TU Wien
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WS 2016

2. Exercise sheet for the course "Modeling with PDEs"
(Traffic flow/Riemann-Problem)

Exercise 1. Consider the weak solutions to the Burgers equation from Examples 1.1 and 1.2 of the lecture notes with initial values

(i) Example 1.1

$$u_0(x) = \begin{cases} 1 & \text{for } x < 0\,, \\ 1 - x & \text{for } 0 \le x < 1\,, \\ 0 & \text{for } x \ge 1\,. \end{cases}$$

(ii) Example 1.2

$$u_0(x) = \begin{cases} 0 & \text{for } x < 0\,, \\ -1 & \text{for } x \ge 0\,. \end{cases}$$

From these weak solutions, for $0 \le t \le 1$, compute the vehicles' trajectories for the LWR model in the scaled variables (x_s, t_s), and draw such trajectories in the x-t plane (also the "virtual" trajectories with $\rho = 0$), under the simplifying assumption that $v_{max} = 1$ and $\rho_{max} = 1$. Compute $v(\rho)$ through the relation $v = \dot{x} = \frac{dx}{dt}$.

Exercise 2. Consider the Burgers equation. Show that, for the Riemann problem with $u_l > u_r$

(i) there is no piecewise-constant weak solution with two shock curves;

(ii) there exist piecewise constant weak solutions with three shock curves. Find the general expression of these weak solutions.

Exercise 3. Show that the function

$$u(x,t) := \begin{cases} u_l & \text{for } x < f'(u_l)\,t\,, \\ (f')^{-1}(\frac{x}{t}) & \text{for } f'(u_l)\,t \le x \le f'(u_r)\,t\,, \\ u_r & \text{for } x > f'(u_r)\,t\,, \end{cases}$$

is a weak solution of the Riemann problem for (1.7), (1.10) with initial condition $u_l < u_r$.

The exercises will be reviewed in class on Wednesday, November 9th, 2016.