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Introduction

Tosio Kato was a giant in a broad area within mathematical analysis and mathematical physics. In

"Tosio Kato (1917–1999)",
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H. Cordes, A. Jensen, S. T. Kuroda, G. Ponce, B. Simon, and M. Taylor wrote essays about Kato and gave insight into the incredible scope and ingenuity of his results. The comments below may be considered as a supplement to those essays. Here we focus on Kato's contributions to nonautonomous evolution equations, a field which he basically created and led for almost half a century.

In the early 1950's, the theory of one–parameter semigroups of linear operators on Banach spaces was in full bloom. It was understood that the initial value problem

$$\begin{aligned} \frac{d}{dt}u(t) &= Au(t), \\ u(0) &= f, \end{aligned}$$

is wellposed for $0 \leq t < \infty$ if and only if A is the generator of a strongly continuous semigroup $T = (T(t))_{t\geq 0}$ (in which case the solution is $u(t) = T(t)f$). The extension of the theory to nonautonomous problems

$$\begin{aligned} &\frac{d}{dt}u(t) = A(t)u(t), \qquad t \geq r, \\ &u(r) = f, \end{aligned} \tag{1}$$

was a natural step which greatly increased the range of applications. The first definitive result was given in Kato's 1953 paper [160]. He used the (ordered) product integral approach. Given $t > r$, consider a partition π, $r = t_0 < t_1 < \cdots < t_n = t$ with intermediate points $t_i' \in [t_{i-1}, t_i]$ and let

$$u_\pi(t) = T(t_n - t_{n-1}; A(t_n')) \cdots T(t_1 - t_0; A(t_1'))f,$$

where $T(s; A(q)) = \exp\{sA(q)\}$ is the semigroup at time s generated by $A(q)$. The solution u is obtained as $u(t) = \lim u_\pi(t)$ as the mesh of π $(= \max_i(t_i - t_{i-1}))$ tends to zero. Kato's result was for the case when each $A(t)$ generates a contraction semigroup and the domain $D = D(A(t))$ is independent of t.

This has proven to be a fundamental result and appears in the current textbook literature. Following this result were several major developments, and Kato played a key role in all of them. He used similarity transformations to extend this result in his 1956 paper [155] to allow $D(A(t))$ to vary widely. He applied this to general second order parabolic equations in one space variable with general boundary conditions. There are now two classes of definite results for parabolic problems where each $A(s)$ generates an analytic semigroup. One goes back to the papers by T. Kato and H. Tanabe in 1962 [123] and 1967 [103], the other is due to P. Acquistapace and B. Terreni [Rend. Padova, 1987].

The hyperbolic case was harder. Kato published the definitive abstract results in 1970 [94] and 1973 [82]. The main applications to general linear symmetric hyperbolic systems with time dependent coefficients and general time dependent boundary conditions were obtained by Kato and by his student F. Massey (together with J. Rauch). Incidentally, in this case as in many others, Kato was very generous in sharing his ideas, and he gave wonderful problems to his students.

The quasilinear problem

$$\begin{aligned} \frac{d}{dt}us &= B(u)u, \\ u(0) &= f, \end{aligned}$$

deals with the case when for each v in the underlying Banach space, $B(v)$ is a generator (and is linear). In order to make an approach by successive approximations work, one must solve the nonautonomous linear problem

$$\begin{aligned} \frac{d}{dt}u_n(t) &= A_n(t)u_n(t), \\ u_n(0) &= f_n, \end{aligned}$$

where $A_n(t) = B(u_{n-1}(t))$. This gives local (in time) wellposedness for an amazingly wide variety of problems, including Navier-Stokes, Korteweg-deVries, systems of hyperbolic conservation laws, etc. The papers that started this were his 1975 articles [72], [74]. He continued to make conceptual and technical breakthroughs in this field. See, for instance, [10] and his 1993 paper [30] (and its review in Math. Rev.[95m:34108]).

Of course, Kato was a leader in autonomous problems as well, and there is not enough room here to give details. Suffice it to mention briefly a few items. Abstract scattering theory deals with the asymptotics of a pair of unitary groups. Kato (with the help of his students S. T. Kuroda and T. Ikebe) was a (or more accurately "the") principal developer of this theory. In particular, the Kato-Kuroda theorem of 1970 [98] finally settled the old problem of asymptotic completeness in the quantum mechanical two body problem. Kato's work with his student H. Fujita on the Navier-Stokes system (1962 [122], 1964 [116]) was a major improvement of the ancient result of J. Leray and it inspired a great deal of work in theoretical fluid dynamics. Kato's work (partly done jointly with G. Ponce) on improved regularity implicit in the KdV equation and other dispersive problems continues to guide much modern research. And Kato greatly clarified and extended Y. Komura's seminal paper on nonlinear semigroups in 1967 [108] leading to much important work by Brezis, Crandall, Dorroh, Liggett, Pazy and others.

Kato's ingenious proof of essential self-adjointness of Schroedinger operators based on what is now call Kato's inequality ([83], see also [24]) is a pearl of mathematical elegance. It also greatly stimulated the systematic investigation of positive semigroups.

As in so many fields, Kato did pioneering work on form methods for evolution equations in Hilbert spaces, documented in the monograph "Perturbation Theory of Linear Operators" ([111]) which has become a classic of modern mathematics while continuing to be a standard reference for current research. The solution of Kato's famous square root problem has only been achieved in recent months, after several decades of research. One important case is proved in the first article of the present issue of JEE.

Kato was a quiet but friendly and serious scholar. The above only gives a hint of the great depth and breadth of his work and his technical genius. It was always a treat to meet him and to learn of his scientific thoughts, which he shared so generously. The evolution equations community, and the mathematics community in general, mourns the loss of one of our greatest teachers and scholars.

This issue of the Journal of Evolution Equations is dedicated to his memory.

Wolfgang Arendt
Jerome A. Goldstein
Rainer Nagel

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