__Recall.__ A C_0-sg T on X is __holomorphic__ if $\exists\, \theta \in (0, \pi/2]$, $M \geq 0$ s.t. T has a hol. extension

$\tilde{T} : \Sigma_\theta \to \mathcal{L}(X)$ such that

$$\|\tilde{T}(z)\| \leq M \quad \text{if} \quad z \in \Sigma_\theta,\ |z| \leq 1.$$

(20.5) __Theorem.__ Let A be a ~~n~~ dd operator on X. Equivalent.

(i) A generates a hol. C_0-sg.

(ii) $\exists\, \omega \in \mathbb{R}$, $M \geq 0$ such $\operatorname{Re}\lambda > \omega \Rightarrow \lambda \in \varrho(A)$ &

$$\|\lambda R(\lambda, A)\| \leq M.$$

__Remarkable__: No powers of the resolvent are needed.

Rk. (ii) $\Rightarrow$ (iii) $\exists\, \theta \in (\frac{\pi}{2}, \pi)$

$\Sigma_\theta \subset \rho(A)$ & $\| \lambda R(\lambda, A) \| \le M'$

$\forall \lambda \in \Sigma_\theta$.

exercise.