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Applied analysis

Exercise sheet 4

Exercise 13.

Let $X = C[0,1]$. Recall that X is a Banach space for the norm

$$\|u\|_\infty = \sup_{t\in[0,1]} |u(t)|.$$

(1) Let $Y = \{u \in X : u(1) = 1\}$. Show that Y is a closed subset of X. Deduce that Y is complete.

(2) Let $T : Y \to Y$ be given by

$$(Tu)(t) = tu(t).$$

Show that for $u, v \in Y$, $u \neq v$,

$$\|Tu - Tv\|_\infty < \|u - v\|_\infty.$$

(3) Show that T does not have a fixed point.

Exercise 14.

Find a mapping $T : [0,\infty) \to [0,\infty)$ such that $|Tx - Ty| < |x - y|$ for all $x, y \in [0,\infty)$, $x \neq y$ and T does not have a fixed point.

Exercise 15.

a: Let X be a metric space and $K \subset X$ be a compact set. Show that K is closed and bounded.

b: Let $p \in [1,\infty]$ and let $(x_n)_n$ with $x_n = (x_n^k)_k$ be a convergent sequence in l^p with limit $x_0 = (x_0^k)_k \in l^p$. Show that for fixed $k \in \mathbb{N}$ the sequence $(x_n^k)_n$ converges in $\mathbb{K}$ to x_0^k.

c: Let X be a metric space and let L be a subset of $C(X)$. Show that if L is bounded then L is pointwise bounded.

Exercise 16.

Show that c_{00} is not dense in l^∞.