Dr. Markus Biegert **Wintersemester 2008/2009**
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Applied analysis

Exercise sheet 6

Exercise 21. Denote by $\overline{\mathbb{R}}$ the set $\mathbb{R} \cup \{-\infty, \infty\}$. Consider the mapping $g : [-1, 1] \to \overline{\mathbb{R}} = [-\infty, \infty]$ defined by

$$g(x) := \frac{x}{(1-x)(1+x)}.$$

Check that $d(y, z) := |g^{-1}(y) - g^{-1}(z)|$ for $y, z \in [-\infty, \infty]$ is a well-defined metric on $[-\infty, \infty]$. Characterize open sets in this metric (these are open intervals plus half-closed intervals with + or - ∞ at the closed end - prove it!). What are convergent sequences in the metric? Show that the Borel σ-algebra on $\overline{R}$ is generated by the sets of the form $[a, \infty]$, where $a \in \overline{\mathbb{R}}$. (8 points)

Exercise 22. Let Ω be a set and let $(\mathcal{A}_\alpha)_{\alpha \in I}$ be a family of σ-algebras on Ω, $I \neq \emptyset$. Show that $\mathcal{A} := \bigcap_{\alpha \in I}$ is a σ-algebra on Ω. (2 points)

Exercise 23. Show that every left-open cell Q is Lebesgue measurable. (5 pts)

Exercise 24.

a: Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be linear. Show that $T : \mathbb{R}^n \to \mathbb{R}^n$ is continuous. (3 pts)

b: Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be linear and invertible. Show that $T^{-1} : \mathbb{R}^n \to \mathbb{R}^n$ is linear. Deduce that T^{-1} is continuous. (2 points)