also $\kappa(t_0)^2 \overset{\text{def.}}{=} |\tilde{k} \circ \varphi(t_0)|^2 = \cancel{|\tilde{k}(0)|^2}$

$$= |\ddot{\tilde{c}}(0)|^2 = |\ddot{c}(t_0)\cdot\dot{\varphi}(0)^2 + \dot{c}(t_0)\ddot{\varphi}(0)|^2$$

$$= \frac{|\ddot{c}(t_0)|^2}{|\dot{c}(t_0)|^4} - \cancel{2}\frac{\langle \ddot{c}(t_0), \dot{c}(t_0)\rangle^2}{|\dot{c}(t_0)|^6} + \cancel{\frac{|\dot{c}(t_0)|^2 \langle \ddot{c}(t_0), \dot{c}(t_0)\rangle^2}{|\dot{c}(t_0)|^8}}$$

$$= \frac{|\ddot{c}(t_0)|^2 |\dot{c}(t_0)|^2 - \langle \ddot{c}(t_0), \dot{c}(t_0)\rangle^2}{|\dot{c}(t_0)|^6} \overset{\text{Hinw.}}{=} \frac{|\dot{c}(t_0) \times \ddot{c}(t_0)|^2}{|\dot{c}(t_0)|^6}.$$

19.) a) $n(t_0) = \tilde{n} \circ \varphi(t_0) = \tilde{n}(0) = \frac{\ddot{\tilde{c}}(0)}{|\ddot{\tilde{c}}(0)|} = \frac{\ddot{\tilde{c}}(0)}{\tilde{\kappa}(0)}$

$$= \frac{1}{\tilde{\kappa}(0)}\left(\frac{\ddot{c}(t_0)}{|\dot{c}(t_0)|^2} - \frac{\langle \ddot{c}(t_0), \dot{c}(t_0)\rangle}{|\dot{c}(t_0)|^4}\dot{c}(t_0)\right)$$

$$\overset{18b.)}{=} \frac{|\dot{c}(t_0)|^{\cancel{3}}}{|\dot{c}(t_0) \times \ddot{c}(t_0)|}\left(\ddot{c}(t_0) - \frac{\langle \ddot{c}(t_0), \dot{c}(t_0)\rangle}{|\dot{c}(t_0)|^2}\dot{c}(t_0)\right)$$

$$b(t_0) = \tilde{b} \circ \varphi(0) = \tilde{b}(0) = \dot{\tilde{c}}(0) \times \tilde{n}(0) = \frac{\dot{c}(t_0)}{|\dot{c}(t_0)|} \times n(t_0)$$

Nun ist $\tau(t_0) = \tilde{\tau}(\varphi(t_0)) = \tilde{\tau}(0) = \langle \dot{\tilde{n}}(0) \cancel{x}, \tilde{b}(0) \rangle$

Wir haben

$$\dot{\tilde{n}}(s) = \frac{d}{ds}\left(\frac{\ddot{\tilde{c}}(s)}{|\ddot{\tilde{c}}(s)|}\right)_{s=0} = \frac{\dddot{\tilde{c}}(0)}{|\ddot{\tilde{c}}(0)|} - \frac{\langle \ddot{\tilde{c}}(0), \dddot{\tilde{c}}(0)\rangle}{|\ddot{\tilde{c}}(0)|^3}\ddot{\tilde{c}}(0)$$

$$= \frac{1}{\kappa(t_0)}\left(\dddot{\tilde{c}}(0) - \langle n(t_0), \dddot{\tilde{c}}(0)\rangle n(t_0)\right)$$

Und $\dddot{\tilde{c}}(s) = \dddot{c}(\varphi(s))(\dot{\varphi}(s))^3 + 3\,\ddot{c}(\varphi(s))\,\dot{\varphi}(s)\ddot{\varphi}(s)$

$$+ \dot{c}(\varphi(s))\,\dddot{\varphi}(s)$$

Also: $\tau(t_0) = \langle \dot{\tilde{n}}(0), \frac{\dot{c}(t_0)}{|\dot{c}(t_0)|} \times n(t_0)\rangle$

Hinw.

$$= \det\left(\dot{\tilde{n}}(0), \frac{\dot{c}(t_0)}{|\dot{c}(t_0)|}, n(t_0)\right)$$

$$= (-1)^2 \det\left(\frac{\dot{c}(t_0)}{|\dot{c}(t_0)|}, n(t_0), \underbrace{\dot{\tilde{n}}(t_0)}_{=\alpha\dddot{\tilde{c}}(0) + \text{l.a.}(n(t_0))}\right)$$

$$= \frac{1}{|\dot{c}(t_0)|\,\kappa(t_0)} \det\left(\dot{c}(t_0), \underbrace{n(t_0)}_{=\alpha\ddot{c}(t_0) + \text{l.a.}(\dot{c}(t_0))}, \dddot{\tilde{c}}(0)\right)$$

$$= \frac{1}{|\dot{c}(t_0)|\,\kappa(t_0)} \det\left(\dot{c}(t_0), \ddot{c}(t_0), \underbrace{\dddot{\tilde{c}}(0)}_{=\alpha\dddot{c}(t_0) + \text{l.a.}(\ddot{c}(t_0), \dot{c}(t_0))}\right) \cdot \frac{|\dot{c}(t_0)|}{|\dot{c}(t_0) \times \ddot{c}(t_0)|}$$

$$= \frac{\cancel{|\dot{c}(t_0)|^3}}{|\dot{c}(t_0) \times \ddot{c}(t_0)|^2} \det\left(\dot{c}(t_0), \ddot{c}(t_0), \dddot{c}(t_0)\right) \cdot \frac{1}{\cancel{|\dot{c}(t_0)|^3}} . \quad \square$$

2 1.) "⇐" Sei $a \in \mathbb{R}^3$ sodass $|c(t)-a|^2 \equiv$ const.

Frenet: $\frac{d}{dt}\begin{pmatrix} \dot c \\ n \\ b \end{pmatrix} = \begin{pmatrix} 0 & -\varkappa & 0 \\ \varkappa & 0 & -\tau \\ 0 & \tau & 0 \end{pmatrix}\begin{pmatrix} \dot c \\ n \\ b \end{pmatrix}$

$$\overset{d/dt}{\Rightarrow} \langle c(t)-a, \dot c(t)\rangle = 0 \quad \forall t$$

$$\overset{d/dt}{\Rightarrow} |\dot c(t)|^2 + \langle c(t)-a, \underbrace{\ddot c(t)}_{= \varkappa \cdot n \text{ (Serret Frenet)}}\rangle = 0$$

$$\Rightarrow -1 = \varkappa(t)\langle c(t)-a, n(t)\rangle$$

$$\Rightarrow \langle c(t)-a, n(t)\rangle = -\frac{1}{\varkappa(t)}$$

$$\overset{d/dt}{\Rightarrow} 0 = \dot\varkappa(t)\langle c(t)-a, n(t)\rangle + \varkappa(t)\underbrace{\cancel{\langle \dot c(t), n(t)\rangle}}_{=0}$$

$$+ \varkappa(t)\langle c(t)-a, \dot n(t)\rangle$$

$$\text{Frenet} \Rightarrow \dot n(t) = -\varkappa(t)\dot c(t) + \tau(t) b(t)$$

$$\text{Also } 0 = \dot\varkappa \underbrace{\langle c(t)-a, n(t)\rangle}_{= -\frac{1}{\varkappa(t)}} + \varkappa(t)\langle \underbrace{c(t)-a, -\varkappa(t)\dot c(t)}_{\text{orthog.}} + \tau(t) b(t)\rangle$$

$$\Rightarrow \varkappa(t)\tau(t)\langle c(t)-a, b(t)\rangle = +\frac{\dot\varkappa(t)}{\varkappa(t)}$$

$$\Rightarrow \langle c(t)-a, b(t)\rangle = \frac{\dot\varkappa(t)}{\varkappa(t)^2}\cdot\frac{1}{\tau(t)}$$

$$\overset{\text{3-Bein}}{\Rightarrow} c(t)-a = -\frac{1}{\varkappa(t)}n(t) + \frac{\dot\varkappa(t)}{\varkappa(t)^2}\cdot\frac{1}{\tau(t)}b(t)$$

$$\Rightarrow \text{const} = |c(t)-a|^2 = \frac{1}{\varkappa(t)^2} + \left(-\frac{\dot\varkappa(t)}{\varkappa(t)^2}\right)^2\frac{1}{\tau(t)^2}$$

$$= R^2 + \dot R^2 T^2.$$

„$\Leftarrow$" Für die Rückrichtung sei $R^2 + (\dot R)^2 T^2 = \text{const}.$

Definiere $\beta(t) := c(t) + R(t)\, n(t) + \dot R(t)\, T(t)\, b(t)$

$$\Rightarrow \dot\beta(t) = \dot c(t) + \dot R(t)\, n(t) + R(t) \underbrace{\dot n(t)}_{= -\kappa \dot c + \tau b} + \big(\dot R(t)\, T(t)\big)^{\cdot}\, b(t) + \dot R(t)\, T(t) \underbrace{\dot b(t)}_{= -\tau n}$$

$$= \cancel{\dot c} + \cancel{\dot R n} \mp \cancel{R \kappa \dot c} + R\tau b + (\dot R T)^{\cdot}\, b \cancel{\pm \dot R T \tau n}$$

$\cancel{= \dot c(t) + R}$

Andererseits $R^2 + (\dot R)^2 T^2 = \text{const} \Rightarrow R \dot R + (\dot R T)^{\cdot} \underbrace{(\dot R T)}_{\neq 0 \text{ n.V.}} = 0$

$$\Rightarrow (\dot R T)^{\cdot} = -\frac{R \dot R}{\dot R T} = -\frac{R}{T} = -R\tau$$

$$\Rightarrow \dot\beta(t) = 0 \Rightarrow \beta = \beta_0 \Rightarrow c(t) + Rn + \dot R T b = \beta_0$$

$$\Rightarrow |c(t) - \beta_0|^2 = R^2 + (\dot R T)^2 = \text{const}. \qquad \square$$