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Applied analysis

Exercise sheet 8

Exercise 29. (a) Let $(\Omega, \mathcal{A})$ be a measurable space and let $f : \Omega \to \overline{\mathbb{R}}$ be a $\mathcal{A}$-measurable function. Show that f^+ and f^- are $\mathcal{A}$-measurable.

(1 point)

(b) Let $(\Omega, \mathcal{A}, \mu)$ be a measure space and let $f : \Omega \to \overline{\mathbb{R}}^+$ be a nonnegative measurable function such that $\int |f|\, d\mu = 0$. Show that $f = 0$ a.e.

(2 points)

Exercise 30. Decide whether the function $f(x) := \frac{\sin x}{x}$ is Lebesgue integrable on the interval $[1, +\infty]$. You may use the fact that $\int_1^{+\infty} \frac{\cos x}{x}\, dx$ exists (as a Riemann integral).

Hint: $y^2 \leq y$ for $y \in [0, 1]$. Also $\cos(2x) = \cos^2 x - \sin^2 x$.

(4 points)

Exercise 31. Compute $\int_a^b f(x)\, dx$ where

i: $f(x) = \sin^n x \cos x, \quad a = 0,\ b = 5\pi$

ii: $f(x) = x^n e^{cx}, \quad a = 0,\ b = 1$

iii: $f(x) = 0$ if x is rational, $f(x) = 4$ if x is irrational, $\quad a = 0,\ b = c$

iv: $f(x) = x^c \quad a = 1,\ b = +\infty$

v: $f(x) = c^x \quad c > 0,\ a = 1,\ b = 2$

n is always a natural number, c is a real parameter. (2 points each)

Exercise 32. Let $\Omega = \mathbb{N}$, $\mathcal{A} = \mathcal{P}(\Omega)$ and let μ be the counting measure on Ω. Prove that f belongs to $L^1(\Omega, \mathcal{A}, \mu)$ if and only if the series $\sum_n f(n)$ is absolutely convergent. Verify that in this case

$$\int f\, d\mu = \sum_n f(n).$$

(3 points)