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Applied Analysis Tutorials

Sheet: 4

Submission: Tuesday, 10.11.2009 during the next tutorial class.

1. Let T be a map of the metric space (X, ρ) into itself such that for a fixed positive integer n,

$$\rho(T^n x, T^n y) \leq \alpha^n \rho(x, y) \ \forall x, y \in X,$$

here α is a positive real number. Show that the function σ defined by

$$\sigma(x, y) := \rho(x, y) + \frac{1}{\alpha}\rho(Tx, Ty) + \frac{1}{\alpha^2}\rho(T^2 x, T^2 y) ... + \frac{1}{\alpha^{n-1}}\rho(T^{n-1}x, T^{n-1}y)$$

is a metric on X and T satisfies

$$\sigma(Tx, Ty) \leq \alpha\sigma(x, y) \ \forall x, y \in X.$$

2. Find a mapping $T : [0, \infty) \to [0, \infty)$ such that $|Tx - Ty| < |x - y|$ for all $x, y \in [0, \infty), x \neq y$ and T does not have a fixed point.

3. (a) Let X be a metric space and $K \subset X$ be a compact set. Show that K is closed and bounded.

   (b) Let g be a continuous mapping from a metric space (X, d) to itself. Suppose that $x_0 \in X$. If the sequence (x_n) defined recursively by $g(x_n) = x_{n+1}$ for every $n \in \mathbb{N}$ is convergent, then the limit of this sequence is a fixed point of g.

4. Show that $\mathbf{c}_{00}$ is not dense in l^∞.