	0/1
th-16	prop-01	—	—	—	—	—	—	—	—	—	—	0/1
th-17	lem-03	—	7.92	—	—	—	—	—	54.47	—	—	0/1
th-18	lem-07	—	5.80	—	—	—	—	—	12.42	—	1.42	0/1
th-19	lem-10	—	1.97	0.08	0.07	1.71	0.22	17.96	0.73	—	3.91	0/1

th-2	add-01	2.00	0.20	0.05	0.04	0.05	0.04	99.68	0.88	—	0.57	0/1
th-20	lem-11	—	2.17	0.03	0.02	0.03	0.02	3.45	1.53	50.23	0.61	0/1
th-21	lem-13	0.01	0.01	0.05	0.07	0.05	0.06	2.98	0.97	—	2.27	0/1
th-22	lem-15	—	—	—	—	—	—	4.52	1.65	62.98	0.53	0/14
th-23	lem-18	0.01	0.01	0.00	0.00	0.00	0.00	1.92	0.25	3.20	0.29	0/1
th-24	lem-19	0.01	0.01	0.00	0.00	0.00	0.00	2.44	0.61	3.16	0.37	0/1
th-25	lem-20	0.02	0.01	0.03	0.03	0.03	0.03	—	29.85	—	1.18	0/1
th-26	lem-21	0.02	0.01	0.01	0.01	0.00	0.00	2.42	0.60	29.72	0.44	0/1
th-27	lem-22	—	1.30	1.31	0.06	—	0.16	2.36	0.71	—	3.71	4/11
th-28	lem-24	—	12.68	1.93	1.70	14.06	1.75	25.42	5.55	—	16.35	5/15
th-29	lem-26	0.04	0.01	0.00	0.00	0.00	0.00	2.41	0.61	2.01	0.38	0/2
th-3	lem-06	2.07	0.18	0.08	0.07	0.08	0.05	99.37	0.91	—	0.58	0/1
th-30	lem-28	0.01	0.01	0.00	0.01	0.00	0.00	2.52	0.58	—	0.37	0/1
th-31	lem-30	0.01	0.01	0.00	0.00	0.00	0.00	2.00	0.31	25.92	0.37	0/1
th-32	lem-31	0.01	0.01	0.02	0.01	0.02	0.01	2.75	0.87	—	1.20	0/2
th-33	prop-02	—	—	—	—	—	—	—	—	—	—	0/1
th-34	prop-03	0.01	0.01	0.00	0.00	0.00	0.00	2.05	0.39	—	1.31	0/1
th-35	prop-04	0.01	0.01	0.03	0.03	0.11	0.09	—	—	—	1.77	0/1
th-36	q-01	3.41	0.48	108.64	74.66	85.08	69.63	—	2.21	—	11.61	0/24
th-37	selax	0.01	0.01	0.00	0.00	0.00	0.00	2.07	0.40	2.17	0.44	0/2
th-38	sub-03	—	—	—	—	—	—	—	—	—	5.02	0/2
th-39	sub-04	—	89.26	—	—	—	—	—	36.70	—	2.25	0/2
th-4	lem-04	8.22	1.79	16.28	5.44	19.34	7.30	—	1.55	—	11.18	0/1
th-40	sub-05	—	—	—	—	—	—	—	—	—	—	0/2
th-41	sub-06	—	—	—	—	—	—	—	—	—	—	0/2
th-42	sub-07	0.34	0.20	5.01	1.30	2.70	1.39	3.36	1.32	—	2.61	0/2
th-43	sub-08	0.41	0.27	1.02	0.74	1.07	0.74	—	—	—	3.14	0/2
th-44	sub-09	5.38	1.76	—	—	—	—	—	—	—	—	0/2
th-45	sub-10	0.01	0.01	0.00	0.01	0.01	0.01	6.88	3.77	109.84	0.81	0/1
th-46	sub-11	0.03	0.04	0.04	0.03	0.05	0.04	3.10	1.14	107.23	0.79	0/1
th-5	lem-12	0.02	0.01	0.06	0.09	0.07	0.07	99.14	0.67	—	0.87	0/1
th-6	lem-17	0.03	0.02	0.08	0.10	0.10	0.07	2.60	0.64	—	0.77	0/1
th-7	lem-02	0.89	0.19	9.17	2.72	—	2.69	—	1.55	—	3.68	1/1
th-8	lem-08	0.66	0.11	0.08	0.08	0.11	0.09	106.15	1.52	—	0.83	0/1
th-9-is	lem-09	—	—	0.65	0.10	0.51	0.11	3.66	1.55	45.73	0.53	1/4
th-9-ib	lem-09	0.01	0.01	0.00	0.00	0.00	0.00	1.63	0.21	1.64	0.26	0/0
th-9-pr	lem-09	—	3.21	0.38	0.06	0.37	0.11	4.94	0.43	1.85	0.35	0/1
Σ		31	39	36	36	34	36	30	40	15	40	44
steps		91	66	65	65	75	65	76	52	102	50	120
ints		10	1	0	0	5	0	1	0	10	0	11

Specification: stopmode from case-study WAM

- Number of theorems: 3
- Proof steps: 4
- Interactions: 0
- Number of noniductive goals: 3
- Noninductive Proof steps: 4
- Noninductive Interactions: 0

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-03	0.01	0.02	???	???	0.00	0.00	0.01	0.02	0.13	0.11	0/2
th-2	lem-01	0.03	0.03	???	???	0.00	0.00	0.02	0.03	0.04	0.04	0/1
th-3	lem-02	0.02	0.02	???	???	0.00	0.01	0.04	0.02	0.05	0.05	0/1
Σ		3	3	0	0	3	3	3	3	3	3	3
steps		3	3	4	4	3	3	3	3	3	3	4
ints		0	0	0	0	0	0	0	0	0	0	0

Specification: term from case-study WAM

- Number of theorems: 16
- Proof steps: 26
- Interactions: 0
- Number of noniductive goals: 16
- Noninductive Proof steps: 26
- Noninductive Interactions: 0

		Se	redSe	Ot	redOt	Ot-ord	redOt-ord	Sp	redSp	Sp-ord	redSp-ord	KIV
th-1	lem-01	—	—	0.18	0.18	0.18	0.18	—	7.70	—	5.04	0/2
th-10	el-var	—	—	0.12	0.08	0.13	0.11	—	9.51	—	4.18	0/2
th-11	lem-04	0.01	0.01	0.00	0.00	0.00	0.00	1.98	0.27	1.95	0.30	0/1
th-12	lem-07	0.01	0.01	0.00	0.00	0.00	0.00	1.95	0.25	1.97	0.28	0/1
th-13	lem-08	0.01	0.01	0.00	0.00	0.00	0.00	1.98	0.28	2.00	0.29	0/1
th-14	lem-10	0.01	0.01	0.00	0.00	0.00	0.00	1.97	0.25	1.98	0.28	0/1
th-15	lem-11	0.01	0.01	0.01	0.00	0.00	0.00	1.95	0.26	1.99	0.29	0/1
th-16	lem-12	0.01	0.01	0.00	0.01	0.00	0.00	1.97	0.27	2.00	0.28	0/1
th-2	lem-02	—	—	0.18	0.18	0.19	0.18	—	6.70	—	4.21	0/2
th-3	lem-03	—	—	0.24	0.17	0.24	0.18	—	6.72	—	5.20	0/2
th-4	lem-05	—	—	0.17	0.18	0.20	0.17	—	7.79	—	5.04	0/2
th-5	lem-06	—	—	0.25	0.18	0.24	0.17	—	21.80	—	6.17	0/2
th-6	lem-09	—	—	0.25	0.19	0.24	0.17	—	6.67	—	5.26	0/2
th-7	el-const	—	—	0.11	0.09	0.13	0.11	—	11.57	—	4.17	0/2
th-8	el-list	—	—	0.14	0.08	0.15	0.12	—	—	—	3.32	0/2
th-9	el-struct	—	—	0.21	0.18	—	85.65	—	85.06	—	4.28	0/2
Σ		6	6	16	16	15	16	6	15	6	16	16
steps		26	26	16	16	17	16	26	17	26	16	26
ints		0	0	0	0	0	0	0	0	0	0	0

References

[ABH+98] W. Ahrendt, B. Beckert, R. Hähnle, W. Menzel, W.Reif, G. Schellhorn, and P. Schmitt. Integrating of Automated and Interactive Theorem Proving. In W. Bibel and P. Schmitt, editors, *Automated Deduction — A Basis for Applications*, volume I: Foundations, chapter 3: Interactive Theorem Proving. Kluwer Academic Publishers, 1998.

[BR95] E. Börger and D. Rosenzweig. The WAM—definition and compiler correctness. In Christoph Beierle and Lutz Plümer, editors, *Logic Programming: Formal Methods and Practical Applications*, volume 11 of *Studies in Computer Science and Artificial Intelligence*. North-Holland, Amsterdam, 1995.

[GLMS94] C. Goller, R. Letz, K. Mayr, and J. Schumann. SETHEO v3.2: Recent developments – system abstract. In A. Bundy, editor, *12th Int. Conf. on Automated Deduction, CADE-12*, Springer LNCS 814. Nancy, France, 1994. for the newest version of SETHEO, see the URL: http://wwwjessen.informatik.tu-muenchen.de/forschung/-reasoning/setheo.html.

[Rei95] W. Reif. The KIV-approach to Software Verification. In M. Broy and S. Jähnichen, editors, *KORSO: Methods, Languages, and Tools for the Construction of Correct Software – Final Report*. Springer LNCS 1009, 1995.

[RH96] C. Weidenbach R. Hähnle, M. Kerber. Common Syntax of the DFG-Schwerpunktprogramm "Deduktion". Technical Report 10/96, Fakultät für Informatik, Universität Karlsruhe, Germany, 1996. current version available from the DFG-Schwerpunktprogramm homepage: http://www.uni-koblenz.de/ag-ki/Deduktion/.

[RS97] W. Reif and G. Schellhorn. Theorem Proving in Large Theories. CADE 97, Workshop on Automated Theorem Proving in Software Engineering, to appear, 1997.

[RS98] W. Reif and G. Schellhorn. Theorem proving in large theories. In W. Bibel and P. Schmitt, editors, *Automated Deduction—A Basis for Applications*, volume I: Foundations. Kluwer Academic Publishers, 1998.

[RSS95] W. Reif, G. Schellhorn, and K. Stenzel. Interactive Correctness Proofs for Software Modules Using KIV. In *Tenth Annual Conference on Computer Assurance*, IEEE press. NIST, Gaithersburg (MD), USA, 1995.

[RSS97] W. Reif, G. Schellhorn, and K. Stenzel. Proving System Correctness with KIV 3.0. In *14th International Conference on Automated Deduction. Proceedings.* Townsville, Australia, Springer LNCS 1249, 1997.

[SA97] G. Schellhorn and W. Ahrendt. Reasoning about Abstract State Machines: The WAM Case Study. *Journal of Universal Computer Science (J.UCS)*, 1997. available at http://hyperg.iicm.tu-graz.ac.at/jucs/.

[SA98] G. Schellhorn and W. Ahrendt. The WAM Case Study: Verifying Compiler Correctness for Prolog with KIV. In W. Bibel and P. Schmitt, editors, *Automated Deduction — A Basis for Applications*, volume III: Applications, chapter 3: Automated Theorem Proving in Software Engineering. Kluwer Academic Publishers, 1998.

[SR97] G. Schellhorn and W. Reif. Proving Properties of Finite Enumerations: A Problem Set for Automated Theorem Provers. Ulmer Informatik-Berichte 97-12, Universität Ulm, Fakultät für Informatik, 1997.

[WGR96] C. Weidenbach, B. Gaede, and G. Rock. SPASS & FLOTTER, version 0.42. In *13th International Conference on Automated Deduction, CADE-13*, Springer LNCS, 1996. for the newest version of Spass, see the URL: http://www.mpi-sb.mpg.de/-guide/software/spass.html.

[WOLB92] L. Wos, R. Overbeek, E. Lusk, and J. Boyle. *Automated Reasoning, Introduction and Applications (2nd ed.).* McGraw Hill, 1992. for the newest version of Otter, see the URL: http://www.mcs.anl.gov/home/mccune/ar/otter/#doc.