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Applied analysis

Exercise sheet 5

Exercise 17. Let $[a, b]$ be a compact (i.e. bounded and closed) interval in $\mathbb{R}$. Show that the unit ball of $C[a, b]$ is not compact.

Exercise 18. Let X be a metric space. Show that $R \subset X$ is relatively compact iff every sequence in R has a convergent subsequence in X.

Exercise 19. Let μ be an outer measure on Ω. Show that the following hold true:

a: Every set $A \subset \Omega$ with $\mu(A) = 0$ is μ-measurable.

b: If $A \subset \Omega$ is μ-measurable then A^c is also μ-measurable.

c: If $A \subset \Omega$ is μ-measurable and $B \subset \Omega$ then A is μ_B-measurable.

Exercise 20. Let Ω be a set and μ an outer measure on Ω. If $A_1, A_2 \subset \Omega$ are μ-measurable then the intersection $A_1 \cap A_2$ is also μ-measurable.