Universität Ulm
Discussion of solutions:
16.02.12

Dr. M. Kunze
M. Gerlach
Winter 2011/12

Exercises for Applied Analysis

Sheet 14

59. The exponential distribution with parameter $\lambda > 0$ is the measure exp_λ on $\mathscr{B}(\mathbb{R})$ given by

$$\mathsf{exp}_\lambda(A) := \int_A \mathbb{1}_{[0,\infty)}(t)\lambda e^{-\lambda t}\,\mathrm{d}t \quad (A \in \mathscr{B}(\mathbb{R}))$$

The Laplace distribution with parameter $\lambda > 0$ is the measure L_λ on $\mathscr{B}(\mathbb{R})$ given by

$$\mathsf{L}_\lambda(A) := \int_A \frac{1}{2\lambda} e^{-\frac{|t|}{\lambda}}\,\mathrm{d}t \quad (A \in \mathscr{B}(\mathbb{R})).$$

For $\lambda > 0$, compute the characteristic functions of exp_λ and L_λ and prove that $X - Y$ has distribution $\mathsf{L}_{1/\lambda}$ if X and Y are independent random variables with distribution $\exp_\lambda$.

60. Let X be a d-dimensional Gaussian random vector with covariance matrix $Q \in \mathbb{R}^{d\times d}$ and $T \in \mathbb{R}^{m\times d}$. Show that TX is a m-dimensional Gaussian vector with covariance matrix TQT^T where T^T denotes the transposed matrix of T.

Moreover, give an example of a 2-dimensional Gaussian random vector whose distribution is not absolutely continuous with respect to the 2-dimensional Lebesgue measure.

61. Let $(X_1, \ldots, X_4)$ be a Gaussian random vector with covariance matrix

$$Q := \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 4 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

and define $Z := X_1 - X_2 + 2X_4$, $Y_1 := X_1 - X_3$ and $Y_2 := X_2 + X_3$. Determine $\mathbb{E}(Z \mid Y_1, Y_2)$.

62. Let X be a standard Gaussian random variable and R be an independent random variable with

$$\mathbb{P}(R = -1) = \mathbb{P}(R = 1) = \frac{1}{2}.$$

Define $Y := RX$.

(a) Compute the characteristic function of the random vector (X, Y).

(b) Show that Y is Gaussian and that X and Y are uncorrelated but not independent.