Universität Ulm

Discussion of solutions:
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Exercises for Applied Analysis

Sheet 3

14. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x,y) := \begin{cases} \frac{xy^2}{x^2+y^6} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } x = y = 0 \end{cases},$$

where $\mathbb{R}^2$ and $\mathbb{R}$ are endowed with the euclidean metric.

(a) Prove that f is not continuous.

(b) Show that the restriction of f to any line $G_{\alpha\beta} = \{(x, y) \in \mathbb{R}^2 : \alpha x + \beta y = 0\}$ is continuous, where $\alpha^2 + \beta^2 \neq 0$ and $G_{\alpha\beta}$ is endowed with the induced metric.

15. Prove that for every set M and d the discrete metric on M, the metric space (M, d) is complete.

16. We consider the real line $\mathbb{R}$ endowed with the metric d_1, given by

$$d_1(x,y) := |\arctan(x) - \arctan(y)| \quad (x, y \in \mathbb{R}),$$

and with the euclidean metric, denoted by d_2.

(a) Prove that a set $U \subset \mathbb{R}$ is open in $(\mathbb{R}, d_1)$ if and only if it is open in $(\mathbb{R}, d_2)$.

(b) Show that $(\mathbb{R}, d_1)$ is not complete.

(c) Decide whether d_1 and d_2 are equivalent.

Hint: You may use that arctan and tan are continuous for the euclidean metric.

17. Show that (ℓ^∞, d_0) is not complete.