The propagation of sound waves through a homogeneous medium may be described mathematically by the wave equation, a prototypical partial differential equation. By invoking tools from areas such as measure theory, functional analysis and Fourier analysis, one can find an infinite-dimensional eigenvalue problem at the heart of the wave equation, whose “eigenvectors” are, in a certain sense, generalisations of sine and cosine functions.

We will outline the derivation and significance of this eigenvalue problem, illustrating how studying its structure allows one to answer questions like why a blind person can hear the dimensions of a room, why most musical instruments are essentially one-dimensional, and what properties of a drum one can “hear”