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(revolution)

29 Invariance for forms.

H Hilbert space, $\mathbb{K} = \mathbb{C}$ or $\mathbb{R}$.

$a : D(a) \times D(a) \longrightarrow \mathbb{K}$ form

$\operatorname{Re} a(u) \geq 0$ $\forall u \in D(a)$ (accretive)

$V := (D(a), (\cdot\,|\,\cdot)_V)$

$(u\,|\,v)_V := \operatorname{Re} a(u,v) + (u\,|\,v)_H$.

Hypothesis: V complete

(i.e., a is a closed form).

Rem: $a : V \times V \longrightarrow \mathbb{K}$ is continuous.

Rem: 1. $\|u_n\|_V \leq c$ $u_n \in V$

$\Rightarrow$ $\exists$ ss $u_{n_k} \rightharpoonup u$ in V

i.e. $(u_{n_k}\,|\,v)_V \longrightarrow (u\,|\,v)_V$ $\forall v \in V$

2. $u_n \to u$ in H, $u_n \in V$

& $\|u_n\|_V \leq c$ $\Rightarrow$ ~~$u_n \longrightarrow u$~~,
$u \in V$ & $u_n \rightharpoonup u$ in V

Rem: $X \hookrightarrow Y$ Banach spaces

$x_n \rightharpoonup x$ in X $\Rightarrow$ $x_n \rightharpoonup x$ in Y

Pf. Let $y' \in Y'$ $\Rightarrow$ $y'|_X \in X'$ $\Rightarrow$

$\langle y', x_n \rangle \longrightarrow \langle y', x \rangle$. $\square$

Pf of 2. 1. $\exists w \in V$ $\exists$ ss $u_{n_k} \rightharpoonup w$

in V $\Rightarrow$ $u_{n_k} \rightharpoonup w$ in H $\Rightarrow$ $w = u$.

2. Suppose $u_n \not\rightharpoonup u$ in V.

$\Rightarrow$ $\exists v \in V$ $(u_n | v)_V \not\to (u|v)_V$

$\Rightarrow$ $\exists \varepsilon > 0$ $\exists$ ss $|(u_{n_k}|v)_V - (u|v)_V| \geq \varepsilon$

But $\exists$ ss $u_{n_{k_\ell}} \rightharpoonup u$ $\lightning$. $\square$

3. Thus $u_n \in V$, $u_n \longrightarrow u$ in H

$\operatorname{Re} a(u_n) \leq c$ $\Rightarrow$ $u \in V$ & $u_n \rightharpoonup u$ in V.

(29.1) <u>Lemma.</u> $u_n \in V$ $u_n \to u$ in H

$v_n \in V$, $\|v_n\|_V \leq C$

$$\mathrm{Re}\, a(u_n, u_n - v_n) \leq 0$$

$$\Rightarrow u \in V \quad \& \quad u_n \to u \text{ in } V.$$

<u>Pf.</u> $\mathrm{Re}\, a(u_n) \leq \mathrm{Re}\, a(u_n, v_n) \leq M \|u_n\|_V \|v_n\|_V$

$$\|u_n\|_V^2 = \|u_n\|_H^2 + \mathrm{Re}\, a(u_n)$$

$$\leq C' + MC \|u_n\|_V = \alpha + 2\beta \|u_n\|_V$$

$$\Rightarrow \sup_{n \in \mathbb{N}} \|u_n\|_V < \infty$$

$$(\|u_n\|_V - \beta)^2 = \|u_n\|_V^2 - 2\beta \|u_n\|_V + \beta^2$$

$$\leq \alpha + \beta^2$$

$$\Rightarrow \|u_n\|_V - \beta \leq \alpha + \beta^2$$

$\overset{3.}{\Rightarrow}$ claim. $\square$

Hyp: $\overline{D(a)} = H$

$a \sim A$; i.e.

$D(A) = \{u \in V : \exists f \in H \quad a(u,v) = (f|v)_H$

$\forall v \in V\}$

$Au := f$.

A is m-sectorial.

$\Rightarrow -A$ generates a C_0-sg T on H.

$C \subset H$ closed, convex, P the minimising projection.

(29.2) <u>Proposition.</u> C invariant $\Rightarrow$

$PD(a) \subset D(a)$.

~~Proof.~~ $\cancel{(I + tA)^{-1}}$

~~(29.3) Lemma. $J_t = (I + tA)^{-1}u \longrightarrow u$ in V~~

~~as $t \downarrow 0$.~~

~~Proof. $J_t u + tAJ_t u = u$~~ $\quad AJ_t u = \frac{u - J_t u}{t}$