Universität Ulm

Abgabe: Mittwoch, 20.01.2016

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Wintersemester 2015/16
Punktzahl: 7+7*

Übungen Dynamische Systeme: Blatt 12

29. Consider the system

$$(*) \quad \begin{cases} \dot{x} &= \gamma x(1 - \frac{x}{K}) - \frac{mx}{a+x} y \\ \dot{y} &= (\frac{mx}{x+a} - d)y, \end{cases}$$

where $a, d, m, K, \gamma > 0$ are parameters. This is a so-called *predator-prey model of Holling type II*; x denotes the prey, y the predator.

(a) Proof that every solution (x, y) of $(*)$ with initial value $(x, y)(0) \in \mathbb{R}^2_+$ exists for all $t \geq 0$, remains in $\mathbb{R}^2_+$ for all $t \geq 0$ and is bounded on the time interval $[0, \infty)$. (2)

(b) Compute all equilibria of $(*)$ in $\mathbb{R}^2_+$ (1)
Hint: if $K(\frac{m}{d} - 1) > a$, then the interior of $\mathbb{R}^2_+$ contains exactly one equilibrium $(\hat{x}, \hat{y})$.

(c) Assume that $K(\frac{m}{d} - 1) > a$. Prove that the equilibrium $(\hat{x}, \hat{y})$ is asymptotically stable if $K - a < 2\hat{x}$ and repulsive if $K - a > 2\hat{x}$. (2)

Remark: By constructing an appropriate Ljapunov function one can even show that the equilibrium $(\hat{x}, \hat{y})$ is *globally* asymptotically stable if $K - a < 2\hat{x}$. We shall, however, not discuss this in detail, here.

(d) Assume that $K(\frac{m}{d} - 1) > a$. Prove that $(*)$ admits a non-trivial periodic solution if $K - a > 2\hat{x}$. (2*)

(e) Give a biological interpretation of the term $\frac{mx}{a+x} y$ which describes the interaction of x and y in the system. (2)

(f) Give an interpretation of the condition $K(\frac{m}{d} - 1) > a$ which we assumed in (c) and (d). Describe the long-term behaviour of all solutions in $\mathbb{R}^2_+$ if we assume $K(\frac{m}{d} - 1) < a$ instead. (5*)