We study a portfolio optimization problem in which the transaction cost structure resembles that of a private investor. One of the main features of this cost structure is that each trade involves a strictly positive cost, leading to the formulation of the optimal investment problem as an impulse control problem, in which the investor trades only finitely many times on any finite time interval. One of the main challenges for the problem under consideration is that the value function turns out to be piecewise but not globally continuous. We establish this result in two steps: (1) We apply the stochastic Perron’s method to show that the value function is a discontinuous viscosity solution of the associated dynamic programming equation, a non-local and fully non-linear second-order partial differential equation. (2) We prove a local comparison principle for viscosity solutions of this PDE, which implies uniqueness of the value function as well as piecewise continuity. With piecewise continuity at hand, we use a characterization of the value functions as the pointwise infimum of a suitable set of superharmonic functions to construct optimal trading strategies. The advantage of this approach is that the pointwise infimum (i.e. the value function) inherits the superharmonicity property, which in turn allows us to prove a verification theorem for candidate optimal strategies requiring only piecewise continuity of the value function. An application of the verification theorem entails the existence of optimal strategies. We conclude with a study of the properties of optimal trading strategies by solving the dynamic programming equation numerically. Our results reveal a rich structure, especially for short investment horizons. This talk is based on joint work with Sören Christensen (CAU Kiel), Lukas Mich (Trier University), and Frank Seifried (Trier University).