

Chapter 5 Invariance and Positivity.

§ 27 Invariant convex sets.

T C_0 -sg on X , gen. A ,

$C \subset X$ closed, convex

C invariant $\Leftrightarrow T(t)C \subset C \quad \forall t \geq 0$

Equ.

(27.1) Proposition. (i) C invariant

(ii) $\exists \omega \in (0, \infty) \cap \rho(A)$ &
 $\lambda R(\lambda, A) \subset C \subset C \quad (\lambda > \omega).$

Rk (iii) $\exists r_0 > 0 \quad \{ \frac{1}{r} : 0 < r < r_0 \} \subset \rho(A)$
 & $(I - rA)^{-1}C \subset C \quad \forall 0 < r < r_0$

(ii) \Leftrightarrow (iii) clear.

(27.2) Lemma. $-\infty < a < b < \infty$

$u \in C([a, b]; X)$, $u(t) \in C \quad \forall t \in [a, b]$

$\varphi \geq 0$, $\int_a^b \varphi(t) dt = 1. \Rightarrow$

$\int_a^b \varphi(t) u(t) dt \in C$