ulm university universität uulm

Geometric Variational Problems

– Ulm University, March 23–27, 2020

Ulm University, March 23–27, 2020 (as at 2020-03-03)

Timetable for the Workshop *Geometric Variational Problems*

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00–10:00		**Schätzle**	**Grunau**	**Kuwert**	**Masnou**
10:00–11:00		**Novaga**	Eichmann Pluda	**Mantegazza**	**Deckelnick**
11:00–11:30		*Coffee break*	*Coffee break*	*Coffee break*	**Closing**
11:30–12:30		**Röger**	**Lee**	**Okabe**	
12:30–14:00	**Registration** in H21 open from 13:00 **Opening** at 13:50 in H20	*Lunch break*	*Lunch break*	*Lunch break*	
14:00–15:00	**Westdickenberg**	**Bellettini**	**Excursions** starting at 14:20 at *Münsterplatz*	**Menne**	
15:00–15:30	Gößwein	Chang		Lue	
15:30–16:00	*Coffee break*	*Coffee break*		*Coffee break*	
16:00–16:30	**Garcke**	Müller		Kröner	
16:30–17:30	**Poster session** and get-together starting at 17:00 in H21	**Blatt**		**Mäder-Baumdicker**	
			Workshop dinner from 18:30 in Restaurant *Drei Kannen*		

Welcome to Ulm!

We are very happy to welcome you to Ulm! This workshop is supported by Ulm University and organized as a part of the DFG research project "Flow of elastic network". We kindly acknowledge the financial support of both parties.

The aim of the workshop is to bring together a range of researchers working on geometric variational problems and related topics. We hope that you will enjoy the scientific programme, the interactions between young researchers and established experts, and of course the charming city of Ulm with its rich history and vibrant culture.

The organizers and local staff will be happy to assist you with any questions or issues during your stay.

The organizers,

Anna Dall'Acqua, Chun-Chi Lin and Paola Pozzi

as well as the local organizers,

Helga Lautenbach, Marius Müller, Fabian Rupp and Kinga Sudhoff

Registration

Please register upon arriving at the workshop. Registration is open from **13:00 on Monday afternoon** in **H21**. Later registration is possible on an individual basis.

Transportation

On the back of the cover, you will find a map of the university and the relevant locations with a link to the campus navigation system.

Public Transport

You can easily reach the University by public transport from anywhere in Ulm. From multiple stops in the city center one can take the **tram number 2** with direction “Science Park II”. For the shortest walk to the lecture hall it is best to exit the tram at the **station “Universität Süd”**. From the “B & B” Hotel the most recommendable stop to enter is “Ehinger Tor” and from the “Goldenes Rad” Hotel it is “Hauptbahnhof”.

If you are planning on using the tram on 5 days for at least two times each day it is recommended to buy a weekly pass for €20.50 at one of the ticket vending machines. The tram number 2 also stops at the central station in between the aforementioned stations.

Taxi and Car

You can also reach the University by car. The only publicly available parking lot is the “Parkhaus Helmholtzstraße” located at Helmholtzstraße 5, 89075 Ulm. Using the parking spot for one day costs €5. Taxis are easily recognizable by the yellow “taxi” signs on top of their roof. You can order a taxi by calling ULM-TAXI +49 731 66066. A taxi from the central station to the university costs about €20.

At the University

Locations

Registration as well as all the lectures and talks will take place in lecture hall **H20** in cross section O27. The coffee breaks and the poster session will take place in **H21**. On the back cover you may find a schematic map that details the important locations: Lecture halls H20/H21, the bus/tram stop “Universität Süd” as well as the Mensa and cafeterias.

Catering

During each of the coffee breaks we are going to provide coffee and some cookies. For lunch we are planning to go to the Mensa. The Mensa provides a variety of dishes and also vegetarian/vegan options. If you only want to have a small snack you can also go to one of the Cafeterias marked on the map. You need to pay for food in cash. As the next ATM is 500 m from campus we recommend to bring enough cash with you. We note that you need to use the checkout (no.1) on the far left in the Mensa to pay for your food.

Internet Access

At most parts of campus you are able to connect to the open WiFi “welcome”. Note that this connection is unencrytped. If you have an eduroam profile installed on your device you should be able to use the WiFi “eduroam”.

Social Events

Poster Session and Get-Together

The poster session will take place on Monday starting from **17:00** in **H21**. During the session drinks and snacks will be offered and we hope for a pleasant atmosphere with many interesting discussions.

You can find a list of posters in this booklet.

Conference Dinner

The conference dinner will take place on Wednesday evening at 18:30. The location is the historical brewery "Drei Kannen" with fine local food.

The address of the restaurant is "Hafenbad 31". It is a 5-minute walk from the "Ulmer Münster" Cathedral. The fastest route is to start in front of the small choir towers on the opposite site of the main tower and then walk straight north in the opposite direction of the "Rathaus" bus station. You will soon enter the "Hafenbad" street which you just have to keep walking along until you see a sign "Drei Kannen" leading into a courtyard. Here you will find the entrance.

For starter a beef broth with patties and fresh herbs or a small mixed salad plate will be served. For the main course you can choose between 3 dishes:

1. Vegetarian whole food plate - various vegetable medallions with wild rice, market vegetables and herb sauce.

2. Oven-fresh roast pork in beer sauce with homemade spaetzle and a small mixed salad

3. "Drei Kannen" trout with apples, raisins, almonds and parsley potatoes

We will ask you for your main dish choice upon arrival.

For dessert please decide between a Bavarian apple strudel coated with fine strudel dough with vanilla sauce or filled curd cheese pancakes with vanilla ice cream and cream.

If you have food allergies or other food restrictions and you have difficulties communicating them to the waiters, please get in touch with the organizers who will be happy to assist you.

Excursion

On Wednesday **14:20–16:00** there will be two tours offered to explore the famous church "Ulmer Münster" – the highest cathedral in the world. The various tours will take place in small groups of approx. 15 people each and will take about 1.5 hours in total. Kindly note that sturdy footwear (e.g. sneakers and heelless shoes) is needed for the tour. In case of rain, please make sure to bring a rain jacket. Please refrain from bringing umbrellas since the passages in the church are too narrow.
We offer two different tours that will all display different highlights of the cathedral

1. **HIDDEN CHAMBERS**

   Introduction with a tour of the church.

   Guided tour to the two chambers of the southern choir towers, walk over the roof vault, Christ chamber, organ chamber, dance balls and paradise. You can see old models of stonemasonry, medieval Jewish tombstones and much more.

2. **AS HEAVEN WOULD OPEN** – the southern choir tower

   Anyone standing directly under the tower helmet of the southern choir tower has the impression that the sky is opening above them. The tour offers rare insights and outlooks in and out of the recently restored southern choir tower - right under the tower helmet.

Schedule

Monday, March 23

13:00–13:50	Registration in H21
13:50–14:00	Opening in H20
14:00–15:00	Maria Westdickenberg
15:00–15:30	Micheal Gößwein
15:30–16:00	Coffee break
16:00–17:00	Harald Garcke
17:00–end	Poster Session and get-together

Tuesday, March 24

09:00–10:00	Reiner Schätzle
10:00–11:00	Matteo Novaga
11:00–11:30	Coffee break
11:30–12:30	Matthias Röger
12:30–14:00	Lunch break
14:00–15:00	Giovanni Belettini
15:00–15:30	Jui-En Chang
15:30–16:00	Coffee break
16:00–16:30	Marius Müller
16:30–17:30	Simon Blatt

Wednesday, March 25

09:00–10:00	Hans-Christoph Grunau
10:00–10:30	Sascha Eichmann
10:30–11:00	Alessandra Pluda
11:00–11:30	Coffee break
11:30–12:30	Yng-Ing Lee
12:30–14:00	Lunch break
14:15–16:00	Excursions
18:30–end	Workshop Dinner

Thursday, March 26

9:0–10:00	Ernst Kuwert
10:00–11:00	Carlo Mantegazza
11:00–11:30	Coffee break
11:30–12:30	Shinya Okabe
12:30–14:00	Lunch break
14:00–15:00	Ulrich Menne
15:00–15:30	Yang-Kai Lue
16:00–16:30	Heiko Kröner
16:30–17:30	Elena Mäder-Baumdicker

Friday, March 27

9:00–10:00	Simon Masnou
10:00–11:00	Klaus Deckelnick
11:00–11:10	Closing

Abstracts

Some results on anisotropic mean curvature flow of partitions

Giovanni Bellettini[1]
University of Siena[1]

We shall discuss a work in progress, in collaboration with Antonin Chambolle (Paris) and Shokhrukh Kholmatov (Wien), which concerns the existence of a weak solution to the anisotropic mean curvature flow of partitions, possibly with mobilities. In the case of the flow of networks in the plane, we shall illustrate some qualitative results on strong solutions obtained with Milena Chermisi (Cremona) and Matteo Novaga (Pisa) a few years ago [1], when one crystalline anisotropy is present.

[1] G. Bellettini, M. Chermisi, M. Novaga, Crystalline curvature flow of planar networks, *Interfaces Free Bound.* **8** (2006), pp. 481–521.

On the Negative Gradient Flow of the *p*-Energy

Simon Blatt[1], Christopher Hopper[2], and Nicole Vorderobermeier[1]
Paris-Lodron University Salzburg[1]; University Konstanz[2]

This talk will be about ongoing work on the negative L^2 gradient flow of p-elastic energies for curves. After an overview over known results we will discuss short- and longtime existence for these evolution equations using de Giorgi's method of minimizing movements. Apart from that the essential new tool in our approach is to write the evolving curves as approximate normal graphs over a fixed smooth curves. In this way we can prove a lower bound for the time of existence that only depends on the $W^{2,p}$ norm of the initial curve. We will shortly discuss the pros and cons of this method and present some open problems.

The uniqueness of self-shrinking networks with 2 enclosed regions

Jui-En Chang[1]

National Taiwan University[1]

To study the formation of singularities in network flow, solutions which move by homothety scaling plays an important role. Such network is called a regular shrinker. Up to now, only the shrinker with less than 2 triple junctions or 1 enclosed region is classified. In this talk, I'll present our recent result about the classification of regular shrinker with 2 enclosed regions. This is a joint work with Dr. Yang-Kai Lue.

A finite element error analysis for axisymmetric mean curvature flow

John W. Barrett[1], Klaus Deckelnick[2] and Robert Nürnberg[3]

John sadly passed away on 30 June 2019[1]; Otto–von–Guericke–Universität Magdeburg[2]; Imperial College London[3]

The talk is concerned with the numerical approximation of axisymmetric mean curvature flow. Translating the underlying evolution law into an equation for the generating curves leads to curve shortening flow with an additional nonlinearity that involves the distance to the axis of rotation. Using DeTurck's trick we are able to formulate this problem in terms of a strictly parabolic system for the position vector of the curves. This system is then discretized with the help of linear finite elements in space and a backward Euler scheme in time. Our main result are optimal error bounds with respect to the L^2– and H^1–norms for closed genus-1 surfaces. We present results of numerical tests which confirm our analysis and use a suitable variant of our approach in order to approximate the Angenent torus. Our numerical method can also be applied to genus-0 surfaces, in which case suitable boundary conditions need to be incorporated. The method works well in practice although a corresponding error analysis is still open.

Minimising the Canham-Helfrich energy

Sascha Eichmann[1]

Eberhard Karls Universität Tübingen[1]

Helfrich [5] and Canham [1] introduced the following geometric curvature energy to modell the shape of human red blood cells and so called vesicles, i.e. lipid bilayers. The main idea is to interpret the boundary layer of such a cell or vesicle as a two dimensional surface Σ, which minimises

$$\int_{\Sigma} |H - H_0|^2 \, dA$$

under suitable constraints on e.g. the enclosed volume and surface area. Here H is the scalar mean curvature of Σ and $H_0 \in \mathbb{R}$ is a parameter called the spontaneous curvature. H_0 induces a prefered curvature of the cell and therefore represents an asymmetry in the boundary layer of the cell itself.
The goal is now to implement the direct method of variational calculus to show existence. Compactness for a minimising sequence under varifold convergence can be easily obtained, but unfortunately lower-semicontinuity of the Helfrich energy under this varifold convergence is in general not correct by a counterexample of Große-Brauckmann [4]. Nevertheless we can actually show a lower-semicontinuity estimate for the minimising sequence itself, see [3]. These kind of estimates can furthermore be used to show existence results for the Helfrich equation, i.e. the Euler-Lagrange equation of the energy, see e.g. [2].
In the last part of the talk, we will discuss directions for future research in this area, i.e. some open problems and some modifications of the Canham-Helfrich energy.

[1] P.B. Canham. The minimum energy of bending as a possible explanation of the biconcave shape of the human red blood cell. *J. Theor. Biol.* (1970), **26(1)** pp. 61–76.

[2] S. Eichmann, The Helfrich Boundary Value Problem, *Calc. Var.* **58:34** (2019) https://doi.org/10.1007/s00526-018-1468-x.

[3] S. Eichmann, Lower-semicontinuity for the Helfrich problem, *arXiv Preprint* (2019) arXiv:1908.11738.

[4] Karsten Große-Brauckmann. New surfaces of constant mean curvature. *Math. Z.* (1993) **214** pp. 527–565.

[5] W. Helfrich. Elastic properties of lipid bilayers: Theory and possible experiments. *Z. Naturforsch. C* (1973) **28** pp. 693–703.

Long Time Existence of Solutions to an Elastic Flow of Networks

Harald Garcke[1], Julia Menzel[1] and Alessandra Pluda[2]
University of Regensburg[1]; University of Pisa[2]

The L^2-gradient flow of the elastic energy of networks leads to a Willmore type evolution law with nontrivial nonlinear boundary conditions. We show local in time existence and uniqueness for this elastic flow of networks in a Sobolev space setting under natural boundary conditions. In addition we show a regularisation property and geometric existence and uniqueness. The main result is a long time existence result using energy methods. Finally, we will show several numerical computations which will illustrate the qualitative behaviour of solutions.

Stability and Parabolic Smoothing for the Surface Diffusion Flow with Triple Junctions in $\mathbb{R}^n$

Harald Garcke[1] and Michael Gößwein[2]
University of Regensburg[1]; University of Duisburg-Essen[2]

The surface diffusion flow is based on a model from Mullins to describe the evolution of thermal grooves on the surface of heated polycrystals. For a surface Γ its evolution by surface diffusion flow is given by

$$V_\Gamma = -\Delta_\Gamma H_\Gamma,$$

where V_Γ denotes the normal velocity of Γ, Δ_Γ the surface Laplace-Beltrami operator and H_Γ the mean curvature. From a mathematical point of view it can be seen as a fourth order (formal) $\mathcal{H}^{-1}$-gradient flow that is volume conserving. In this talk we discuss our results for the evolution of clusters with triple junctions, which means that three hypersurfaces meet in a common boundary. We focus on the stability analysis of energy minimizers, which are standard double bubbles, and the parabolic smoothing of the flow. For the first result we explain how a ojasiewicz-Simon gradient inequality can be used in the higher dimensional case. For the second one, we develop a strategy with a parameter trick dealing with the fully-nonlinear boundary conditions.

[1] H. Garcke and M. Gößwein, On the Surface Diffusion Flow with Triple Junctions in Higher Space Dimensions, *arXiv:1907.11682*.
[2] H. Garcke and M. Gößwein, Non-linear Stability of Double Bubbles under Surface Diffusion, *arXiv:1910.01041*.

Boundary value problems for the Willmore and the Helfrich functional for surfaces of revolution

Hans-Christoph Grunau[1]

Otto-von-Guericke-Universität Magdeburg[1]

This talk concerns joint works with A. Dall'Acqua, K. Deckelnick, M. Doemeland, S. Eichmann, and S. Okabe.

A special form of the Helfrich energy for a sufficiently smooth (two dimensional) surface $S \subset \mathbb{R}^3$ (with or without boundary) is defined by

$$\mathscr{H}_\varepsilon(S) := \int_S H^2\, dS + \varepsilon \int_S dS,$$

where H denotes the mean curvature of S. The first integral may be considered as a bending energy and the second as surface (stretching) energy. $\mathscr{W}(S) := \mathscr{H}_0(S)$ is called the Willmore functional. We consider surfaces of revolution S

$$(x,\varphi) \mapsto \big(x, u(x)\cos\varphi, u(x)\sin\varphi\big), \quad x \in [-1,1],\ \varphi \in [0,2\pi],$$

with smooth strictly positive profile curve u subject to Dirichlet boundary conditions

$$u(-1) = \alpha, \quad u(1) = \beta, \quad u'(\pm 1) = 0$$

and aim at minimising $\mathscr{H}_\varepsilon$. Thanks to these boundary conditions the Gauss curvature integral $\int_S K\, dS$ becomes a constant and needs not to be considered.

In the first part of the lecture I shall consider the Willmore case, i.e. $\varepsilon = 0$. After briefly recalling minimisation in the symmetric case $\alpha = \beta$ (see [1,4]) I shall show how much more complicated the problem becomes for $\alpha \neq \beta$. Only when α and β do not differ too much, the profile curve will remain a graph while in general it will become a nonprojectable curve, see [3].

In the second part, $\mathscr{H}_\varepsilon$ is considered for $\varepsilon \in [0,\infty)$, but again in the symmetric setting $\alpha = \beta$. For $\alpha \geq \alpha_m = c_m \cosh(\frac{1}{c_m}) \approx 1.895$ with $c_m \approx 1.564$ the unique solution of the equation $\frac{2}{c} = 1 + e^{-2/c}$, when one has a catenoid v_α which is stable with respect to the surface energy, we find minimisers u_ε for any $\varepsilon \geq 0$ and show uniform convergence $u_\varepsilon \to v_\alpha$ under the singular limit $\varepsilon \to \infty$. These results will be collected in [2].

At the end I shall briefly mention ongoing work on obstacle problems [5].

[1] A. Dall'Acqua, K. Deckelnick, and H.-Ch. Grunau, Classical solutions to the Dirichlet problem for Willmore surfaces of revolution, *Adv. Calc. Var.* **1** (2008), 379-397.

[2] K. Deckelnick, H.-Ch. Grunau, and M. Doemeland, Boundary value problems for the Helfrich functional for surfaces of revolution - Existence and asymptotic behaviour, in preparation.

[3] S. Eichmann and H.-Ch. Grunau, Existence for Willmore surfaces of revolution satisfying non-symmetric Dirichlet boundary conditions, *Adv. Calc. Var.* **12** (2019), 333–361.

[4] H.-Ch. Grunau, The asymptotic shape of a boundary layer of symmetric Willmore surfaces of revolution. In: C. Bandle et al. (eds.), Inequalities and Applications 2010. *International Series of Numerical Mathematics* **161** (2012), 19-29.

[5] H.-Ch. Grunau and S. Okabe, Willmore obstacle problems under Dirichlet boundary conditions, in preparation.

Flowing the leaves of a foliation with the logarithm of the mean curvature in the rotationally symmetric case

Heiko Kröner[1]

University Duisburg-Essen[1]

Adapting results from [1] where a logarithmic Gauss curvature type flow is used to give a variational reproof of the Minkowski problem we show the existence of a critical leave in any rotationally symmetric foliation of $\mathbb{R}^{n+1} \setminus \{0\}$ by uniformly convex hypersurfaces with mild assumptions concerning the asymptotical behavior of a 'logarithmic mean curvature flow': When starting the flow from a leave of the foliation inside the critical one it shrinks to a point while starting the flow from a leave outside the critical one its diameter goes to infinity. In the critical case the flow converges to a translating solution of the flow equation.

[1] K.-S. Chou and X.-J. Wang, *A* logarithmic Gauss curvature flow and the Minkowski problem, Ann. Inst. H. Poincaré Anal. Non Linéaire **17** (2000), no. 6, 733-751.

Asymptotic Estimates for the Willmore Flow with small Energy

Ernst Kuwert[1]

University of Freiburg[1]

It is well-known that if a 2-sphere immersed in $\mathbb{R}^n$ has small Willmore energy, then its Willmore flow converges smoothly to a round sphere. For this limit, we discuss stability estimates for geometric quantities, such as the barycenter and the averaged mean curvature (for $n = 3$). (joint work with Julian Scheuer, Freiburg)

Lagrangian Mean Curvature Flow

Yng-Ing Lee[1]

National Taiwan University[1]

Mean Curvature Flow (MCF) is a canonical way to deform sub-manifolds to minimal submanifolds. The condition of being Lagrangian is preserved for compact smooth solutions of MCF in a Kahler-Einstein manifold. The case is thus called Lagrangian mean curvature flow (LMCF) which is a potential way to construct special Lagrangians or minimal Lagrangians.

In this talk, I will discuss results and difficulties in LMCF, and report some of our progress in this direction.

The second-order L^2-flow for inextensible elastic networks

Chun-Chi Lin[1] and Yang-Kai Lue[2]

National Taiwan Normal University and National Center for Theoretical Sciences[1]; National Taiwan Normal University[2]

In this talk, I will introduce some results about the geometric flow for the inextensible elastic networks in the plane. The flow is described by a second-order parabolic PDE system for the set of tangent indicatrices of curves forming a network. The bending energy of networks decreases during the flow. Under mild assumptions on the boundary conditions at the junction points of networks and smoothness of the initial data, we show the short-time existence of solutions to the flow.

The Morse Index of Willmore spheres and its relation to the geometry of minimal surfaces

Elena Mäder-Baumdicker[1]

Technical University of Darmstadt[1]

Together with Jonas Hirsch we recently studied the Morse Index of Willmore spheres. This Index is the number of linearly independent variational directions that locally decrease the Willmore energy up to second order. The crucial point is to carefully use the conformal invariance of the Willmore functional for the computation of the second variation. It turned out that there is a strong connection to results about Jocobi fields on minimal surfaces proven by Montiel, Pérez and Ros in the 1990's. We will explain these relations and why this is of interest for the so-called Sphere Eversion.

Evolution by curvature of networks in the plane

Carlo Mantegazza[1]

University of Naples[1]

I will present the state-of-the-art of the study of the motion by curvature of a network of curves in the plane, discussing the current open problems.

A self-avoiding approximate mean curvature flow

Elie Bretin[1], Chih-Kang Huang[2], and Simon Masnou[3]

INSA Lyon, France[1]; École normale supérieure de Lyon, France[2]; Université Claude Bernard Lyon 1, France[3]

The starting point for this work is the following observation: there is a phase field approximation of the Willmore flow that seems to prevent, at least numerically, the appearance of self-intersections. From the "active principle" of this approximation prohibiting self-intersections, we can deduce a simpler term which, added to the phase field approximation of mean curvature flow, acts as a dynamic obstacle that promotes self-avoidance.

We will describe some preliminary theoretical properties of this term and a numerical scheme for the self-avoiding approximate mean curvature flow. We will show some numerical simulations, in particular for the approximation of solutions to Steiner's or Plateau's problems.

A priori geodesic diameter bounds for solutions to a variety of Plateau problems

Ulrich Menne[1] and Christian Scharrer[2]

National Taiwan Normal University and National Center for Theoretical Sciences[1]; University of Warwick[2]

Plateau's problem in Euclidean space may be given many distinct formulations with solutions to most of them admitting an associated varifold. This includes Reifenberg's approach based on sets and Čech homology as well as Federer and Fleming's approach using integral currents and their homology. Thus, we employ the setting of varifolds to prove a priori bounds on the geodesic diameter in terms of boundary behaviour.

An obstacle problem for elastic curves

Marius Müller[1]

Ulm University[1]

In this talk we seek to minimize Euler's elastic bending energy

$$\mathcal{W}(\gamma) := \int_\gamma \kappa^2 \, \mathrm{ds} \tag{0.1}$$

among *graph curves* with fixed ends in $\mathbb{R}^2$, i.e. $\gamma(x) = (x, u(x))$ for some $u \in W^{2,2}(0,1) \cap W_0^{1,2}(0,1)$. Further, the minimization is subject to an *obstacle constraint*, i.e. we require that γ lies above the graph of a certain *obstacle function* ψ. While existence of minimizers has been shown in [1] for small obstacles ψ, it will in general fail for large obstacles.

What "large" means can be quantified: In 1744 Euler drew the *critical curves* of $\mathcal{W}$. The critical height of the obstacle can be read off of one of those: the *wavelike free elastica*. One can ask why minimization fails: Numerical studies expose that minimizing sequences develop *vertical parts* and thus leave the class of graph curves.

Since straight vertical lines have vanishing bending energy, one could rather look at them as a natural phenomenon than as obstruction to minimization. This leads to a slightly different framework in which we allow admissible curves to have vertical parts. This new framework can also be understood as a *relaxation* of the problem. We will show that minimizers with vertical parts exist for arbitrarily large obstacles.

[1] A. Dall'Acqua and K. Deckelnick, An obstacle problem for elastic graphs, *SIAM J. Math. Anal.* **50** (2018).

[2] M. Müller, An obstacle problem for elastic curves: Existence results, *Interfaces Free Bound.* **21** (2019)

Asymptotics of the fractional perimeter and of the fractional mean curvature flow

Matteo Novaga[1]

University of Pisa[1]

I will discuss the asymptotic expansion of the fractional perimeter P_s, as $s \to 0^+$ and $s \to 1^-$, in the sense of Γ-convergence. I will also consider the convergence of the corresponding evolutions.

The results are based on joint works with A. Cesaroni, L. De Luca and M. Ponsiglione [1,2].

[1] L. De Luca, M. Novaga and M. Ponsiglione, The 0-fractional perimeter between fractional perimeters and Riesz potentials, Preprint (2019).

[2] A. Cesaroni and M. Novaga, Second-order asymptotics of the fractional perimeter as $s \to 1$, Preprint (2020).

On the isoperimetric inequality and surface diffusion flow for multiply winding curves

Tatsuya Miura[1] and Shinya Okabe[2]

Tokyo Institute of Technology[1]; Tohoku University[2]

In this talk we consider the Cauchy problem of the curve diffusion flow:

$$\begin{cases} \partial_t \gamma = -(\partial_s^2 \kappa)\nu, \\ \gamma(\cdot, 0) = \gamma_0(\cdot), \end{cases} \tag{0.2}$$

where $\gamma : \mathbb{S}^1 \times [0, T) \to \mathbb{R}^2$ is a one-parameter family of immersed curves, and κ, ν, and s denote the signed curvature, the unit normal vector, and the arc length parameter of each time-slice curve $\gamma(t) := \gamma(\cdot, t)$, respectively. Since all the stationary solutions to (1) are circles possibly multiply covered, it is expected that the solution of (1) converges to one of the equilibria as $t \to \infty$, if the solution exists globally in time. Indeed, for the singly winding case, [2, 4, 7] proved the existence of a global-in-time solution to (1) provided that an initial curve is close to a circle. The purpose of this talk is to show such a sufficient condition on initial data for the multiply winding case.

To this end, we impose a rotational symmetry on initial curves, because of a main difference from the singly winding case: multiply covered circles are not dynamically stable. In this talk, we prove that if an initial curve is H^2-close to an n-times covered circle and m-th rotationally symmetric for $1 \leq n \leq m$, then there exists a unique global-in-time solution to (1). The crucial point in our argument is to prove a generalized isoperimetric inequality for rotationally symmetric curves without local convexity. Indeed, although for locally convex curves the same kind of isoperimetric inequality under rotational symmetry has been already proved in [1, 3, 6], this is not directly applicable to the curve diffusion flow because the flow may lose the convexity ([5]). Combining our generalized isoperimetric inequality with Wheeler's variational argument, we prove the existence of a global-in-time solution to (1).

[1] K.-S. Chou, A blow-up criterion for the curve shortening flow by surface diffusion, *Hokkaido Math. J.* **32** (2003), pp. 1–19.

[2] C. M. Elliott and H. Garcke, Existence results for diffusive surface motion laws, *Adv. Math. Sci. Appl.* **7** (1997), pp. 467–490.

[3] C. L. Epstein and M. Gage, The curve shortening flow, in *Wave motion: theory, modelling, and computation (Berkeley, Calif., 1986)*, Math. Sci. Res. Inst. Publ., **7**, Springer, New York, 1987, pp. 15–59.

[4] J. Escher, U. F. Mayer and G. Simonett, The surface diffusion flow for immersed hypersurfaces, *SIAM J. Math. Anal.* **29** (1998), pp. 1419–1433.

[5] Y. Giga and K. Ito, Loss of convexity of simple closed curves moved by surface diffusion, in *Topics in nonlinear analysis, Progr. Nonlinear Differential Equations Appl.*, **35**, Birkhäuser, Basel, 1999, pp. 305–320.

[6] B. Süssmann, Isoperimetric inequalities for special classes of curves, *Differential Geom. Appl.* **29** (2011), pp. 1–6.

[7] G. Wheeler, On the curve diffusion flow of closed plane curves, *Ann. Mat. Pura Appl. (4)* **44** (2013), pp. 931–950.

Calibrations for minimal partition problems

Alessandra Pluda[1]

University of Pisa[1]

The Steiner problem in its classical formulation reads as follows: given a finite collection of points S in the plane, find the connected set that contains S with minimal length. Although existence and regularity of minimizers is well known, in general finding explicitly a solution is extremely challenging, even numerically. For this reason every method to determine solutions is welcome. A possible tool is the notion of calibrations. In this talk I will define calibrations for the Steiner problem within the framework of covering spaces. Moreover I will give some example of both existence and non–existence of calibrations and to overcome this second unlucky case I will introduce the notion of calibration in families. I will also show how with the same tools one can handle the minimal partition problem in the plane. If time allows, I will consider generalizations of the problem to higher dimensions and to the case of non–local perimeters.

Stochastic mean curvature flow for graphs

Matthias Röger[1] and Nils Dabrock[1] and Martina Hofmanova[2]
Technische Universität Dortmund[1]; Universität Bielefeld[2]

We consider the evolution of graphs under a stochastically perturbed mean curvature flow. The noise term is white in time and possibly colored in space. We show suitable a priori estimates and prove existence of solutions in arbitrary dimensions. In the case of spatially homogeneous noise we obtain a uniform gradient bound and prove that solutions become asymptotically constant in space and behave like the driving noise in time.

The umbilic set of Willmore surfaces

Reiner Schätzle[1]
University of Tübingen[1]

It is well known that the umbilic points of minimal surfaces in spaces of constant sectional curvature consist only of isolated points unless the surface is totally umbilic on some connected component, as for example the Hopf form is holomorphic. We prove that on connected not totally umbilic Willmore surfaces in codimension one the umbilic set is locally a one dimensional real-analytic submanifold without boundary or an isolated point, and we mention examples that one dimensional submanifolds of umbilic points indeed occur.

Energetic methods for capturing sharp convergence rates of nonconvex gradient flows

Maria G. Westdickenberg[1]
RWTH Aachen University[1]

Together with Felix Otto and collaborators, we have developed energy-based methods to capture rates of convergence to equilibrium and used them to establish optimal, algebraic convergence for the Mullins-Sekerka problem in the plane and the Cahn-Hilliard equation on the line. The first method is based on the observation of Brezis that the gradient flow with respect to a convex (but not strictly convex) energy satisfies

$$E \leq (H\,D)^{\frac{1}{2}}, \quad \dot{E} = -D, \quad \dot{H} \leq 0, \quad \dot{D} \leq 0,$$

where E, H, and D represent the energy, squared-distance to equilibrium, and dissipation, respectively. From this information it is not hard to deduce

$$E \leq \frac{H_0}{t}.$$

Our first result shows that it is possible to adapt this idea to the (mildly) nonconvex setting.

A second method that we have developed has at its heart a Nash-type inequality of the form

$$E \leq D^{\frac{1}{3}}(V+1)^{\frac{4}{3}},$$

where V represents the L^1-distance from equilibrium. Critical in order to make use of this information is a duality argument that establishes *boundedness* of the L^1-distance for all positive time.

Work in progress with Richard Schubert (postdoc, RWTH Aachen University) combines elements from each framework.

[1] O. Chugreeva, F. Otto, and M. G. Westdickenberg, Relaxation to a planar interface in the Mullins-Sekerka problem, *Interfaces and Free Bound.* **21** (2019), pp. 21–40.

[2] F. Otto and M. G. Westdickenberg, Relaxation to equilibrium in the one-dimensional Cahn-Hilliard equation, *SIAM J. Math. Anal.* **46** (2014), pp. 720–756.

[3] S. Scholtes, F. Otto, and M. G. Westdickenberg, Optimal L^1-type relaxation rates for the Cahn–Hilliard equation on the line, *SIAM J. Math. Anal.* **51** (2019), pp. 4645–4682.

Poster Session

Name	**Affiliation**	**Title**
Arian Berdellima	University of Göttingen	*Weak topology in Hadamard spaces*
Katharina Brazda	University of Vienna	*Variational models for multiphase biomembranes*
Marco Doemeland	RWTH Aachen University	*Gradient flows of closed differential forms*
Van Phu Chong Le	University of Trento	*On the obstacle problem for fractional semilinear wave equations*
Jiri Minarcik	CTU Prague	*Minimal surface generating flow for space curves of non-vanishing torsion*
Tatsuya Miura	Tokyo Institute of Technology	*Geometric inequalities involving mean curvature for convex surfaces*
Nobuhito Miyake	Tohoku University	*Positive solutions to the Cauchy problem for linear and semilinear biharmonic heat equation*
Marius Müller	Ulm Unversity	*A biharmonic Bernoulli problem*
Sven Pistre	RWTH Aachen University	*The Plateau problem for the Busemann–Hausdorff area in arbitrary codimension*
Jesse Ratzkin	University of Würzburg	*Constant Q-curvature metrics and conformal invariants*
Fabian Rupp	Ulm University	*The constrained Lojasiewicz-Simon gradient inequality*
Christian Scharrer	Warwick University	*Existence and regularity of Helfrich spheres*
Philip Schrader	Tohoku University	*Morse theory for elastica*
Daniel Steenebrügge	RWTH Aachen University	*A speed preserving Hilbert gradient flow for generalized integral Menger curvature*
Kathrin Stollenwerk	RWTH Aachen University	*Optimality conditions for the buckling load of a clamped plate*
Dmitry Vorotnikov	Universidade de Coimbra	*Inextensible strings and variational evolution of arcs*
Kensuke Yoshizawa	Tohoku University	*A remark on elastic graphs with the symmetric cone obstacle*

List of Participants

Sophie Atzpodien (LMU Munich)

Giovanni Bellettini (University of Siena)

Abderrazek Benhassine (Higher Institute of Computer Sciences and Mathematics of Monastir)

Simon Blatt (University of Salzburg)

Omar Boussaid (Hassiba Benbouali University of Chlef)

Katharina Brazda (University of Vienna)

Emma Brink (LMU Munich)

Jui-En Chang (National Taiwan University)

Anna Dall'Acqua (Ulm University)

Klaus Deckelnick (University of Magdeburg)

Nicola De Nitti (University of Bari)

Marco Doemeland (RWTH Aachen University)

Wenkui Du (University of Toronto)

Sascha Eichmann (University of Tübingen)

Alexander Friedrich (University of Copenhagen)

Harald Garcke (University of Regensburg)

Rabeh Ghoudi (University of Gabes)

Michael Gößwein (University of Duisburg-Essen)

Hans-Christoph Grunau (University of Magdeburg)

Boris Gulyak (University of Magdeburg)

Ide Ibrahim (Saratov State University)

Kennedy Idu (University of Pisa)

Sholeh Karimghasemi (RWTH Aachen University)

Daehwan Kim (Korea Institute for Advanced Study)

Jan Knappmann (RWTH Aachen University)

Yulia Kostina (LMU Munich)

Heiko Kröner (University of Duisburg-Essen)

Ernst Kuwert (University of Freiburg)

Van Phu Cuong Le (University of Trento)

Yng-Ing Lee (National Taiwan University)

Oluwakemi Oluwabusayo Lewis (Obafemi Awolowo University)

Chun-Chi Lin (National Taiwan Normal University)

Yang-Kai Lue (National Taiwan Normal University)

Manivannan M (Central University of Rajasthan)

Elena Mäder-Baumdicker (TU Darmstadt)

Carlo Mantegazza (University of Napoli)

Simon Masnou (University of Lyon 1)

Ulrich Menne (National Taiwan Normal University)

Marius Müller (Ulm University)

Amar Megrous (Ecole Supérieure de Comptabilité et de financ)

Jiri Minarcik (CTU Prague)

Tatsuya Miura (Tokyo Institute of Technology)

Nobuhito Miyake (Tohoku University)

Abdulghani Muhyi (Aligarh Muslim University)

Matteo Novaga (University of Pisa)

Shinya Okabe (Tohoku University)

Ashraf Owis (Cairo University)

Waheed Prince Oyesetan (ASFAO Sports and Academy association)

Sven Pistre (RWTH Aachen University)

Alessandra Pluda (University of Pisa)

Marco Pozzetta (University of Pisa)

Paola Pozzi (University of Duisburg-Essen)

Jesse Ratzkin (University of Würzburg)

Matthias Röger (TU Dortmund)

Fabian Rupp (Ulm University)

Noussaiba Saadoudi (UMBB)

Nermin Saber (British University in Egypt)

Mario Santilli (Augsburg University)

Manfred Sauter (Ulm University)

Reiner Schätzle (University of Tübingen)

Christian Scharrer (University of Warwick)

Philip Schrader (Tohoku University)

Solomon Oluwatosin Somoye (Obafemi Awolowo University)

Gbolahan Moses Somoye (Obafemi Awolowo University)

Daniel Steenebrügge (RWTH Aachen University)

Kathrin Stollenwerk (RWTH Aachen University)

Dmitry Vorotnikov (University of Coimbra)

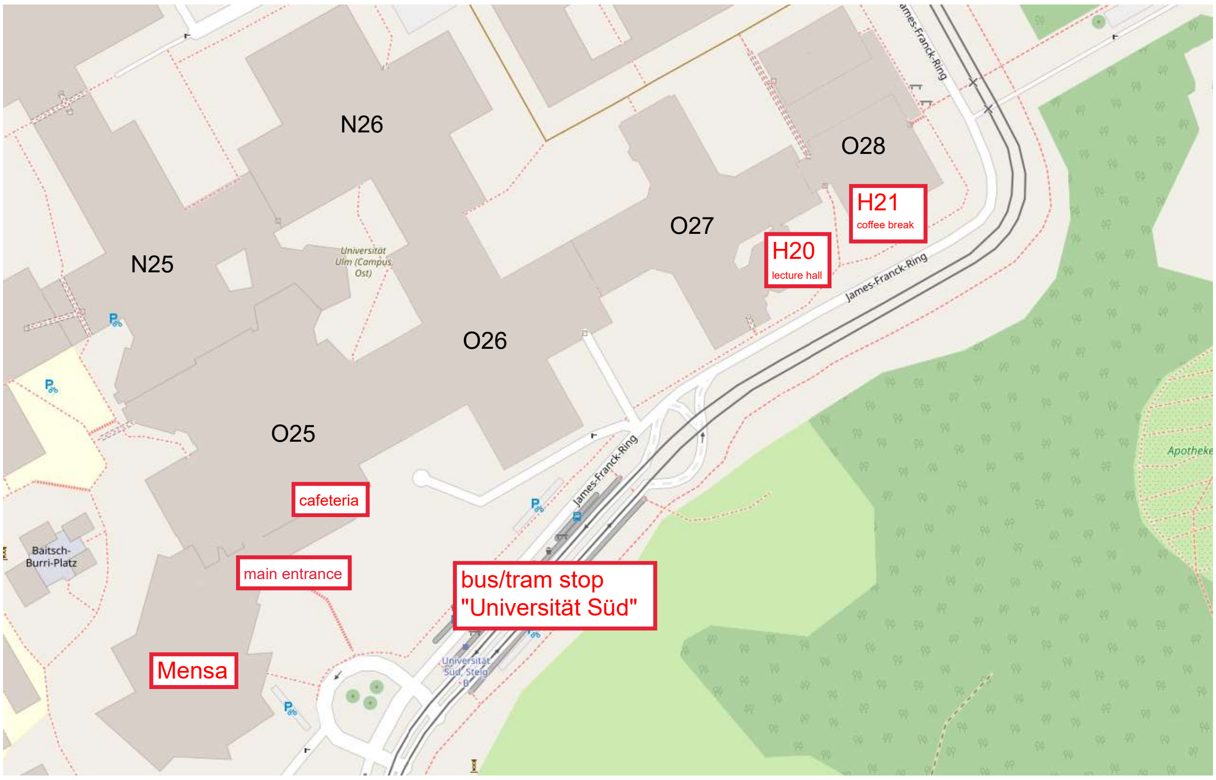
Liangjun Weng (University of Freiburg)

Maria Westdickenberg (RWTH Aachen University)

Zhipeng Yang (University of Göttingen)

Kensuke Yoshizawa (Tohoku University)

Map of the University



For a more detailed map see `https://www.uni-ulm.de/en/einrichtungen/kiz/further-information/campus-navigation/`.