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Applied analysis

Exercise sheet 10

Exercise 37. Use Lebesgue's dominated convergence theorem to compute the following limits:

$$(a)\quad \lim_{n\to\infty}\int_0^1 \frac{\sqrt{n^3}x}{1+n^2x^2}\,dx$$

$$(b)\quad \lim_{n\to\infty}\int_0^1 \frac{nx}{1+n^2x^2}\,dx$$

$$(c)\quad \lim_{n\to\infty}\int_0^1 nxe^{-nx^2}\,dx$$

$$(d)\quad \lim_{n\to\infty}\int_0^\infty nxe^{-nx^2}\,dx$$

$$(e)\quad \lim_{n\to\infty}\int_0^1 nx^{-nx^2}\,dx$$

$$(f)\quad \lim_{n\to\infty}\int_0^\infty \frac{\log(n+x)}{n}e^{-x}\,dx$$

$$(g)\quad \lim_{n\to\infty}\int_0^\infty e^{-xy}\cos x\,dx$$

$$(h)\quad \lim_{n\to\infty}\int_0^1 \frac{e^{-x^2(t^2+1)}}{t^2+1}\,dt$$

(each 1 point)

Exercise 38. Show that the following functions are continuous (as functions of a) on the given interval.

$$(a)\quad \int_0^\infty \frac{e^{-ax}}{1+x^2}\,dx \text{ on } [0,\infty)$$

$$(b)\quad \int_0^\infty e^{-ax}\,dx \text{ on } (0,\infty)$$

$$(c)\quad \int_{-1}^1 \sqrt{a^2+x^2}\,dx \text{ on } \mathbb{R}$$

$$(d)\quad \int_0^1 \frac{1}{\sqrt{a^2+x^2}}\,dx \text{ on } (0,\infty)$$

$$(e)\quad \int_0^2 x^2\cos ax\,dx \text{ on } \mathbb{R}$$

$$(f)\quad \int_{\frac{1}{2}}^\infty \frac{\cos x}{x^a}\,dx \text{ on } (1,\infty)$$

(each 2 points)

Exercise 35. Check that the substitution $\sin x = y$ is not allowed on the whole interval of integration for

$$\int_0^{5\pi} \sin^2 x \cos^2 x \, dx.$$

Compute the integral!

(2+4 points)

Exercise 36. Prove the following statement: If f is Laplace transformable, then the Laplace transform $\mathcal{L}(f)$ is infinitely many times differentiable on $(abs(f), \infty)$ and

$$(\mathcal{L}f)^{(n)}(x) = \mathcal{L}[(-t)^n f(t)](x) \quad \forall x > abs(f).$$

(3 points)