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Total point: 40

Exercise Sheet 1 – Analysis II

(Homework solutions will be handed in and discussed at 14:00-16:00, 25.10.2018, N24 - H15)

*(Please submit only the exercises **1a, 2a-b, 3a-b, 4, 5a-b**).*

1. Prove the following identities:
   (a) $\cos(\arcsin x) = \sin(\arccos x) = \sqrt{1-x^2}$ für alle $x \in \mathbb{R},\ |x| \leq 1,$ [4]
   (b) $2\arctan x = \arcsin \frac{2x}{1+x^2}$ für alle $x \in \mathbb{R},\ |x| < 1.$
2. Determine the antiderivatives of the following functions by integration by parts: [4+5]
   (a) $\arctan x$ für $|x| < 1,$ (b) $e^{-x}\cos(5x)$ (c) $\dfrac{x}{\sin^2 x}$ für $x \neq k\pi,\ k \in \mathbb{Z}.$
3. Determine the antiderivatives of the following functions by substitution:
   (a) $\dfrac{x \arcsin x}{\sqrt{1-x^2}}$ for $|x| < 1,$ (b) $x^2\sqrt{a^2-x^2}$ for $a \in \mathbb{R},\ a > 0,\ |x| < a.$ [5+5]
   (c) $\dfrac{1}{x^2}\sin\dfrac{1}{x}$ for $x > \dfrac{2}{\pi},$ (d) $\dfrac{1}{x\log x \cdot \log(\log x)}$ for $x > 1,\ x \neq e.$
4. Let $P(x) = \sum\limits_{k=0}^{n} a_k x^k$ be a polynomial of degree n and $a \in \mathbb{R} \setminus \{0\}$. [3+3+2]
   (a) Prove that there exists a polynomial $Q(x) = \sum\limits_{k=0}^{n} b_k x^k$ also of degree n such that
   $$\int P(x)e^{ax}dx = Q(x)e^{ax} \quad \text{for all } x \in \mathbb{R}.$$
   (b) Determine a recursion formula to calculate b_k from a_k.
   (c) Use the results above to calculate
   $$\int (13x^4 + 5x - 3)e^{2x}dx.$$
5. Determine the antiderivatives of the following functions:
   (a) $\dfrac{x^6+1}{x^4-x^2-2x+2}$ for $x \neq 1,$ (b) $\dfrac{x+\sqrt{1+x^2}}{x-\sqrt{1+x^2}},$ [5+4]
   (c) $\dfrac{1}{x\sqrt{1-x^2}}$ for $|x| < 1,\ x \neq 0,$ *Hint*: Integral (b) is of Type 1 (see Page 270)

BEMERKUNGEN:

1. The 1st exercise class takes place on 25.10.
2. Your solutions must be submitted individually before the exercise class (14:00-14:15 every Thursday).
3. If you have multiple sheets, you should staple them together.