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Applied analysis

Exercise sheet 12

Exercise 40.
For $y \in l^1$ we let $\varphi_y : \mathbf{c_0} \to \mathbb{K}$ be given by

$$\varphi_y(x) := \sum_{n=1}^{\infty} x_n y_n.$$

a: Show that $\varphi_y \in \mathbf{c_0}'$ and $\|\varphi_y\|_{\mathbf{c_0}'} = \|y\|_{l^1}$.

b: Show that $y \mapsto \varphi_y : l^1 \to \mathbf{c_0}'$ is an isometric isomorphism.

(2/3 points)

Exercise 41.
For $y \in l^\infty$ we let $\varphi_y : l^1 \to \mathbb{K}$ be given by

$$\varphi_y(x) := \sum_{n=1}^{\infty} x_n y_n.$$

Show that $\varphi_y \in (l^1)'$ for all $y \in l^\infty$ and that $y \mapsto \varphi_y : l^\infty \to (l^1)'$ is an isometric isomorphism.

(2/3 points)

Exercise 42.
Let $a < b$ be real numbers, $E := C([a,b])$ and let $k \in E$. Verify that $\varphi_k : C([a,b]) \to \mathbb{K}$ given by

$$\varphi_k(f) := \int_a^b f(t)k(t)\,dt$$

is in E' and that $\|\varphi_k\|_{E'} = \|k\|_{L^1([a,b])}$.

(5 points)

Exercise 43.
Let $a, b \in \mathbb{R}$ with $a < b$, $E := C([a,b])$ and let $k \in C([a,b] \times [a,b])$. Then we define $T_k : C([a,b]) \to C([a,b])$ by

$$(T_k f)(x) := \int_a^b k(x,y)f(y)\,dy.$$

Prove that T_k is well-defined, $T_k \in \mathcal{L}(E)$ and that

$$\|T_k\|_{\mathcal{L}(E)} \leq \sup_{x \in [a,b]} \int_a^b |k(x,y)|\,dy.$$

(5 points)