Dr. Markus Biegert **Wintersemester 2008/2009**
Dipl. Math. Michal Chovanec **Ulm, den 26.11.2008**

Applied analysis

Exercise sheet 7

Exercise 25. Let $\Omega \subset \mathbb{R}^n$ be a set, $\mathcal{B}(\Omega)$ the Borel σ-algebra on Ω and let $x_0 \in \Omega$. For $A \in \mathcal{B}(\Omega)$ we define

$$\delta_{x_0} := \begin{cases} 1 \text{ if } x_0 \in A \\ 0 \text{ else} \end{cases}$$

Verify that δ_{x_0} is a measure on $(\Omega, \mathcal{B}(\Omega))$.

(3 points)

Exercise 26.

a: Let $(\Omega, \mathcal{A}, \nu)$ be a measure space and let $B \in \mathcal{A}$. Show that $(\Omega, \mathcal{A}, \nu_B)$ is a measure space.

(2 point)

b: Let $\nu_1, \nu_2, ..., \nu_n$ be measures on a fixed measurable space $(\Omega, \mathcal{A})$ and let $\lambda_1, ..., \lambda_n \in \mathbb{R}^+$. Show that $\nu := \sum_{k=1}^n \lambda_k \nu_k$ is a measure on $(\Omega, \mathcal{A})$ where

$$\nu(A) := \nu := \sum_{k=1}^n \lambda_k \nu_k(A), \ A \in \mathcal{A}.$$

(3 points)

Exercise 27. Let $(\Omega, \mathcal{A})$ be a measurable space. Show that

a: Every constant function is measurable.

(3 points)

b: Let $A \subset \Omega$. Then A is $\mathcal{A}$-measurable iff χ_A is $\mathcal{A}$-measurable.

(3 points)

Exercise 28.

a): Let $A \subset \mathbb{R}^n$, $x \in \mathbb{R}^n$ and $\alpha > 0$. Show that A is Lebesgue-measurable iff $x + \alpha A$ is Lebesgue-measureable. (2 points)

b): Show that in order to prove that every left-open cell Q is Lebesgue-measurable it suffices to consider $Q = H$, where H is a (left-open) half-space. (4 points)