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Applied analysis

Exercise 1. Let (X, d) be a metric space, $(x_n)_{n\in\mathbb{N}}$ be a sequence in X and $x_0 \in X$. Define a new metric $d_1(x, y) := \min\{1, d(x, y)\}$ for $x, y \in X$. Show that (X, d_1) is a metric space and that $x_n \to x_0$ in (X, d) if and only if $x_n \to x_0$ in (X, d_1).

Exercise 2.

a: Let q be an arbitrary rational number. What are the accumulation points of the sequence $(sin(qn\pi))_{n\in\mathbb{N}}$? (**) What would be the analogous result if q were irrational?

b: What is the boundary of the set $S := \{1/n,\ n \in \mathbb{N}\}$?

Exercise 3. Let (M, d) be a metric space.

a: Show that $B(x_0, r) := \{y \in M;\ d(x_0, y) < r\}$ is an open set.

b: Let $A_\alpha \subset M$ be closed for any $\alpha \in I$, where I is an arbitrary index set. Show that $\bigcap_{\alpha\in I} A_\alpha$ is closed.

c: Find a metric space (X, d) and a sequence of closed sets in X such that their union is **not** closed.

d: Is the union of the points in the sequence from (2a) closed? Is the set S in (2b) closed? Is it open?

e: Is the set $\{n;\ n \in \mathbb{N}\}$ closed in $\mathbb{R}$?

Exercise 4.

a: Find a metric d for the set S in (2b) so that S equipped with this metric become a **complete** metric space!

b: Is the new space (S, d) compact?

c: Denote by Q_1 the set of all rational numbers within the closed interval $[0, 1]$. Is Q_1 closed in $\mathbb{R}$? Is it open? Is it complete? Is it compact? Determine the closure, the interior and the boundary of Q_1.